Fractal structure of sprites

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Abstract. A large scale model of sprites based on a phenomenological probabilistic approach to modeling of streamer corona discharges is developed. The model utilizes the experimentally documented macroscopic properties of positive and negative streamer corona in air and allows a realistic determination of the propagation of multiple breakdown branches in a self-consistent electric field. The model results reproduce the large scale volumetric shapes of sprites, agree with the experimentally documented thundercloud charge moment changes in sprite producing cloud to ground lightning discharges (CGs), and demonstrate fundamental asymmetries between sprites generated by CGs of positive and negative polarity.

Introduction

Sprites are spectacular luminous glows which occupy volumes in excess of thousands of cubic kilometers in clear air above thunderstorms [e.g., *Sentman et al.,* 1995]. Recent high spatial resolution telescopic imaging of this phenomena has uncovered a fascinating complex of internal sprite structure, including fine scale, highly branched luminous filaments with spatial scales ranging from tens to several hundreds of meters [*Inan et al.,* 1998; *Gerken et al.,* 1998]. The analysis of the temporal and spatial scales which characterize the electrical breakdown at different altitudes above sprite producing thunderstorms has demonstrated that the upper extremities of sprites always appear as amorphous diffuse glows, while the lower portions exhibit a complex streamer structure [*Pasko et al.,* 1998a]. The direct large scale modeling of portions of sprites dominated by streamers using a standard set of continuity fluid equations for charged particles is computationally not possible at present due to the extremely fine spatial resolution which is required to resolve individual streamer channels (i.e., the needed grid resolution is ~1 cm to resolve streamers with spatial scales ~10 m at 70 km altitude [*Pasko et al.,* 1998a]). The purpose of this paper is to report results from a new model of sprites which circumvents this difficulty by using experimentally documented macroscopic properties of positive and negative streamer corona in air and reproduces well the general observed shapes of sprites as well as their internal structure.

Critical Fields for Streamer Breakdown

Recent publications on sprite theory [*Pasko et al.,* 1998a; *Raizer et al.,* 1998, *Petrov and Petrova,* 1999], although different in some details, all support the general idea that highly structured regions of sprites are simply scaled analogs of the streamer electrical breakdown processes observed at ground pressure. A remarkable feature of the streamer corona is that in spite of its internal structural complexity, involving multiple highly branched streamer channels, its macroscopic (large scale) characteristics remain relatively stable under a variety of external conditions. The minimum field required for the propagation of positive streamers in air at ground pressure has been extensively documented experimentally and usually stays close to the value E_{cr}^{+}=4.4 kV/cm [*Allen and Ghaffar*, 1995], in agreement with recent results of numerical simulations of positive streamers [*Babaeva and Naidis*, 1997; *Morrow and Lowke*, 1997]. The absolute value of the similar field E_{cr}^{-} for negative streamers is a factor of 2-3 higher [e.g., *Raizer*, 1991, p. 361; *Babaeva and Naidis*, 1997]. In our modeling we assume E_{cr}^{-}=-12.5 kV/cm in accordance with Fig. 7 of *Babaeva and Naidis* [1997]. The field measurements inside of the streamer zone of positive [*Petrov et al.*, 1994] and negative [*Petrov and Petrova,* 1993] leaders indicate that E_{cr}^{+} and E_{cr}^{-} are also close to the integral fields which positive and negative corona establish in regions of space through which they propagate.

The fields E_{cr}^{+} and E_{cr}^{-} are the minimum fields needed for the propagation of individual positive and negative streamers, but not for their initiation [e.g., *Petrov and Petrova*, 1999]. Streamers can be launched by individual electron avalanches in large fields exceeding the conventional breakdown threshold E_k, defined by equality of the ionization and dissociative attachment coefficients in air [e.g., *Raizer*, 1991, p. 135; *Pasko et al.,* 1998a], or by initial sharp points creating localized field enhancements, which is a typical case for point to plane discharge geometries [e.g., *Raizer et al.,* 1998]. The possibility of simultaneous (in opposite directions) launching of positive and negative streamers from a single midgap electron avalanche is well documented experimentally [e.g., *Loeb and Meek*, 1940; *Raizer*, 1991, p. 335] and reproduced in numerical experiments [e.g., *Vitello et al.,* 1993]. In our modeling we assume E_k=32 kV/cm [*Raizer,* 1991, p.135]. Fig. 1a shows an altitude scan of the electric field created by static charges of 10 C and 100 C placed at 10 km altitude between two perfectly conducting planes at the ground (0 km) and the ionosphere (95 km), as well as the critical fields E_{cr}^{+}, E_{cr}^{-}, and E_k, which scale with altitude proportionally to the neutral atmospheric density.

