JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 96, NO. A7, PAGES 11,379–11,388, JULY 1, 1991

Lower Hybrid Waves Excited Through Linear Mode Coupling and the Heating of Ions in the Auroral and Subauroral Magnetosphere

T. F. BELL AND R. A. HELLIWELL

STAR Laboratory, Stanford University, Stanford, California

M. K. HUDSON

Department of Physics and Astronomy, Dartmouth College, Hanover, New Hampshire

Experimental observations on the ISIS 1, ISIS 2, ISEE 1, and DE 1 spacecraft demonstrate that strong lower hybrid (LH) waves can be excited by VLF electromagnetic (em) whistler mode waves as the em waves propagate through regions of the ionosphere and magnetosphere where small-scale magnetic-field-aligned irregularities exist in the mean plasma density. There is strong evidence that the LH waves are excited by linear mode coupling as the em waves scatter from the irregularities. The present paper considers two related aspects of the linear mode conversion mechanism: (1) the controlled heating of suprathermal ions in the ionosphere and magnetosphere over a powerful VLF/ELF transmitter using LH waves excited by the em transmitter signals through linear mode conversion; (2) the excitation of intense LH waves in the auroral regions through the linear conversion of VLF/ELF em auroral hiss, and the subsequent heating of ions by the excited LH waves. In addressing both aspects a critical feature is the behavior of the mode coupling mechanism at frequencies less than 10.2 kHz, the lowest value observed in earlier experiments. Using the results of controlled experiments carried out at Siple Station, Antarctica, it is demonstrated that strong LH waves can be excited by em waves down to frequencies as low as 2 kHz in the subauroral low-altitude magnetosphere. Extrapolating from these observations and making use of ISEE 1 satellite wave amplitude data, we conclude that a ~250 kW VLF/ELF transmitter operating in the subauroral region at 2 kHz could excite sufficiently intense LH waves to heat suprathermal 1 eV H^+ ions and 16 eV O^+ ions to roughly 50 eV in a region extending in altitude from 1000 to 5000 km above the transmitter with a horizontal scale of ~500 km. Furthermore, if coherent wave stochastic heating occurs, the energy gain of O^+ ions could be as large as 200 eV. This effect would be readily measurable with available satellite instrumentation. Using a recently developed model of the linear mode coupling mechanism, we furthermore conclude that a significant portion of the LH waves observed in the auroral zone in regions of ion conic development may be excited by em VLF/ELF auroral hiss as the hiss propagates through the irregular background plasma commonly observed in the auroral regions.

1. INTRODUCTION

The purpose of this paper is to consider two aspects of the linear mode conversion of electromagnetic whistler mode waves into quasi-electrostatic lower hybrid waves in regions of the ionosphere and magnetosphere where magnetic-field-aligned irregularities exist in the mean plasma density. The first aspect concerns the controlled heating of ions over powerful VLF transmitters using lower hybrid waves produced by linear mode conversion. The second concerns the production of intense lower hybrid waves in the auroral regions through the linear conversion of VLF electromagnetic auroral hiss and the subsequent heating of ions by the excited lower hybrid waves.

Lower hybrid waves are an important plasma wave mode in a number of diverse fields of plasma physics. They have been proposed as a means of plasma heating in fusion devices [*Tripathi et al.*, 1977; *Antani and Kaup*, 1984], as a means of heating ions and creating ion conics in the auroral region [*Lysak et al.*, 1980; *Chang and Coppi*, 1981; *Koskinen*, 1985; *Kintner and Scales*, 1989; *Retterer et al.*, 1986, 1989; *McWilliams*, 1989; *Lysak*, 1989] and as a means of thermalizing the incoming energetic ions at the bow shock [*Mellott and Greenstadt*, 1988]. They are also apparently involved in ionospheric modification experiments [*Wong et al.*, 1981] and in the ion heating observed over powerful VLF transmitters [*Dzhordzhio et al.*, 1987].

Paper number 91JA00568.
0148-0227/91/91JA-00568$05.00

Although lower hybrid waves have been studied extensively in magnetospheric physics in the last decade, their origin is still unclear. In the auroral region it has been proposed that lower hybrid waves are produced either as a result of a drift instability or a beam-driven instability [*Chang and Coppi*, 1981; *Koskinen*, 1985]. In the foot of the bow shock their origin has been attributed to cross-field instabilities [*Gary et al.*, 1987]. In addition, the generation of lower hybrid waves through nonlinear parametric interactions has been extensively studied [*Tripathi et al.*, 1976; *Lee and Kuo*, 1984; *Riggin and Kelley*, 1982; *Koskinen*, 1985] in recent years. Although much of the past work on the generation of lower hybrid waves in the magnetosphere focused on specific regions of the ionosphere and magnetosphere, such as the auroral regions or bow shock, it has become clear from recent satellite data that high-amplitude lower hybrid waves are commonly stimulated throughout large regions of the ionosphere and magnetosphere by ordinary electromagnetic (em) VLF whistler mode waves, such as whistlers and signals from VLF transmitters, as they propagate through regions where small scale magnetic-field-aligned plasma density irregularities exist [*Bell et al.*, 1983*b*; *Titova et al.*, 1984; *Tanaka et al.*, 1984; *Inan and Bell*, 1985; *James and Bell*, 1987; *Tanaka et al.*, 1987; *Bell and Ngo*, 1988, 1990]. We believe that these irregularities do not arise from random temporal thermal fluctuations but are actually relatively time independent spatial changes in the mean plasma density which occur transverse to the Earth's magnetic field $\vec{B}_0$. These irregularities may be similar to HF ducts [*Gross and Muldrew*, 1984] in terms of their extension along $\vec{B}_0$ and peak density enhancement, but their scale transverse to $\vec{B}_0$ is apparently smaller. The known characteristics of these irregularities are discussed below in section 4. Since the amplitude of the excited lower hybrid waves can exceed that of the input electromagnetic waves by as much as 30 dB [*Bell et al.*, 1983*b*; *Bell and Ngo*, 1990], understanding the mechanism of excitation of the lower hybrid waves is crucially important for the understanding of wave-particle interactions in the ionosphere and magnetosphere.

Although other explanations have been proposed [*Titova et al.*, 1984; *Groves et al.*, 1988], we suggest that the lower hybrid waves are excited by a passive linear mode coupling mechanism as the em waves are scattered by the small scale irregularities [*Bell and Ngo*, 1988, 1990]. According to this model, the lower hybrid waves are a type of quasi-electrostatic whistler mode wave whose wave vector lies close to the resonance cone direction.

In the linear mode conversion model [*Bell and Ngo*, 1990], for the case of planar irregularities, high-amplitude lower hybrid waves are excited whenever the following two conditions are satisfied:

$$f_{LHR} \leq f \ll f_{He} \tag{1}$$

$$\lambda N^{-1} \frac{dN}{dx} \geq 1 \tag{2}$$

where f is the frequency of the input electromagnetic whistler mode wave, λ is the wavelength of the electromagnetic whistler mode wave, f_{LHR} is the lower hybrid resonance frequency, f_{He} is the electron gyrofrequency, N is the mean thermal plasma density, and x is the direction perpendicular to both the Earth's magnetic field $\vec{B}_0$ and the planar irregularities. When (2) holds, the WKB approximation fails for the electromagnetic wave and strong mode coupling occurs between the electromagnetic whistler mode waves and the quasi-electrostatic lower hybrid waves.

Within the plasmasphere, (2) is readily satisfied for the case of small-scale magnetic-field-aligned irregularities. For example, a 5% change in the mean plasma density occurring over a distance of 50 m gives $N^{-1}dN/dx \sim 10^{-3}\text{m}^{-1}$, whereas typical values for the whistler mode wavelength are $\lambda \sim 1-3$ km. Relation (2) can also be readily satisfied in the auroral and subauroral regions where large gradients in N are known to commonly exist.

In addition, in both the auroral and subauroral regions a significant background spectrum of electomagnetic whistler mode waves exists which could act as the input energy source for exciting the lower hybrid waves. Thus it would appear that all the conditions necessary to produce high-amplitude lower hybrid waves by linear mode coupling are present throughout those large portions of the magnetosphere where lower hybrid waves are actually observed.

