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Triggering of whistler mode emissions by the band-limited impulse (BLI) associated with amplified Vlf signals from Siple Station, Antarctica

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Abstract.
At termination of an amplified and saturated (B_{sat}) Vlf pulse from Siple Station, Antarctica,a band-limited (50-500 Hz range) impulse (BLI) is generated which in turn triggers a narrowband emission, of about the same amplitude, that starts at about 50-150 Hz above the frequency of B_{sat}, called the positive frequency offset (PFO). The BLI creates the PFO by reducing the parallel velocities of existing phase-bunched cyclotron resonant electrons that form the source currents ($J_{\perp}$) of the stimulated radiation. The PFO is proportional to the magnetic field intensity B_{BLI} of the BLI. Since $B_{BLI} \approx B_{sat}$ the PFO then becomes a measure of the saturated field B_{sat}. An example for L = 4 and a pitch angle of 60°, gives $B_{sat} \cong 5$pT, in good agreement with previous estimates of B_{sat}.

Introduction

When coherent amplified VLF pulses transmitted from Siple Station, Antarctica [*Helliwell*, 1988] are terminated, they often trigger a band-limited impulse (BLI) that in turn triggers an emission with a positive frequency offset (PFO), as sketched in Figure 1. We propose an explanation for this combination of events, called the BLI/PFO, based on Doppler-shifted cyclotron resonance between coherent VLF signals and counter-streaming energetic electrons [*Helliwell*, 1967]. The BLI/PFO is a key factor in a range of largely unexplained triggered emission phenomena. Often seen in association with the BLI [*Helliwell*, 1979] at the end of an amplified Siple pulse, the PFO is a rapid (< 10 ms) step up in frequency (~ 30 - 150 Hz) that separates a triggered emission from its triggering source, usually a coherent Siple pulse, in the 2-6 kHz range. Our proposed BLI/PFO model provides a new tool for measuring the *in situ* saturation intensity B_{sat} using ground-based measurement of a small frequency change (PFO).

A sketch of an idealized dynamic spectrum of the BLI/PFO phenomenon as seen on the ground at the conjugate point (Lake Mistissini, Quebec) to the transmitter is shown in Figure 1a (the vertical width of the signal trace is proportional to the log of the amplitude of signal intensity). The spatial relations of the generation regions of the signals sketched in Figure 1a are shown in Figure 1b as a function of the distance z from the magnetic equator, based on our small-signal model of coherent wave growth [*Carlson et al.*, 1990] and our hypothesis of the origin of the BLI [*Helliwell*, 1979]. Unstable exponential growth of the input signal B_{in} takes place mainly in the second-order resonance (SOR) region through feedback between waves and electrons. Saturation (at B_{sat}) occurs when the gain G in the feedback loop falls to unity [*Helliwell*, 1967; *Helliwell & Inan*, 1982].

The sequence of events sketched in Figure 1 is as follows: B_{in} stimulates exponential growth in the interaction region (IR) for SOR. When B_{in} terminates (at 1 sec.) it excites, in the BLI generation region (BLI GR), the natural response of the previously phase-bunched electrons. In the SOR region, the perpendicular vector velocities are averaged to give an effective perpendicular velocity $\bar{V}_{\perp}$ corresponding to a transverse current density $J_{\perp}$ that is the source of the transverse stimulated wave magnetic field B_s. Since the phase at the onset of the natural response of each electron comprising the BLI is the same as at the end of the driven response (no instantaneous change of phase is permitted) the initial wave intensity B_{BLI} of the BLI is simply the sum of all the driven responses excited by B_{in}. However, as shown in the next section, the natural frequencies are determined by the local values of the electron parallel velocities and the medium parameters ($f_N \& f_H$). While these radiating electrons remain partially phase-bunched, their frequencies rise at different rates, depending primarily on $f_H(z)$, causing their wave trains to phase mix away, after a time that is approximately the reciprocal of the bandwidth. As a result, the BLI has detectable amplitude only at the termination of B_{in}. Its duration t_i, is the reciprocal of its bandwidth f_i. Because the BLI components are advanced in phase with respect to $\bar{V}_{\perp}$ by roughly 270° [*Paschal*, 1988], the frequencies of the effective transverse currents $\bar{J}_{\perp}$ in the SOR region are raised by the PFO as explained below. The triggered emission (TE) starts out at the same intensity as the triggering signal. But in the absence of B_{in} the location of the SOR region drifts upstream ($-z$ direction) because the signal gain G is now too low to