Fractal Model of Streamer Corona

We use a phenomenological probabilistic approach which was proposed in [*Niemeyer et al.,* 1989] for modeling of a streamer corona. The model solves Poisson equation for the electric field in a cylindrical coordinate system (r, z, ϕ) with the z axis representing altitude, and assumes azimuthal symmetry ($\partial/\partial\phi$=0). The field is calculated on a two dimensional (r,z) grid using the source charge placed at altitude z=10 km between two perfectly conducting planes

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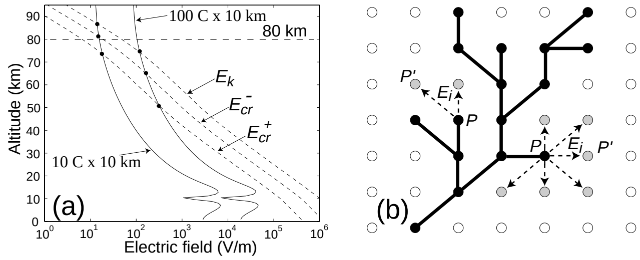

Figure 1. (a) Altitude scans at r=0 of the electrostatic field corresponding to different charges (10 C and 100 C) having Gaussian spatial distribution with scale 3 km and placed at altitude 10 km. The characteristic fields E_k, E_{cr}^+, and E_{cr}^- are shown by the dashed lines; (b) Schematics of the discharge model used in calculations.

(the ground z=0 km and the ionosphere z=95 km). The side boundary (placed at r=95 km) is also assumed to be perfectly conducting. The effect of this artificial boundary on the fields inside the system is small (i.e., 0.03%, 0.2% and 2% at r=0, 25 and 50 km, respectively at altitude 50 km). The field self-consistently accounts for the distortions caused by the propagation of the streamer corona which is modeled as a sequence of links between grid points. The growth of the discharge tree is initiated by a single electron avalanche in a region in which the field exceeds the threshold E_k (i.e., at altitude 80 km for 100 C charge as shown in Fig. 1a). The potential of this initial point is fixed and the discharge is propagated by adding additional links. The potential drop along every new link is assumed to be defined by E_{cr}^+ or E_{cr}^- depending on the field polarity. The grid points which belong to active links are then assumed to retain the potential which they acquired until the end of the simulation (unless a return stroke develops, see below). Only one link is added at any time step and the field is recalculated on the entire grid after each new single link added. At every step, every point of the discharge tree P is scanned for its neighbors P' (Fig. 1b). Only those points P' which form a potential difference with P with the corresponding field E_i such that $E_i \leq E_{cr}^-$ or $E_i \geq E_{cr}^+$ are added to the list of candidates for the next step. Each candidate link i is then characterized by a probability $p_i = |E_i - E_{cr}^{\pm}|^\eta / \sum_i |E_i - E_{cr}^{\pm}|^\eta$. Based on these probabilities a random choice is made and a new step is realized. The growth of the tree stops self-consistently when all probabilities become zero (i.e., the field around all active points of the tree falls between E_{cr}^+ and E_{cr}^-). In the region of field values typical for streamer tips ($2.5E_k$-$5E_k$) the effective ionization coefficient of air scales linearly with the field and we assume η=1, following *Niemeyer et al.* [1989] and *Femia et al.* [1993]. However, the streamer tip field $\sim 5E_k$ can not be resolved on coarse grids typically used in fractal models and therefore η should be viewed as one of the phenomenological input parameters in our model. The parameter η can eventually be evaluated more accurately on the basis of experimental measurements of the sprite fractal dimension.

The accumulated drop of potential along each of the discharge branches leads to substantial differences between potentials on the tips of channels and the ionospheric potential when these channels reach the upper boundary. The difference should lead to the development of a return wave with a strong field (i.e., return stroke), additional ionization in the channel and establishment of effective channel potential close to that of the ionosphere, similar to spark discharges in laboratory experiments [*Raizer*, 1991, p. 343]. In our model we assume that every channel developing return stroke acquires the ionospheric potential along its body.