These facts underscore the need to understand the linear mode coupling mechanism in order to evaluate its importance with respect to the other proposed lower hybrid wave generation mechanisms and to determine the effect of the excited waves upon the energetic and thermal particle population in the auroral and subauroral magnetosphere.

Spacecraft observations of lower hybrid waves excited by em whistler mode waves clearly define their characteristics. For example, because the group velocity of the the stimulated lower hybrid waves is small compared to that of the em waves which stimulated them and because they are able to propagate thousands of kilometers, they are observed on a spacecraft with time delays as large as 500 ms with respect to the em whistler mode waves which excite them. Furthermore, since the lower hybrid waves have a relatively short wavelength (5 m$\leq \lambda \leq$100 m), they are observed on a moving spacecraft with a large Doppler shift [*Bell et al.* 1983*b*, *Bell and Ngo*, 1988, 1990]. Because of these characteristics, the lower hybrid (LH) waves are most readily observable when the input em whistler mode wave is coherent with known frequency-time characteristics. In particular, lightning-generated whistlers and signals from VLF transmitters are useful.

2. The Heating of Ions Using VLF/ELF Transmitters

The first topic we treat is the heating of ions by lower hybrid waves which are excited through linear mode coupling by signals from VLF/ELF transmitters. We base our arguments on the results of controlled experiments designed to study the lower hybrid wave excitation effect using the VLF/ELF transmitter at Siple Station, Antarctica, and the Canadian ISIS 2 spacecraft. We use these results to predict the lower hybrid wave field and ion heating that might be produced in the subauroral region over a wave injection facility.

The Siple experiments were part of a series of joint VLF/ELF wave injection experiments involving the Communications Research Centre at Ottawa, Canada, and the Stanford University STAR Laboratory. The joint wave-injection experiments had four main components: (1) broadband VLF/ELF receivers on the ISIS 1 and ISIS 2 satellites; (2) a broadband (1-20 kHz) controllable VLF transmitter located at Siple Station, Antarctica [*Helliwell and Katsufrakis*, 1974]; (3) various VLF navigation and communication transmitters, such as those of the worldwide Omega network; and (4) ground stations in the Antarctic and Canada.

The main goal of these, and similar, experiments is to acquire understanding of interactions between coherent VLF waves and energetic particles in the magnetosphere and ionosphere [*Helliwell and Katsufrakis*, 1974; *McPherson et al.*, 1974; *Dowden et al.*, 1978; *Bell et al.*, 1981, 1983*a*; *Helliwell*, 1988; *Bell and Ngo*, 1988, 1990]. Sources of the coherent waves involved in these studies include VLF transmitters, large-scale power grids, whistlers, and other natural coherent VLF signals.

The VLF wave source in the wave-injection study reported herein was the Siple Station VLF transmitter at $L \sim$4.3 (76°S, 84°W geographic). One of the major advantages of the Siple transmitter was its ability to produce controlled VLF signals at frequencies in the range 1.5-6 kHz. This is the approximate range of the lower hybrid resonance frequency (LHR) at ISIS 2 altitudes ($\sim$1400 km) on magnetic shells in the range $4 \leq L \leq 6$. Thus Siple transmitter signals can be used to study changes in lower hybrid wave excitation characteristics that occur as the wave frequency is swept through the local LHR frequency. These changes include bandwidth increases [*Bell et al.*, 1983*b*], as well as amplitude increases [*Bell et al.*, 1985].

The ISIS 2 spacecraft was in a nearly circular orbit at approximately 1400 km altitude with an inclination of 89° prograde. The 75-m (tip-to-tip) receiving dipole antenna fed a VLF receiver operating in the 50-Hz to 30-kHz range. The receiver featured a linear amplifier with an automatic gain control (AGC) providing a total dynamic range of 70 dB. Signals were telemetered to ground and recorded in real time using a conventional FM modulation scheme. Telemetry of data was arranged through the courtesy of H. G. James.

Receivers, recorders, and antennas necessary to acquire the telemetry signal from the analog portion of the ISIS 2 VLF wave experiment were installed at Siple Station in January 1982. Because of funding limitations, equipment necessary to acquire the digital telemetry from the spacecraft was not available. As a result of this limitation, information concerning the absolute gain level of the satellite VLF receiver was not recovered, although the wave spectral distribution was fully recovered. Furthermore, data from other operable spacecraft experiments, such as the topside sounder, which might have provided important correlative information, were also not recovered because they were carried on the digital telemetry signal. As a consequence of this situation, the absolute amplitude of the excited electrostatic waves reported here could not be determined. However, their relative amplitude with respect to the input signals was generally readily measurable. In addition to the ISIS 2 observations made at 1400 km over Siple Station, a few observations of the effect were also carried out in the northern hemisphere near the conjugate point of Siple Station using the ISEE 1 satellite (as well as the ISIS 2 spacecraft).

2.1. *Observations*

Figure 1*a* shows the typical experimental configuration during periods in which strong lower hybrid waves were excited by pulses from the Siple transmitter as these pulses propagated along the short 1400-km path to the spacecraft through the ionosphere and lower magnetosphere. In all cases the ISIS 2 satellite was located near the Siple Station magnetic meridian at an invariant latitude in the range 58° to 75° and at an altitude of approximately 1400 km. The distance in the Earth-ionosphere wave guide between the subsatellite point and the Siple transmitter location (T) was $\leq$1100 km.

The lower hybrid wave excitation experiments at Siple Station were carried out from late April through early August in 1982. Transmissions to the ISIS 2 spacecraft were attempted on 28 separate passes during this time interval, and during 20 of these passes lower hybrid waves excited by transmitter pulses were observed on the spacecraft.

Figure 1*b* shows the invariant latitude and geomagnetic longitude of the ISIS 2 satellite for representative passes when excited electrostatic waves were observed. Most observations of the effect were made in the subauroral region poleward of the plasmapause location ($67° > \lambda_l > 60°$) and were spread with rough uniformity over 16° of geomagnetic longitude, centered approximately on the transmitter meridian.

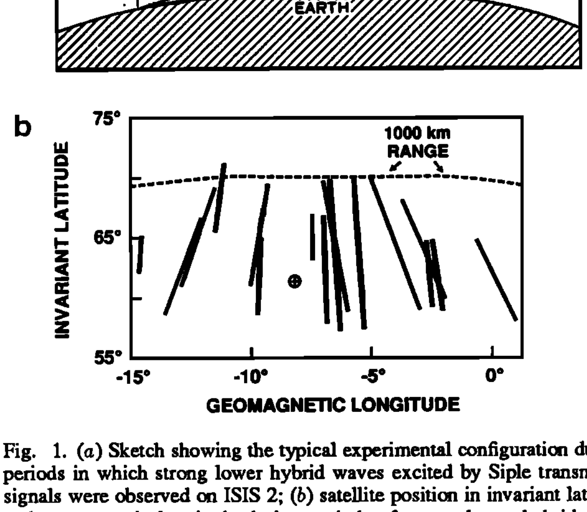

Fig. 1. (*a*) Sketch showing the typical experimental configuration during periods in which strong lower hybrid waves excited by Siple transmitter signals were observed on ISIS 2; (*b*) satellite position in invariant latitude and geomagnetic longitude during periods of strong lower hybrid wave excitation. The circled cross represents the position of Siple Station.

Typical examples of excited electrostatic waves are shown in the spectrograms of Figure 2. The upper panel of Figure 2*a* shows six swept-frequency transmissions from Siple when the local LHR frequency at the satellite was below 3 kHz. The lower panel of Figure 2*a* shows the signal format as transmitted at Siple. The bandwidths of the received signals lie in the range 200-500 Hz, more than 2 orders of magnitude larger than the nominal 1-Hz bandwidth of the transmitted signals. The upper panel of Figure 2*b* shows six swept-frequency transmissions from Siple when the local LHR frequency was above the highest transmitted frequency. The local LHR frequency is equal to the lower cutoff frequency of the LHR noise band and is marked by an arrow on the frequency axis at the beginning of the spectrogram [*Laaspere et al.*, 1971]. The lower panel of Figure 2*b* shows the signals as transmitted at Siple. For this case, measurements show that the bandwidths of the transmitted and received signals are comparable.