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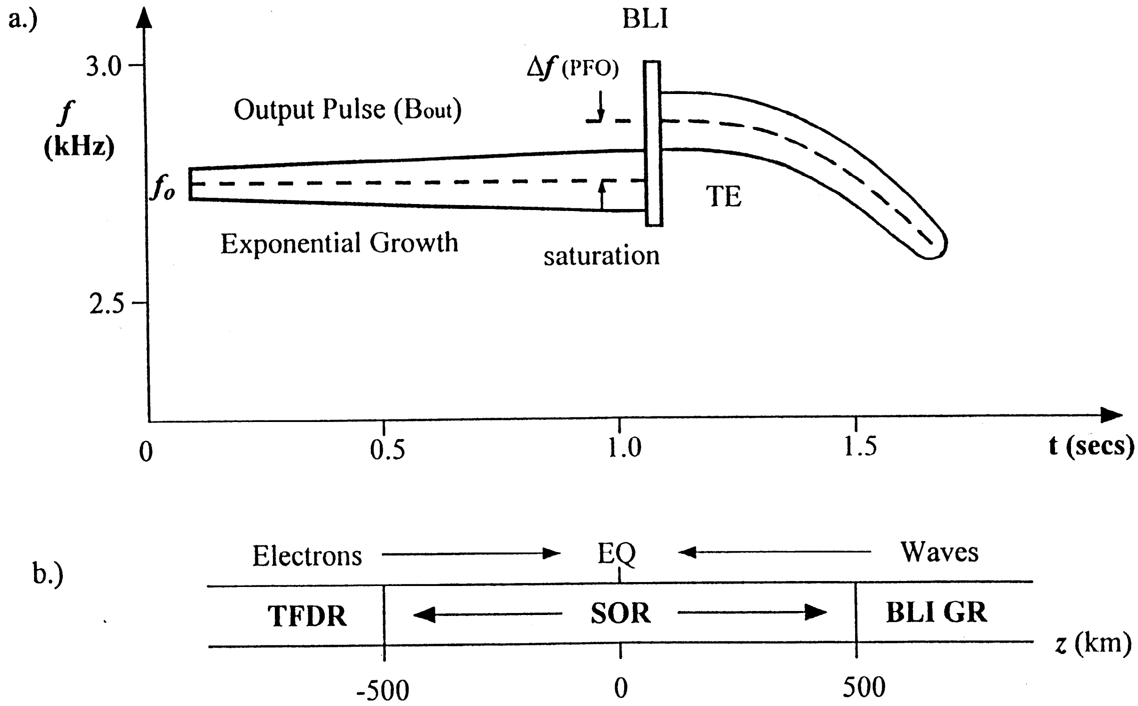

Figure 1. a.) Idealized dynamic spectrum of a one-second long amplified pulse and its associated emission (TE) which is triggered by the BLI and starts at the Δf (PFO) above the amplified frequency f_0, with an initial amplitude equal to the saturated intensity B_{sat} of the amplified one-second Siple pulse. The positive *BLI* peak intensity $B_{BLI} \approx B_{sat}$ of the amplified Siple pulse. b.) Illustration of field line near the magnetic equator (EQ) showing second order resonance (SOR) region, the generation region (GR) of the band-limited impulse (BLI), and the triggered faller drift region (TFDR). Coherent input signal B_{in} is amplified in the SOR region by a factor G of about 4, becoming B_{out} at the output. The BLI is postulated to be created from previously phase-bunched currents emanating from the BLI GR (see text).

keep the SOR generation region (IR) on the equator [*Helliwell*, 1967]. As the IR drifts further upstream, the natural frequency of the TE falls [*Helliwell*, 1967, 1970]. This causes the number density of the resonant electrons in a typical distribution function to fall as a result of the increase in the required velocity caused by the off-equatorial rise in the electron gyro frequency f_H. As a result, eventually G falls below unity, so that the triggered emission decays to zero.

A remarkable feature of the Siple pulse and its following emission is the continuity in signal intensity before and after the BLI/PFO. This means that the electrons comprising $J_\perp$ are effectively fully phase-bunched and their $v_\perp$'s are nearly all in phase in the radiating region [*Helliwell & Inan*, 1982].