Validation of the Fractal Model

The above model can be used to follow the propagation of multiple branches of streamer corona of both polarities in a complex field which is self-consistently distorted by each of these branches. Although models of this type have produced results in good agreement with many features of electrical discharges observed in real experiments [e.g., *Femia et al.*, 1993], we note that the length of individual links in the discharge tree is defined by the size of numerical grids, not by the internal physics of streamers. Although one may argue that discharge trees in fact obey laws of fractal geometry [e.g., *Niemeyer et al.*, 1984] and that every link can be viewed as representing multiple internal branches on lower spatial scale, the independence of the macroscopic characteristics of modeled streamer corona of purely mathematical discretization should be demonstrated before such a model can be applied to more complex systems such as large scale sprite modeling.

We performed a series of numerical experiments to test the sensitivity of the model to spatial discretization. Fig. 2a shows the result of a simulation of positive streamer corona which develops in the field of a charge 10^{-9} C centered at z=1 cm, r=0 inside of a metallic cylindrical box with radius 1 cm and height 2 cm filled with air at atmospheric pressure. The charge is assumed to have a Gaussian spatial distribution with scale 1 mm and fractal trees were allowed to grow from a surface of a smaller cylinder with radius 1 mm, and height 2 mm shown in the center of the Fig. 2a. The potential is kept unchanged in all points inside of the smaller cylinder during the simulation. This procedure allowed the specification of identical initial field distributions (shown in Fig. 2b by open circles) on grids with different spatial resolution. A structure similar to that shown in Fig. 2a was recently obtained by *Valdivia et al.* [1998] who used a similar approach to model horizontal lightning. Simulations were performed with resolutions 20, 30, 40 and 50 grid points per 1 cm and repeated 10 times for each resolution. Fig. 2b shows the resultant radial scan of the field at z=1 cm averaged over ten runs. We also monitored the Hausdorff dimension of resultant fractal trees D which was found from the power law relation $N(R) \sim R^D$ between the number of links $N(R)$ enclosed inside the circle with radius R

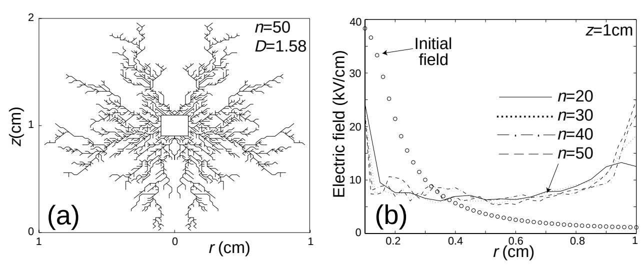

Figure 2. (a) An example of the model generated positive streamer corona; (b) The radial scan of the electric field corresponding to different spatial resolutions.

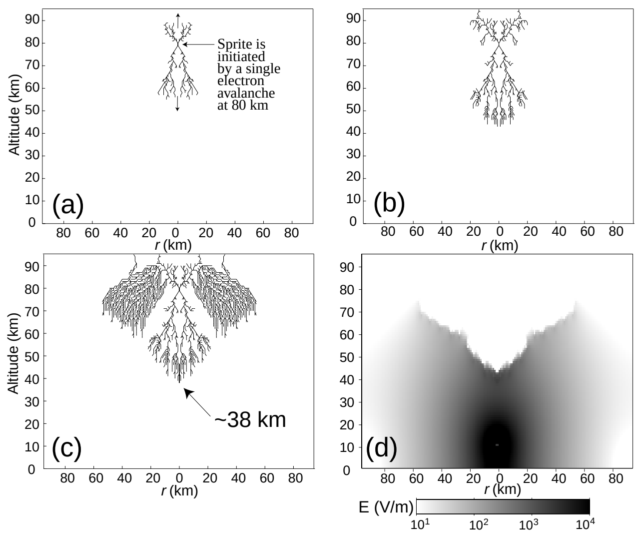

Figure 3. The dynamics of sprite development produced by 100 C of charge removed from 10 km altitude. The sequence of figures (a)(b)(c) shows cross-sectional view of discharge trees; (d) A cross-sectional view of the distribution of the absolute values of the electric field corresponding to the structure shown in (c).