This figure illustrates a characteristic feature of the Siple/ISIS 2 data, i.e., lower hybrid waves were excited only when the frequency of the input signal f equaled or exceeded the local lower hybrid resonance frequency. This condition is identical to that found during earlier observations at frequencies above 10 kHz [*Bell et al.*, 1983*b*; *Titova et al.*, 1984, *Tanaka et al.*, 1984; *Inan and Bell*, 1985; *Bell and Ngo*, 1988] and helps to identify the excited waves as lower hybrid waves.

A unique feature of the 1982 ISIS 2 observations was the fact that they included the subauroral region outside the plasmapause ($4 \leq L \leq 6$) where the local f_{LHR} at the satellite location was often as low as 2 kHz, a value comparable to those obtaining in the low-altitude auroral region. As a result of this circumstance it was possible to demonstrate that lower hybrid waves could be excited at these frequencies, as long as the condition $f_{LHR} \leq f$ was satisfied.

An example of lower hybrid wave excitation for f<3 kHz is shown in Figure 3. At this time satellite data indicated that f_{LHR} ~2.5 kHz. The spectrogram in the figure shows lower hybrid waves excited within an apparent bandwidth of ~300 Hz centered on the instantaneous frequency of the input pulse as the frequency decreases linearly with time from 4 kHz down to ~2.5 kHz, the approximate local value of f_{LHR}. For $f < f_{LHR}$, no lower hybrid waves are excited and the signal bandwidth is ~1 Hz. The format of the transmitted input pulse is similar to those shown in Figure 2. As the frequency of the input signal approaches the value $f \sim f_{LHR}$, shortly before the 4 s. mark, there is an abrupt increase in apparent bandwidth, from ~300 to ~500 Hz, as well as an increase of ~3 dB in total signal amplitude. This increase of apparent bandwidth and amplitude of the lower hybrid waves when $f \rightarrow f_{LHR}$ from higher frequencies was a common occurrence during our study and may be a characteristic feature of the excitation mechanism.

When lower hybrid waves were excited by input signals from the Siple transmitter, their total electric field intensity with respect to the input pulse ranged from a few dB up to as much as 30 dB. Figure 4 shows a few typical examples of the dynamic power spectral density (PSD) of the electric field of lower hybrid waves excited by Siple transmitter pulses. The units of the plot are (volts)^2/Hertz. The input signals for Figures 4*a* and 4*b* were fixed frequency pulses at 4.45 and 4.95 kHz, respectively. The PSD of each pulse was sampled every 50 ms using a digital spectrum analyzer with 4-Hz frequency resolution, and the results were plotted on moving photographic paper. Each PSD sample was

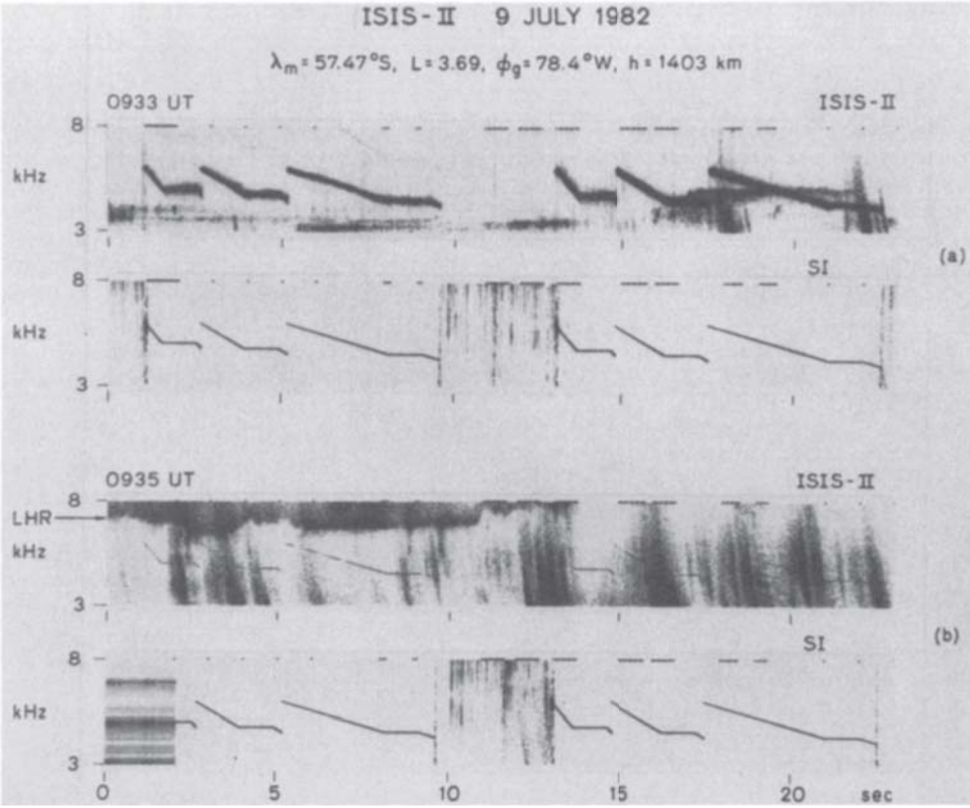

Fig. 2. (a) Lower hybrid waves excited by signals from the Siple Station transmitter when $f > f_{LHR}$. (b) lower hybrid waves are not excited when $f < f_{LHR}$.

plotted across the paper in 8 ms and new samples were obtained every 50 ms. Thus each panel shows approximately 300 ms of data. The intensity is measured on a logarithmic scale with respect to the sloping baseline. Figure 4a shows the most common case in which the spectrum of lower hybrid waves was distributed roughly symmetrically about the input pulse frequency. In this example the lower hybrid wave intensity exceeded that of the input pulse by roughly 10 to 20 dB over the 300 ms time interval. Figure 4b shows a less common case in which the lower hybrid waves appear predominantly at lower apparent frequencies than the input pulse. Here the lower hybrid wave intensity was roughly 5 to 15 dB above that of the input pulse.

2.2. *Antenna Response*

According to theory [*Bell et al.*, 1983b; *James and Bell*, 1987; *Bell and Ngo*, 1988; 1990], the frequency difference $\Delta\omega$, between any given lower hybrid wave component and the input pulse, is given by the well-known Doppler shift expression:

$$\Delta\omega_D = \vec{k}_{LH} \cdot \vec{V}_s \quad (3)$$

where $\vec{k}_{LH}$ is the wave vector of the lower hybrid wave and $\vec{V}_s$ is the spacecraft velocity vector. As a consequence of (3), the PSD plot of Figure 4 can be used to determine the electric field amplitude of lower hybrid waves as a function of wave length.

In general, the PSD approaches the background noise levels fairly smoothly as $|\Delta\omega_D|$ increases beyond 100 Hz. However, there is good reason to believe that this intensity decrease is a feature of the satellite dipole antenna response rather than a characteristic of the lower hybrid wave spectrum.

Physically, we would expect the response of an electric dipole antenna to the electric field of a lower hybrid wave might be much reduced whenever the wavelength λ_{LH} of the lower hybrid wave was much less than the antenna length $|\vec{l}|$. This condition can be

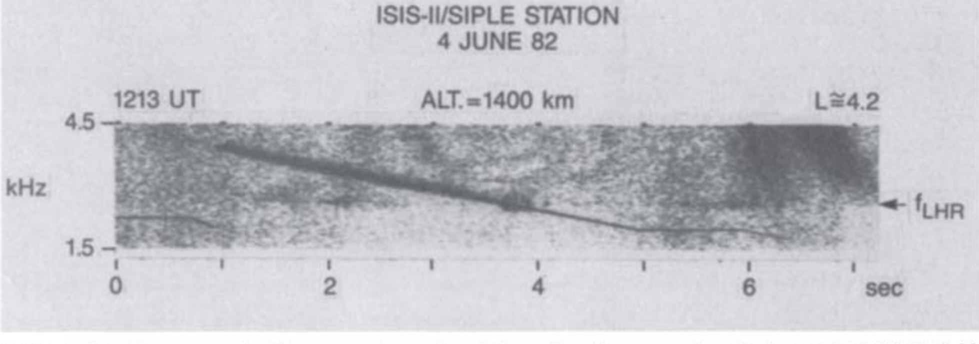

Fig. 3. Lower hybrid waves excited by a transmitter pulse of descending frequency. Amplitude and apparent bandwidth of the lower hybrid waves increase as $f \rightarrow f_{LHR}$.