It will be shown that at termination the pre-existing quasi-steady state phase-bunched currents ($J_\perp$) located in the second order resonance (SOR) region have their parallel velocities $V_\parallel$ reduced by the BLI, so as to cause a small positive frequency offset (PFO) in the frequency of the $J_\perp$ currents (which themselves have already been shifted slightly upwards ($\cong$ 1 - 4 Hz) during the growth process [Paschal, 1988]). These frequency-shifted transverse currents then continue as a self-excited emission (labeled "TE" in Figure 1) at the new (higher) frequency. Since all transverse currents see essentially the same applied signal (the BLI), the resulting ensemble, that has been up-shifted in frequency by the BLI constitutes a self-excited narrow-band triggered emission (TE) of constant frequency, much like the exciting pulse, but with no external input. As time progresses, the interaction region (IR) of the TE may slowly drift downstream, creating a riser (R), or upstream, creating a faller (F) [*Helliwell*, 1967, 1970].

In developing our proposed model of triggering we recognize its inherently nonlinear character where new frequencies are created that were not present in the input signal. We employ the results of the "small-signal" theory developed much earlier [*Brice*, 1963; *Helliwell*, 1967, 1970] and recently confirmed by a realistic inhomogeneous simulation [*Carlson et al.*, 1990]. Then we fit the experimental data on the BLI/PFO to our proposed impulse excitation model. We show that the BLI/PFO model predicts a saturation value of wave magnetic field intensity, B_{sat}, in the interaction region which compares favorably to several independent estimates of B_{sat} for ducted signals inside the plasmasphere [*Helliwell & Walworth*, 1996]. Another theory of wave growth [*Nunn*, 1974] (that we shall call the "large signal" theory) is based on the assumption that the resonant electrons are trapped by the initial wave's magnetic field, leading to much higher values of B_{sat}. Thus the proposed model of the BLI/PFO provides, for the first time, an independent test of these two models.

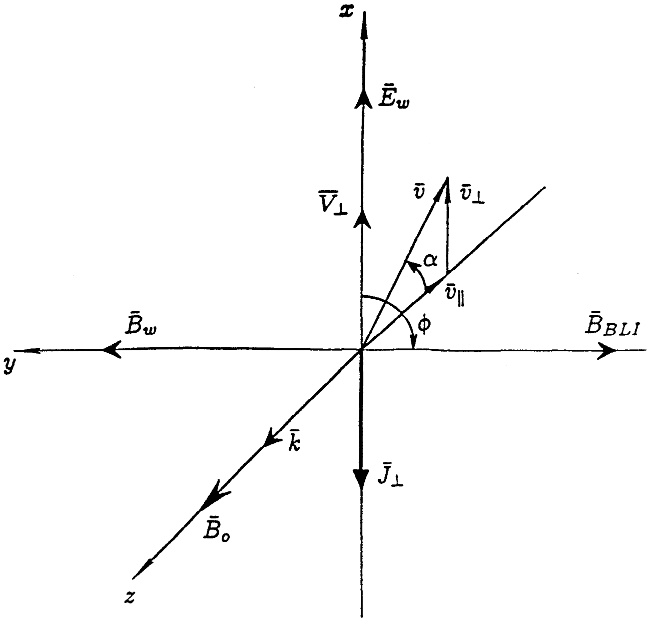

Figure 2. Right hand circularly polarized wave, $\bar{E}_w, \bar{B}_w$ propagating in the $-z$ direction with the wave vector $\bar{k}$. Interacting electrons spiraling around the $+z$ axis in the reverse direction with parallel velocity $v_\parallel$, perpendicular velocity $\bar{v}_\perp$ and pitch angle α, are phase-bunched by B_w, producing transverse currents $J_\perp$ that radiate the waves (E_w, B_w). The BLI, whose wave magnetic field B_{BLI} lags E_w by $\cong 90°$, acts on the average perpendicular velocity $\bar{V}_\perp$ (opposed to $\bar{J}_\perp$ because of minus sign in q) to produce the PFO (see text).