centered at z=1 cm, r=0 [e.g., *Niemeyer et al.,* 1984]. We observe from the Fig. 2b that the field inside the streamer corona remains quasi-constant as a function of r in agreement with earlier findings of *Niemeyer et al.* [1989] and close to ∼5 kV/cm observed in experiments [*Pertov et al.,* 1994]. More importantly, results of runs with different spatial resolution show very similar integral field distributions inside the streamer corona. Results also show similar fractal dimensions D: 1.44±0.16; 1.31±0.16; 1.46±0.18 and 1.39±0.18 for n=20, 30, 40 and 50, respectively. It should be noted that Fig. 2a shows a sectional view of the streamer corona and actual three dimensional (3D) structure can be obtained as figure of rotation of the shown sectional contours around the central axis of the system. Although the two-dimensional azimythally symmetric model does not allow the modeling of streamer corona as true line channels it nevertheless correctly reproduces salient macroscopic properties of the real 3D streamer coronas observed in experiments.

Results and Discussion

Fig. 3a, 3b and 3c show the dynamics of development of a sprite initiated by a single electron avalanche at altitude 80 km. The starting field is defined by the removal of +100 C of charge by a positive CG lightning from 10 km altitude (Fig. 1a). The sprite is characterized by the upward development of negative, and downward development of positive, streamers. This behavior agrees with high speed video observations of initial sprite development [*Stanley et al.,* 1999]. We note that the model can not provide information on the velocity of streamers. The velocity of a streamer propagating in fields greater than E_{cr} is an increasing function of the streamer length and can reach values ≥10 km/ms [e.g., *Babaeva and Naidis*, 1997; *Raizer et al.,* 1998] in agreement with sprite observations [*Stanley et al.,* 1999]. Fig. 3b illustrates the moment of attachment of negative streamers to the lower ionosphere leading to the return stroke. Fig. 3c shows the final configuration after the arresting of propagation of all streamer trees. A large "jelly fish" sprite is formed with its lower extremities reaching an altitude of ∼38 km. Fig. 3d illustrates the corresponding configuration of the field characterized by the expulsion of the field from the body of sprite and the enhancement of the field on the sharp edges of streamer trees. Calculations were repeated ten times with the same initial conditions leading to quite a variety of structure of sprites, with two other examples shown in Fig. 4a and 4b. Nevertheless, the total volume occupied by sprites, their lowest altitude (39.7±1 km) and fractal dimension (D=1.37±0.13) remained very similar in different runs. The lowest altitude 39.7±1 km obtained in our model for the charge moment 1000 C km agrees well with the stopping altitude for positive streamers obtained by *Raizer et al.* [1998] (∼50 km for 350 C km), when scaled to the same charge moment. The lower portions of sprites always demonstrate the highly branched structure of positive streamer corona, while upward portions usually show just several channels formed by negative streamers which attach sprites to the lower ionospheric boundary. The model can not determine the physical width of the channels which in real sprites typically ranges from tens to several hundreds of meters [*Inan et al.,* 1998; *Gerken et al.,* 1998]. Results of our model calculations in terms of total volume occupied by sprites, their lowest altitude and the structure of upward/downward branching appeared to be insensitive to the location of starting point(s) of sprite development and spatial resolution of the model.

Fig. 4c shows the result of a model run in which a sprite was initiated by a negative CG lightning removing -100 C of charge from 10 km altitude. The negative sprites in our model generally extend to altitudes ∼10 km higher than positive ones for identical absolute amounts of the removed charge (e.g., to ∼47 km in Fig. 4c). They also grow wider in lateral dimension, forming numerous attachment points to the lower ionosphere. Otherwise, for large amounts of charge (∼100C) sprites produced by positive and negative discharges are quite similar.