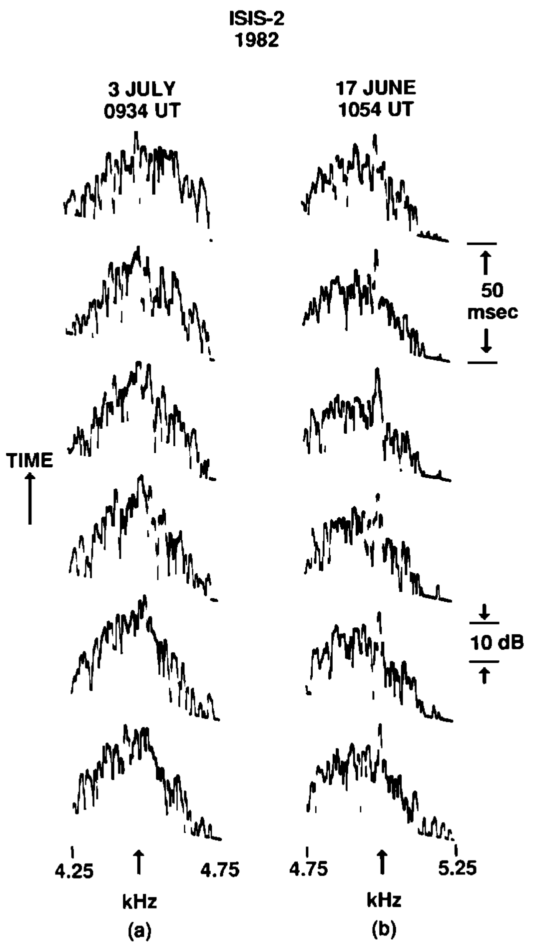

Fig. 4. Power spectral density of lower hybrid waves excited by two transmitter pulses. (*a*) lower hybrid spectrum roughly symmetric about input pulse frequency (denoted by arrow on frequency axis) (*b*) lower hybrid spectrum observed predominantly at lower apparent frequencies than that of the input pulse.

written:

$$l_x \gg \lambda_{LH} \quad (4)$$

where l_x is the component of $\vec{l}$ along $\vec{E}$. When (4) is satisfied the phase of the wave electric field will go through a large number of complete cycles along $\vec{l}$ and the antenna voltage produced by the wave electric field will tend to average toward zero. On the other hand, using similar arguments, we expect the antenna response to be relatively independent of wavelength when the wavelengths in question are such that

$$l_x \leq \lambda_{LH}/2 \quad (5)$$

Equation (5) restricts the antenna length in the direction of $\vec{E}$ to values less than, or equal to, one half of the wavelength of the lower hybrid wave. Applying (5) to the case of the ISIS 2 dipole antenna whose total length is 75 m, we conclude that the antenna response may decrease for lower hybrid waves whose wavelength is less than 150 m. The expected maximum Doppler shift for λ_{LH} ~150 m can be calculated from (3), using V_s ~7 km/s:

$$|\Delta f_D| \sim 50\text{Hz} \quad (6)$$

Equation (6) suggests that, in the case of excited lower hybrid waves whose apparent frequency differs by more than 50 Hz from the input wave frequency, the measured amplitude must be corrected for the reduced antenna response in order to determine the true wave amplitude. This correction is not straightforward since at the present time there is no generally accepted theory which describes the response of an electrically long antenna in a magnetoplasma. However it is of interest to consider a model of antenna response which has been successfully applied to the case of short wavelength electrostatic waves in the magnetosheath [*Gallagher*, 1985]. In this model the antenna response for ISIS 2 has the form

$$R = \cos^2\alpha\left(\frac{\sin^4 x}{x^4}\right) \quad (7)$$

where α is the angle between $\vec{k}_{LH}$ and $\vec{l}$, and $x = (\vec{k}_{LH}\cdot\vec{l})/4$.

In the limit of long wavelengths, $x \ll 1$, and $R \sim \cos^2\alpha$ as expected. In the short wavelength limit, $k_{LH}l \gg 1$, and two cases need to be considered. First, if $\alpha \sim 0$, then:

$$R_0 \sim \left(\frac{k_{LH}l}{4}\right)^{-4} \quad (8)$$

On the other hand, R has an absolute maximum value when $\alpha = \cos^{-1}[(k_{LH}l)^{-1}]$:

$$R_m \sim \left(\frac{k_{LH}l}{4}\right)^{-2} \quad (9)$$

and the maximum value R_m occurs when the projection of the antenna length along the wave electric field vector is roughly one-half wavelength.

Comparing (8) and (9), it can be seen that the antenna response can vary greatly as a function of antenna orientation in the lower hybrid wave field. This variation is believed to be the cause of the large changes in the apparent bandwidth of the lower hybrid waves which have been correlated with antenna orientation [*Bell et al.*, 1983*b*]. Examples of lower hybrid wave bandwidth variation with antenna orientation can be found in the work by *Bell et al.* [1983*b*] where it is shown that maximum bandwidths of as much as 2 kHz can be observed when strong transmitter signals are involved.

To understand how the bandwidth can be controlled by the antenna orientation with respect to $\vec{B}_0$, consider the following argument. When the antenna is roughly parallel to $\vec{k}_{LH}$, lower hybrid waves for which $k_{LH}l \gg 1$ will produce only a small antenna response, according to (6) and will be suppressed. For some given value of $k_{LH} = k_{LH}^N$, the measured intensity of the lower hybrid waves will just equal the intensity of the noise background, and for $k_{LH} > k_{LH}^N$ it will be below that of the noise background. When the antenna approaches the position where the maximum response of the antenna occurs, as shown in (9), the short wavelengths are not suppressed as strongly and a range of lower hybrid wave components with $k_{LH} > k_{LH}^N$ will now be measured at intensity levels above that of the noise. Since these short wavelengths are associated with larger Doppler shifts, the apparent bandwidth of the lower hybrid waves will then increase.

It is of interest to apply (8) to the data of Figure 4, which shows that the lower hybrid wave intensity reaches the background noise level when $|\Delta f_D|$ ~150 Hz. From (3) this implies a maximum wavelength of λ_L ~45 m, and for this value R ~0.03. Thus if (7) is correct, the PSD plot in Figure 3 actually underestimates the true intensity of the lower hybrid waves with $|f_D|$ ~150 Hz by roughly 15 dB. These considerations suggest that the ratio of total lower hybrid wave intensity to input wave intensity is larger than the spacecraft observations at first sight appear to indicate.

With the data of Figure 3 it is not possible to determine the existence of lower hybrid waves with λ <45 m. However, satellite observations of lower hybrid waves excited by signals from more powerful transmitters show lower hybrid waves with Doppler shifts of up to ±1 kHz. For V_s = 7 km/s this implies a wavelength of roughly 7 m [*Bell et al.*, 1983*b*].

2.3. *Ion Energy Gain*

The fact that lower hybrid waves can be readily excited by electromagnetic whistler mode waves from VLF/ELF transmitters and that the lower hybrid wave amplitude can exceed the amplitude of the input wave by as much as 30 dB suggests interesting possibilities for using this phenomenon to produce significant ion heating in the ionosphere over a powerful VLF/ELF transmitter. In particular, it might prove possible to produce ion conics under controlled conditions and thereby shed light on this important auroral phenomenon. Experimental measurements of lower hybrid waves in regions of ion conic formation give values generally in the range 0.2-10 mV/m for the lower hybrid wave electric field amplitude [*Kintner and Garney*, 1984; *Peterson et al.*, 1988].