Analysis of the BLI/PFO model

The proposed mechanism of the PFO can be understood in terms of the vector diagram of Figure 2 showing the coordinate system, the particle velocity components $v_\perp \& v_\parallel$, the effective "phase-bunched" velocity $\bar{V}_\perp$, the pitch angle α, the components $E_w \& B_w$ of the right hand circularly polarized whistler mode wave and static magnetic field $\bar{B}_o$. Also shown is the phase and magnitude (not to scale) of the BLI in relation to $J_\perp(z)$ as estimated from the single stream model of [*Helliwell & Inan*, 1982]. The most important feature of the relative phasing of these quantities is the relation between $\bar{B}_{BLI}$ and the $\bar{V}_\perp$ of the macro electron representing the signal current at saturation. The magnitude of the BLI is measured from the spectral data, while its phase shift is taken from phase measurements of Siple triggered emissions [*Paschal*, 1988, Fig. 4.5, 3rd 200 ms pulse] which show a phase advance of about π radians in the phase of $\bar{B}_{BLI}$ relative to the phase of the amplified Siple pulse (B_{max}) at its end point. Note that because of the continous phase advance of $\bar{B}_{out}$, at the rate of 3.2 Hz [*Paschal*, 1988] in this case, the phase of B_{max} is $\sim 2\pi$ radians ahead of $\bar{B}_{in}$ at pulse termination.

As the BLI encounters the oppositely traveling electrons that constitute the $J_\perp(z)$ it lowers the parallel velocity $v_\parallel$ of each resonant electron (because of the negative value of the electron charge q in the $q\bar{v}_\perp \times \bar{B}_{BLI}$ product). This reduction in $v_\parallel$ is enough to produce the observed PFO, according to the condition for transverse resonance where the electron gyrofrequency is given by $f_H = f + (k/2\pi)v_\parallel$, and the wave number by $k = 2\pi f_N/c(f/(f_H - f))^{\frac{1}{2}}$; f = wave frequency, f_N = electron plasma frequency, and $v_\parallel$ = parallel velocity of the resonant electrons and c = velocity of light.

We calculate the PFO using the same equation of motion for the parallel velocity $v_\parallel$ of a resonant electron that has been employed previously [*Carlson et al.*, 1990]. In terms of the $\bar{V}_\perp$ of the phase-bunched ensemble this equation can be written

$$m\frac{d\bar{v}_\parallel}{dt} = q\bar{V}_\perp \times \bar{B}_{BLI} = qv_\parallel \tan\alpha \hat{r}_\perp \times \bar{B}_{BLI} \qquad (1)$$

where

m = mass of electron

$v_\parallel$ = resonant velocity of the interacting electrons = 11, 400 km/s, from Table 1, [*H & I*, '82], p. 3542, for L = 4.

$\bar{V}_\perp$ = perpendicular velocity of the single "macro" electron that represents $< \bar{v}_\perp > = v_\parallel \tan\alpha\hat{r}_\perp$; where $\hat{r}_\perp$ = average unit vector representing the direction of $\bar{V}_\perp$

α = pitch angle of resonant electrons

Rearranging (1) and replacing d by Δ, we have

$$\frac{\Delta v_\parallel}{v_\parallel} = \left(\frac{q}{m}\tan\alpha\right)\left|\hat{r}_\perp \times \bar{B}_{BLI}\right|\Delta t \qquad (2)$$

A second equation in $\frac{\Delta v_\parallel}{v_\parallel}$ can be derived from the expressions for f_H and k, above, assuming small Δf; the result is easily shown to be

$$\frac{\Delta v_\parallel}{v_\parallel} \approx -\frac{1}{2}\frac{\Delta f}{f_o}\left(\frac{f_H/f_o + 2}{f_H/f_o - 1}\right) \qquad (3)$$