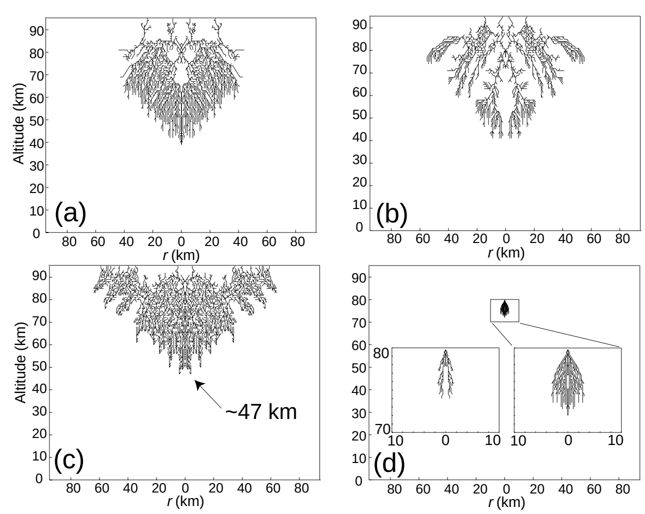

Figure 4. (a)(b) Results of two different runs for the same input parameters as in Fig. 3; (c) The sprite produced by a negative CG lightning removing -100 C from 10 km altitude; (d) The sprite produced by a positive CG lightning removing 10 C from 10 km altitude.

In the above modeled cases we assumed that the upper boundary of the simulation region is placed at 95 km altitude and is assumed to be a perfect conductor. The structure of upper portions of real sprites is significantly affected by the ambient ionospheric conductivity and usually takes a form of diffuse glows with no streamer structure [*Pasko et al.,* 1998a], which can not be modeled with the fractal model. Physically, the assumption which we used corresponds to a very sharp lower ionospheric boundary, when the diffuse region of sprites [*Pasko et al.,* 1998a] occupies a negligibly small volume near the upper boundary.

In addition to the quite complex interplay between different local parameters [*Pasko et al.,* 1998a], the altitude structure of sprites may also depend on the dynamics of charge removal by the causative CG lightning. For a relatively slow ($\geq$1 ms) charge removal the effective time dependent boundary separating regions dominated by displacement (free space) and conduction (good conductor) currents can be used as a dynamic reference for the lower ionosphere [e.g., *Pasko et al.,* 1998b; 1999]. For high conductivities of the lower ionosphere this boundary may very quickly (in $\sim$1 ms) slide to $\sim$80 km (e.g., [*Pasko et al.,* 1999, Fig. 3]). Fig. 1a shows schematically how sprite breakdown can be affected by reducing the altitude of the lower ionospheric boundary to 80 km (shown by the horizontal dashed line). The inspection of Fig. 1a shows, in particular, that the asymmetry between sprites produced by positive and negative CGs may become much more pronounced for lower values of the removed thundercloud charge and the reduced height of the lower ionospheric boundary, when the field exceeds E^+_{cr}, but not E^-_{cr}. In this case sprites can only be initiated from initial inhomogeneities providing localized field enhancements, but not from individual electron avalanches. For a positive CG lightning removing 10 C of charge from 10 km altitude, for example, sprites no longer can be initiated by electron avalanches (since $E<E_k$) and can not support negative streamers (since $|E|<|E^-_{cr}|$). Fig. 4d illustrates an example of this kind of sprite which is assumed to be initiated at the upper boundary (80 km) due to a localized lower ionospheric inhomogeneity [e.g., *Raizer et al.,* 1998]. A small (8 km tall) sprite is formed. No negative sprites are observed if the polarity of the CG lightning is changed to negative for the same amount of the removed charge. For a -20 C case we still did not observe any sprites for the negative polarity, while for the positive polarity sprites were able to extend 20 km down to altitudes $\sim$60 km (not shown). We suggest that negative CG lightning usually does not lead to sprites because of two reasons: (1) There is a fundamental asymmetry between requirements for propagation of positive and negative streamers which is characterized by a factor of three difference in critical fields E^+_{cr} and E^-_{cr} and which prevents production of negative sprites for a wide range of low charge values (which nevertheless lead to the well developed sprites in the case of positive polarity). (2) This asymmetry is reduced with the increasing of the magnitude of the applied field [e.g., *Babaeva and Naidis*, 1997] and negative CGs are able to form sprites in the same way as positive discharges do (i.e., Fig. 4c). However, the removal of large quantities of charge (i.e., $\sim$100 C from 10 km altitude) is required in this case, which itself is extremely rare for negative CG lightning [*Uman*, 1987, p. 123]. The documented cases of negative sprites [*Barrington-Leigh et al.,* 1999] generally do not contradict to the above statements.

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