To reach the upper/end of this amplitude range, 10 mV/m, would require an input electromagnetic whistler mode wave amplitude of roughly 1 mV/m, assuming a 20 dB ratio of lower hybrid wave amplitude to input electromagnetic wave amplitude. Measurements on the ISEE 1 spacecraft indicate that when the radiated power is roughly 1 kW, the amplitude of 2-5 kHz signals from the Siple Station transmitter is at least 0.2 mV/m up to an altitude of 5000 km directly over the station and over a horizontal scale of ~500 km centered on the station. Thus a radiated power of roughly 25 kW would be needed to produce an input wave amplitude of 1 mV/m over the transmitter. Experience with the Siple transmitter suggests that radiation efficiencies of 10% are achievable with a multielement horizontal antenna array even for frequencies as low as 1 kHz. Thus interesting ion heating experiments should be possible with antenna input powers in the 250-kW range.

To illustrate these possibilities, in the appendix we calculate the energy gain of suprathermal ions moving through the excited lower hybrid wave field using a model similar to that descussed by *Lysak et al.* [1980] and developed by *Chang and Coppi* [1981]. This model treats the suprathermal ions as unmagnetized, in which case the ions resonate with a given lower hybrid wave whenever the condition holds

$$v_\perp \cong \frac{\omega}{k_\perp} \tag{10}$$

where $v_\perp$ is the ion velocity perpendicular to $\vec{B}_0$ and $k_\perp$ is the wave number of the lower hybrid wave perpendicular to $\vec{B}_0$. Since satellite observations suggest that the minimum wavelength for excited lower hybrid waves is roughly 7 m, the smallest value of $v_\perp$ which will satisfy (10) for a 2-kHz frequency is $v_\perp \sim 14$ km/s, or roughly the velocity of a 1 eV H^+ ion or a 16 eV O^+ ion.

As discussed in the Appendix, if it is assumed that the LH wave field consists of waves with random phases (other simplifying assumptions are also discussed in the Appendix) the energy gain $\Delta\xi$ of an ion moving through the lower hybrid wave field has the approximate value:

$$\Delta\xi \cong [4r\ell]^{\frac{1}{4}} \quad [\frac{\pi}{2}q^2 P_A(k_\perp)]^{\frac{1}{2}} \tag{11}$$

where r is the radial distance from the center of the Earth to the center of the scattering region, ℓ is the length of the scattering region along $\vec{B}_0$, q is the ion charge, and $P_A(k_\perp)$ is the average power spectral density over the band $0 \le k_\perp \le \frac{\omega}{v_{\perp 0}}$, and $v_{\perp 0}$ is the initial ion velocity. From (A14) of the appendix it is found that $P_A(k_\perp) \sim 10^2$ $(\mathrm{mV})^2$/m for an rms value of 10 mV/m for the LH wave electric field.

Assuming a scattering region extending from 1000 to 5000 km above the transmitter and a value of 10 mV/m for the lower hybrid wave RMS electric field, it is found that the energy gain of both 1 eV H^+ ions and 16 eV O^+ ions is approximately

$$\Delta\xi \sim 50 \quad \mathrm{eV} \tag{12}$$

This same energy increase will occur for H^+ ions of energy greater than 1 eV since (11) is independent of initial ion energy as long as (10) is satisfied. Larger energy changes are possible if the minimum lower hybrid wave length λ_m is larger than 7 m since then the range of $k_\perp$ will be decreased and $P_A(k_\perp)$ must be larger to produce the same rms electric field amplitude of 10 mV/m. For example, if λ_m = 14 m, H^+ ions below 4 eV will not be resonantly heated, but H^+ ions of energy 4 eV and above will increase in energy by approximately 70 eV. Similarly if λ_m = 28 m, H^+ ions below 16 eV will not be heated, but H^+ ions above 16 eV will increase in energy by approximately 100 eV.

Clearly, many interesting possibilities for ion heating appear to exist if the required lower hybrid wave amplitude can be produced above the transmitter. One method of enhancing the amplitude of these waves might be to artificially increase the plasma turbulence above the transmitter by chemical releases from rockets, since theory [*Bell and Ngo*, 1990] predicts that the amplitude of the lower hybrid waves will be proportional to the amplitude of the small-scale magnetic-field-aligned plasma density fluctuations that exist in the plasma over the transmitter. However, we know of no experimental data which demonstrates that the required small-scale irregularities can be created during chemical releases.

The foregoing analysis assumes that H^+ and O^+ ions are heated quasilinearly, that is to say, the cumulative heating is due to a superposition of accelerations by a large number of waves with random phases. While this assumption of random phase agrees with the bulk of the observations concerning excited LH waves [*Bell et al*, 1983*b*; *Bell and Ngo*, 1988], it is nevertheless common to find that energy in the LH wave field is concentrated at just a few values of $k_\perp$ (for example, see Figure 11 of *Bell et al*, [1983*b*]). In these cases the wave field can be modeled as a sum of a few coherent LH waves, and if an ion maintains a definite phase relationship with respect to one or more coherent wave modes, its phase space trajectory can become stochastic. This implies that the particle trajectory will eventually fill the available phase space and have access to energies up to some maximum value which has been calculated [*Karney and Bers*, 1977; *Karney*, 1978; *Papadopoulos et al.*, 1980]. This type of acceleration, which amounts to the coherent addition of accelerations by $E_R \cos(k_x x - \omega t - \xi)$ versus the addition of random fields as E_R^2, has a threshold for the stochastic filling of available phase space given by *Karney* [1978] for coherent LH waves:

$$E_0 = \frac{1}{4}\left(\frac{\omega_{Hi}}{\omega}\right)^{\frac{1}{3}} \frac{\omega}{k_\perp} B_0 \tag{13}$$

where E_0 is the threshold field, ω_{Hi} is the ion gyrofrequency, and B_0 is the magnitude of the Earth's magnetic field. When the LH field exceeds (13), ions of mass M and charge q satisfying the resonance condition (10) can then be heated to a maximum energy [*Karney*, 1977; Papadopoulos et al., 1980]:

$$\frac{\xi_{max}}{T_H} = 4\left(\frac{M}{M_H}\right)^{\frac{5}{3}} \left(\frac{q}{e}\right) \left[\left(\frac{k_\perp \rho_H}{\pi}\right) \left(\frac{e\phi}{T_H}\right)^2 \left(\frac{f}{f_H}\right)^2\right]^{\frac{2}{3}} \tag{14}$$

where ϕ is the potentical of the LH wave and M_H, T_H, ρ_H, and f_H are the mass, temperature, gyroradius, and gyrofrequency, respectively, of hydrogen ions. The energy gain of (14) is achieved after ~1000 cyclotron periods of the heated ions [*Karney*, 1978].

For O^+ ions between 1000 and 5000 km altitude this represents an interaction time of roughly 30 s. During this short a period the heated O^+ ions will traverse only a small altitude range (~200 km) and thus (14) represents a relatively local heating effect as compared with the quasi-linear heating of (11) which requires a ~4000 km altitude interaction region. According to (13), for given values of $k_\perp$ and ω lower values of E_0 will occur at higher altitudes where B_0 and ω_{Hi} are smaller. Furthermore, according to (14), greater stochastic heating will also occur at higher altitudes for the same reason. At 5000 km altitude near $L \sim 5$, $f_H \sim 150$ Hz and for $\lambda_\perp = 7$ m and $f = 2$ kHz we find $E_0 \sim 15$ mV/m for 1 eV H^+ ions and $E_0 \sim 6$ mV/m for 16 eV O^+ ions. According to (14) the maximum stochastic heating for the 16 eV O^+ ions when $E \sim 10$ mV/m, keeping other parameter constant, is $\xi_{max} \sim 90$ eV. Since the mass dependence factor in (14) is unity for H^+ but $\sim 10^2$ for O^+, it is clear that the coherent wave stochastic heating is much more efficient for heavy ions. For instance, (14) predicts that even a large LH field amplitude of 40 mV/m gives for H^+ ions an energy gain of only $\xi_{max} \sim 6$ eV, while (11) predicts that the same rms field would heat H^+ to 200 eV. Thus it can be seen that the quasi linear acceleration is much more efficient than the coherent wave acceleration in heating H^+, while the coherent acceleration is more efficient in heating O^+.