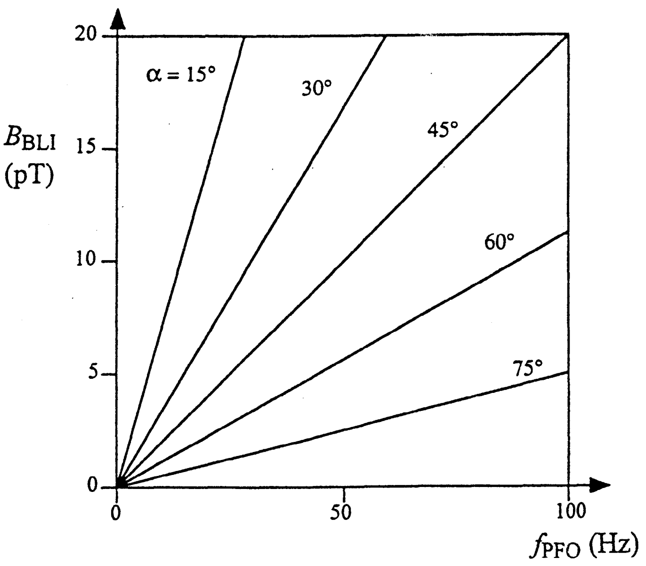

Figure 3. Amplitude B_{BLI} of the band-limited impulse (BLI) versus the positive frequency offset (PFO) caused by the BLI, parametric in the pitch angle α of the cyclotron resonant electrons.

where

f_o = output frequency of $\bar{B}_s$ (1 - 4 Hz above f_{in})

f_{in} = input frequency of B_{in}

Now we can replace $\frac{\Delta v_{\parallel}}{v_{\parallel}}$ in (2) using (3); then using the expressions for f_H and k, we can eliminate $\frac{\Delta v_{\parallel}}{v_{\parallel}}$ and neglecting small quantities obtain

$$B_{BLI} = \frac{1}{2\Delta t_i}\left(\frac{\Delta f}{f_o}\right)\left(\frac{f_H/f_o+2}{f_H/f_o-1}\right)\left(\frac{m/|q|}{\tan\alpha\sin\phi}\right) \quad (4)$$

where ϕ = angle between $\bar{V}_{\perp}$ and $\bar{B}_{BLI}$.

Curves of B_{BLI} vs $f_{PFO} = \Delta f$ are shown in Figure 3 parametric in α, assuming Δt_i = 10 ms and calculated using (4) for L = 4 and $\phi = \pi/2$ and a typical model of the interaction region (IR) taken from [*H & I*, '82]. As we show below, our estimate of B_{sat} depends on the ratio of the relative values of B_{BLI} and B_{sat} as measured on the received spectrum.

Comparison of theory with observations

We have derived a triggering model based on the termination type of triggering since it appears simpler than other types. In this case the features labeled in the idealized spectrogram of Figure 1 have typical values of $f_i \cong 100$ Hz, $f_{PFO} \cong 50$ Hz and the ratio of B_{sat} to the initial intensity of the TE $\cong$ 1. Using Fig. 3, we find that for the assumed pitch angle in (*H & I* '82) of 30°, $B_{max} \approx B_{BLI}$ = 16pT, a value significantly larger than that estimated in *Helliwell & Walworth* [1996]. However if we use the recent data from Polar [*Bell et al.*, 1999] showing a pitch angle of 60° near the magnetic equator we obtain, from Fig.3, a value of $B_{BLI} \approx B_{max} \cong$ 5pT, in close agreement with Helliwell & Walworth (1996).

Discussion

We have used experimental observations of magnetospheric Vlf whistler mode emissions triggered by coherent waves from Siple Station, Antarctica to derive an explanation of this important but poorly understood process. We have shown that the step termination of an amplified Siple pulse launches a transient band-limited impulse (the *BLI*) that in turn produces a frequency increase (called the *PFO*) of the existing transverse phase-bunched electron currents that amplify the input pulse. A significant feature of this triggering mechanism is its dependence on the amplitude of the BLI which in turn provides a direct measure of the saturation value B_{sat} of the amplified input signal as well as the following emission, assuming a pitch angle distribution. This is the first time that the *in situ* B_{max} of the CWI has been measured using ground-based observations. These saturation values must be accurately known if we are to correctly calculate the wave-induced precipitation of energetic particles (WIPP) from the radiation belts (e.g. *Inan et al., J. Geophys. Res., 90(A1)*, 359, 1985).

The next step is to compare available Vlf wave data on emission triggering with representative measured energetic electron distribution functions, including the pitch angle distributions, in order to separate the effects of pitch angle and the wave saturation intensity (such as shown in Figure 3). In particular, we need to confirm the above result showing agreement between the model and a measured pitch angle of 60°. We suggest that the mechanism of "termination" triggering presented here can be applied to the other principal type called pre-termination triggering, with the aid of the triggering window (TW) concept [*Helliwell & Inan*, 1982]. Further discussion of these points is beyond the scope of this letter.

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