Which heating mechanism operates depends on whether the LH waves are sufficiently coherent and of large enough amplitude. If coherent, the quasi-linear assumption of random phases breaks down, and one would expect that waves whose amplitudes are above threshold for stochastic heating would heat oxygen more than hydrogen. There is evidence for higher perpendicular fluxes of oxygen than helium and hydrogen in the observations of *Dzhordzhio et al.* [1987] concerning the presence of heated ions at altitudes near 1700 km over a powerful VLF transmitter. In this experiment the transmitter frequency was 15 kHz, the radiated power was 300 kW, and the enhanced fluxes of ~300 eV H^+, He^+, and O^+ ions were observed for roughly 1 min as the ARCAD 3 satellite passed over the transmitter location (64°N, 42°E geomagnetic). The heating was attributed to the presence of an intense lower hybrid noise band with amplitude ~30 mV/m near 4.5 kHz which was produced by the 15-kHz transmissions. *Dzhordzhio et al.*, [1987] suggested that the lower hybrid waves were excited through a parametric conversion of the transmitter signals into lower hybrid waves and electrostatic ion cyclotron waves.

Are the Dzhordzhio et al. results due either to quasi-linear or stochastic heating by the lower hybrid waves at 4.5 kHz? Near the satellite location we have $f_H \sim 400$ Hz and assuming $\lambda \sim 7$ m we find from (13) a threshold value of $E_0 \sim 90$ mV/m for H^+ ions and $E_0 \sim 40$ mV/m for O^+ ions. Thus the measured LH wave amplitude does not appear large enough to have produced stochastic heating effects. To calculate the possible quasi-linear heating for the 30 mV/m LH wave amplitude, we use $\ell \sim 1000$ km in (11) (since the upgoing ions were heated below the satellite) to obtain $\Delta\xi \sim 100$ eV, a value too small to explain the observations. Thus we conclude that an additional heating mechanism may have been operating in the Dzhordzhio et al. experiments.

Although the mechanism of excitation of the lower hybrid waves is not certain in the Dzhordzhio et al. experiment, experience with the Siple transmitter suggests that a VLF wave injection facility capable of radiating 25 kW at 2 kHz could be readily modified to radiate ~300 kW at ~15 kHz. Thus considerable flexibility in the means of producing lower hybrid waves for ion heating would appear to be available using VLF transmitters. In particular, in an experiment similar to that of Dzhordzhio et al. the radiated power from the transmitter could be varied in order to determine under controlled conditions the threshold LH amplitude level at which the transition from quasi-linear to mass dependent heating takes place. This intriguing possibility could aid our understanding of why preferential heating of oxygen is often observed in naturally occuring ion conics.

In addition to the heating mechanisms discussed above, it may also be possible to heat suprathermal ions by lower hybrid waves which are generated by parametric instabilities in which the transmitter signal acts as the input energy source. A three-wave parametric interaction of this kind has been discussed by *Riggin and Kelley* [1982], and a four-wave interaction has been discussed by *Groves et al.* [1988]. An input wave amplitude of 1 mV/m appears sufficient to satisfy the threshold requirements for these interactions. However, neither *Riggin and Kelley* [1982] nor *Groves et al.* [1988] show that the wavelengths of LH waves produced in their models will be small enough (<10 m) to resonantly heat 1-eV ions. Thus these interactions need further study in order to determine their efficacy in ion heating.

3. Lower Hybrid Wave Excitation in the Auroral Regions

The origin of the lower hybrid waves observed in regions where ion heating and ion conics are found is not clear. Thus we ask the question: Can a significant portion of the lower hybrid waves observed in the auroral regions be excited by linear mode conversion as electromagnetic VLF/ELF auroral hiss scatters from the magnetic-field-aligned irregularities which exist in the mean plasma density in the auroral region?

In order to estimate the level of lower hybrid wave activity in the auroral regions that might be produced though linear mode coupling, we assume that the auroral irregularities are planar and roughly magnetic-field-aligned, perhaps analogous to the sheets of ionization produced by auroral energetic particle precipitation. In this case we can represent the mean cold plasma density $N(x)$ in any given region containing irregularities by the Fourier series:

$$N(x) = N_0 + \sum_{m=\infty}^{\infty} \Delta N_m \quad e^{-ik_m x} \tag{15}$$

where $k_m = 2\pi m/W$, $m = \pm 1, \pm 2$, etc., and W is the width of the irregular region along the x axis.

It is useful to identify each term in (15) so that our formulation is clear. As stated above, $N(x)$ is the mean plasma density, i.e., the time averaged value of the local plasma density. Consequently, $N(x)$ has no explicit time variation and is a function only of distance transverse to the ambient magnetic field direction. The terms on the right-hand side of (15) are defined by the usual relations for Fourier series:

$$N_0 = \frac{1}{W} \int_0^W N(x) dx$$

$$\Delta N_m = \frac{1}{W} \int_0^W N(x) e^{ik_m x} dx$$

Thus N_0 is a constant which is equal to the average value of the mean plasma density, and the ΔN_m are the time-independent amplitudes of the spatial Fourier components which together contain all the information concerning the gradients of $N(x)$ within the region of width W.

According to the theory of *Bell and Ngo* [1990], an electromagnetic whistler mode wave propagating across the irregular region will excite lower hybrid waves whenever conditions (1) and (2) are satisfied. The amplitude of the excited lower hybrid waves will maximize whenever the matching condition is satisfied:

$$k_x^{EM} + k_m = k_x^{LH} \tag{16}$$

where k_x^{EM} and k_x^{LH} are the x components of the wave vectors of the electromagnetic and lower hybrid waves respectively. Since $k_x^{EM} \ll k_x^{LH}$, the matching condition becomes $k_x^{LH} \sim k_m$, and thus the excited lower hybrid wave amplitude will maximize whenever the wavelength of the lower hybrid wave matches that of any of the Fourier components of $N(x)$.

This behavior is illustrated in Figure 17 of *Bell and Ngo* [1990], where an electromagnetic whistler mode wave scattering from a series of 10 equally spaced Gaussian mean plasma density enhancements is shown to excite lower hybrid waves whose amplitude increases linearly with distance. The amplitude increase in this case occurs because the spacing between the Gaussian density enhancements is approximately equal to the wavelength of the lower hybrid wave excited by the input whistler mode wave. Thus the lower hybrid waves excited on each enhancement add coherently along the x axis and the total lower hybrid wave amplitude increases with distance. In this case, (16) is just the condition necessary for the coherent addition of amplitudes for the lower hybrid waves from each enhancement. One way to understand this result is to think of $N(x)$ as a time-independent diffraction grating with spatially periodic components discribed by the second terms in (15). Waves scattered (excited) by this grating will add coherently whenever (16) is satisfied. Otherwise, the scattered (excited) waves will mutually interfere and lower intensity will result.

Equation (A5) of *Bell and Ngo* [1990] gives the lower hybrid wave amplitude (in the electrostatic limit) excited by an input electromagnetic whistler mode wave scattering from the general mean plasma density profile given in (15). For present purposes we simplify this expression by assuming that there is a dominent spatial Fourier component with wave number k_m and that: $N(x) \sim N_0 + \Delta N_m \cos(k_m x + \phi)$, where ϕ is an arbitrary constant. In this case it can be shown from (A8) of *Bell and Ngo* [1990] that when (16) is satisfied, the amplitude E_{LH} of the excited lower hybrid wave is related to the spatial mean density change by the expression:

$$|E_{LH}|^2 = \frac{1}{4}\left(k_m W\right)^2 \left(\frac{D}{S}\right)^2 \left(\frac{\Delta N_m}{N_0}\right)^2 |E_{EM}|^2 \tag{17}$$

where E_{EM} is the electric field amplitude of the input electromagnetic wave, and D and S are components of the cold plasma dielectric tensor as defined by *Stix* [1962].

For frequencies near ω_{LHR},

$$\frac{D}{S} \cong \frac{f}{f_{He}} \left(\frac{k_x^{LH}}{k_z^{EM}}\right)^2 \tag{18}$$

where k_z^{EM} is the wave number of the input electromagnetic wave along $\vec{B}_0$.

Now consider typical low-altitude (~2000 km) auroral conditions with $f_{LHR} \sim 2$ kHz, $f_{He} \sim 800$ kHz and $k_z^{EM} \sim 10^{-3} m^{-1}$. In this case if we assume $\Delta N_m / N_0 \sim 0.1\%$, $W \sim 500$ m, and $\lambda_m \sim 10$ m, where $k_m = 2\pi/\lambda_m$, (17) becomes

$$|E_{LH}|^2 \cong 3 \times 10^4 |E_{EM}|^2 \tag{19}$$

The choice of W=500 m is consistent with the data shown in Figure 2 [*Bell and Ngo*, 1990]. However, the appropriate value in the auroral regions is not known.

Typical values for the electric field spectral density of electromagnetic auroral hiss range from 10^{-10} to 10^{-12}(V/m)^{2}Hz^{-1} [*Laaspere et al.*, 1971; *Gurnett and Frank*, 1972] at frequencies within a few kHz of ω_{LHR}. Taking the midpoint of this range and using a bandwidth of 2 kHz, we obtain

$$|E_{EM}|^2 \sim 2 \times 10^{-2} \left(\frac{\text{mV}}{\text{m}}\right)^2 \tag{20}$$

Using (19) and (20), we obtain

$$|E_{LH}|^2 \sim 6 \times 10^2 \left(\frac{\text{mV}}{\text{m}}\right)^2 \tag{21}$$

The prediction of (19) lies at the upper end of the range of lower hybrid wave amplitude in regions of ion conic formation and is also comparable to the lower hybrid wave amplitudes required to produce ion conics according to the theory of resonant ion heating suggested by *Lysak et al.* [1980] and developed by *Chang and Coppi* [1981].

Thus it appears possible that in the auroral regions significant ion heating may be produced by lower hybrid waves which are excited through linear mode coupling as electromagnetic VLF auroral hiss scatters from spatial irregularities in the mean plasma density. The necessary condition is that the mean plasma density must possess some periodicity with a spatial wavelength of the order of 10 m or less transverse to the ambient magnetic field direction.

4. Discussion and Conclusions

An important unresolved question in the foregoing development is the nature of the small-scale irregularities which are associated with the excitation of the LH waves. We do not believe that they are ordinarily produced through random temporal thermal fluctuations since LH wave excitation by whistler mode waves is not always observed at mid-latitudes [*Bell et al.*, 1983*b*] within the plasma sphere, a region in which the temperatures of ions and electrons do not usually vary considerably in space or time. However thermal fluctuations could lead to a low level of LH wave excitation which might appear as random noise in the wave detector. To treat the problem of thermal fluctuations, a formulation different from the one given above is required, but this is beyond the scope of the present paper. However, a mathematical treatment of this effect might be analogous to that of *Sentman* [1982].

In the linear mode coupling theory [*Bell and Ngo*, 1990] these irregularities are assumed to consist of spatial changes in the mean plasma density. They maybe similar to HF ducts [*Gross and Muldrew*, 1984] in terms of their extension along $\vec{B}_0$, peak density enhancement, and sheetlike geometry. However, their scale in at least one dimension transverse to $\vec{B}_0$ is generally much smaller. Satellite observations of variations in the local value of f_{LHR} during periods of LH wave excitation suggest that the mean plasma density varies by as much as a few percent in a distance of ~5 m [*Bell et al.*, 1983*b*]. This supports the assumption of the theory. Attempts to determine the physical properties of the irregularities using satellite observations in conjunction with theory have yielded information concerning the amplitude of the irregularities at scales as small as 13 m [*Bell and Ngo*, 1990]. However, satellite observations with smaller electric antennas (<10 m) are needed to determine irregularity amplitudes at scales less than 13 m. Such measurements have recently been carried out on the DE 1 spacecraft [*Bell et al.*, 1991].

The origin of the irregularities is also not known at the present time, but it is possible that they are produced by a feed back mechanism involving the precipitation of energetic particles by the excited LH waves [*Bell and Ngo*, 1988].

We conclude that significant heating of suprathermal ions in the low-altitude subauroral magnetosphere can be achieved using lower hybrid waves excited by VLF/ELF transmitters through linear mode coupling. Thus controlled experiments of this important auroral phenomenon may be possible using a VLF/ELF wave injection facility. In particular it may be possible to determine the threshold value of lower hybrid wave amplitude at which the transition from quasi-linear heating to mass-dependent heating takes place. Furthermore, we conclude that a significant component of the lower hybrid waves associated with ion heating and ion conics in the auroral and subauroral magnetosphere may be generated by a linear mode coupling mechanism as electromagnetic VLF auroral hiss scatters from magnetic-field-aligned irregularities in the background mean plasma density.

5. Appendix: Heating Calculations

Given that a lower hybrid wave amplitude of roughly 10 mV/m can be excited up to 5000 km above a 250 kW VLF transmitter, what heating rates can be expected for suprathermal ions? This question is difficult to answer precisely since there is presently no commonly accepted mechanism for the heating of ions by lower hybrid waves, and both resonant [*Chang and Coppi*, 1981] and nonresonant [*Singh and Schunk*, 1984] heating mechanisms have been proposed, as well as nonlinear resonant heating [*Karney and Bers*, 1977; *Karney*, 1978; *Papadopoulos et al.*, 1980].

For the sake of argument, we assume that linear resonant heating is the dominant mechanism and that the effect of the Earth's magnetic field on the motion of the ions can be neglected [*Chang and Coppi*, 1981]. For simplicity, we also assume that the plasma density irregularities within which the lower hybrid waves are excited are planar and extend roughly parallel to $\vec{B}_0$ and that consequently the electric field vectors of the excited lower hybrid waves lie on the resonance cone in a direction roughly normal to the irregularities [*Bell and Ngo*, 1990]. In this case an ion will respond to any given component of the lower hybrid wave field $\vec{E}$ according to the Lorentz relation:

$$\dot{\vec{v}} = \frac{q}{m}\vec{E}e^{i(\omega t - \vec{k}\cdot\vec{r})} \tag{A1}$$

where $\frac{q}{m}$ is the ion charge to mass ratio, and $\vec{E}$ is directed along $\vec{k}$. For $f \sim 2$ Hz, the resonance cone angle $\Theta_r \sim 90^\circ$, $k_z \ll k_x$ and $|E_z| \ll |E_x|$; thus to a good approximation (A1) can be written

$$\dot{v}_x = \frac{q}{m}E_x e^{i(\omega t - k_x x)} \tag{A2}$$

where the x axis is normal to the planar irregularities.

For a given amplitude E_x the largest changes in v_x will occur when the resonance conditon is satisfied:

$$v_x = \frac{\omega}{k_x} \tag{A3}$$

Furthermore, the ion will also interact strongly with lower hybrid waves with k_x values close to those defined in (A3) as long as the phase factor in (A2) remains small, i.e.,

$$|(\omega - k_x v_x)\Delta t| \leq 2\pi \tag{A4}$$

where Δt is the interaction time.

Expanding k_x in (A4) about the value defined in (A3) we obtain:

$$|v_x \Delta k_x \Delta t| \cong 2\pi \tag{A5}$$

Equation (A5) defines the interaction time for an ion in resonance with a set of lower hybrid waves with wave number spread Δk_x. The mean square energy change of the ion due to this interaction over a time Δt can be obtained from (A2):

$$< (\Delta\xi)^2 > \; = \frac{2\pi^2 q^2 E_x^2}{(\Delta k_x)^2} \tag{A6}$$

where $\xi = 1/2mv_x^2$.

From (A6) the diffusion rate for energy can be calculated:

$$D = \frac{< (\Delta\xi)^2 >}{\Delta t} = \pi q^2 P(k_x) v_x \tag{A7}$$

where we have eliminated Δt on the right-hand side using (A5), and where $P(k_x) = E_x^2/\Delta k_x$ is the power spectral density defined by the relation

$$E_R^2 = \int P(k_x) dk_x \tag{A8}$$

where E_R is the root-mean-square value of the lower hybrid wave electric field.

After moving in the lower hybrid wave field for a time τ, ions can experience a net energy increase of roughly

$$\Delta\xi \sim (\tau D)^{\frac{1}{2}} \tag{A9}$$

where we have assumed that $P(k_x)$ is roughly constant over the range of k_x. An approximate value for τ can be found from the equation which describes the adiabatic motion of the ions in the Earth's dipole magetic field:

$$\dot{v}_z = -\frac{1}{2}v_\perp^2 \frac{\partial}{\partial z}[\log B_0] \cong \frac{3v_\perp^2}{2r} \tag{A10}$$

where $B_0 = |\vec{B}_0|$ and r is radial distance of the ion from the Earth's center. If we assume that $v_\perp^2$ increases linearly with time and that r does not vary significantly over the altitude range of the lower hybrid waves, we can integrate (A10) up through the scattering region to obtain the result:

$$\tau \cong \frac{(4r\ell)^{\frac{1}{2}}}{v_{\perp f}} \tag{A11}$$

where ℓ is the length of the scattering region along $\vec{B}_0$ and $v_{\perp f}$ is the final velocity of the heated ion as it leaves the scattering region.

We now use the average value of $v_\perp$ in (A7) and combine this with (A9) and (A11) to arrive at

$$\Delta\xi \sim [4r\ell]^{\frac{1}{4}} \quad [\frac{\pi}{2}e^2 P_A(k_x)]^{\frac{1}{2}} \tag{A12}$$

where $P_A(k_x)$ is the average value of $P(k_x)$. Note from (A12) that the energy gain is independent of ion mass for a given power spectral density $P(k_x)$.

What energy gain can we expect from a lower hybrid wave field of 10 mV/m? For the heating to take place as hypothesized we must have k_x values which cover the range:

$$\frac{\omega}{v_{\perp 0}} \geq k_x \geq \frac{\omega}{v_{\perp f}} \tag{A13}$$

Since $v_{\perp f} \gg v_{\perp 0}$, the necessary total bandwidth in $k_\perp$ is $\sim \frac{\omega}{v_{\perp 0}}$.

Thus we have from (A8)

$$E_R^2 = P_A(k_x)(\frac{\omega}{v_{\perp 0}}) \tag{A14}$$

If we now assume that the scattering region extends from 1000 to 5000 km above the transmitter operating at 2 kHz and that the ions are originally 1 eV H^+ ions, we have $\ell \sim 4000$ km, $r \sim 9000$ km, $v_{\perp 0} \sim 14$ km/s, and $P_A = 1.1\times10^2$ $(mV)^2/m$. In this case (A12) becomes

$$\Delta\xi \sim 50 \quad \text{eV} \tag{A15}$$

This is also the energy gain for 4 eV H_e^+ ions and 16 eV O^+

ions since these ions will also have an initial velocity of 14 km/s and can satisfy (A3) throughout the interaction region.

Acknowledgments. We wish to acknowledge the many valuable discussions we have held with our colleagues in the STAR Laboratory during the course of this work. We are especially indebted to H. G. James for arranging for the reception of ISIS satellite data at Siple Station. The manuscript was typed by Zheng Xu. This research was sponsored by the National Aeronautics and Space Administration under grants NAGW-1680-1 (T. F. Bell and R. A. Helliwell) and DII89-02856 (M. K. Hudson).

The Editor thanks M. J. Keskinen and another referee for their assistance in evaluating this paper.

REFERENCES

Antani, S. N., and D. J. Kaup, Whistler scattering from density fluctuations in magnetized plasmas, *Phys. Fluids, 27,* 1169, 1984.

Bell, T. F., and H. D. Ngo, Electrostatic waves stimulated by coherent VLF signals propagating in and near the inner radiation belt, *J. Geophys. Res., 88,* 2599, 1988.

Bell, T. F., and H. D. Ngo, Electrostatic lower hybrid waves excited by electromagnetic whistler mode waves scattering from planar magnetic-field-aligned plasma density Irregularities, *J. Geophys Res., 95,* 149, 1990.

Bell, T. F., U. S. Inan, and R. A. Helliwell, Nonducted coherent VLF waves and associated triggered emissions observed on the ISEE 1 satellite, *J. Geophys. Res., 86,* 4649, 1981.

Bell, T. F., U. S. Inan, I. Kimura, H. Matsumoto, T. Mukai, and K. Hashimoto, EXOS-B/Siple Station VLF wave-particle interaction experiments, 2, Transmitter signals and associated emission, *J. Geophys. Res., 88,* 295, 1983*a*.

Bell, T. F., H. G. James, U. S. Inan, and J. P. Katsufrakis, The apparent spectral broadening of VLF transmitter signals during transionospheric propagation, *J. Geophys. Res., 88,* 4813, 1983*b*.

Bell, T. F., J. P. Katsufrakis, and H. G. James, A new type of VLF emission triggered at low altitude in the subauroral region by Siple Station VLF transmitter signals, *J. Geophys. Res., 90,* 12,183, 1985.

Bell, T. F., U. S. Inan, V. S. Sonwalkar, and R. A. Helliwell, DE-1 observations of lower hybrid waves excited by VLF whistler mode waves, *Geophys. Res. Lett., 18,* 393, 1991.

Chang, T., and B. Coppi, Lower hybrid acceleration and ion evolution in the supra-auroral region, *Geophys. Res. Lett., 8,* 1253, 1981.

Dowden, R. L., A. C. McKey, L. E. S. Amon, H. C. Koons, and M. H. Dazey, Linear and nonlinear amplification in the magnetosphere during a 6.6 kHz transmission, *J. Geophys. Res., 83,* 169, 1978.

Dzhordzhio, N. V., M. M. Mogilevskii, V. M. Chmyrev, R. A. Kavrazhkin, O. A. Molchanov, Yu. I. Galperin, J. M. Baske, and J. L. Rocke, Acceleration of ions in the the plasma environment of the earth by the radiation from a low-frequency transmitter on the ground, *JETP Lett., 46,* 405, 1987.

Gallagher, D. L., Short wavelength electrostatic waves in the Earth's magnetosheath, *J. Geophys. Res., 90,* 1435, 1985.

Gary, S. P., R. L. Takar, and D. Winske, Ion/ion and electron/ion cross field instabilities near the lower hybrid frequency, *J. Geophys. Res., 92,* 10,029, 1987.

Gross, S. H., and D. B. Muldrew, Uniformly spaced field-aligned ionization ducts, *J. Geophys. Res., 89,* 8986, 1984.

Groves, K. M., M. C. Lee, and S. P. Kuo, Spectral broadening of VLF radio signals traversing the ionosphere, *J. Geophys. Res., 93,* 14,683, 1988.

Gurnett, D. A., and L. A. Frank, VLF hiss and related plasma observations in the Polar magnetosphere, *J. Geophys. Res., 77,* 172, 1972.

Helliwell, R. A., VLF wave-injection experiments from Siple Station, Antarctica, *Adv. Space Res., 8,* 279, 1988.

Helliwell, R. A., and J. P. Katsufrakis, VLF wave injection into the magnetosphere from Siple Station, Antarctica, *J. Geophys. Res., 79,* 2511, 1974.

Inan, U. S., and T. F. Bell, Spectral broadening of VLF transmitter signals observed on DE 1: A quasi-electrostatic phenomenon?, *J. Geophys. Res., 90,* 1771, 1985.

James, H. G., and T. F. Bell Spin modulation of spectrally broadened VLF signals, *J. Geophys. Res., 92,* 7560, 1987.

Karney, C. F. F., Stochastic ion heating by a lower hybrid wave, *Phys. Fluids, 21,* 1584, 1978.

Karney, C. F. F., and A. Bers, Stochastic ion heating by a perpendicularly propagating electrostatic wave, *Phys. Rev. Lett., 39,* 550, 1977.

Kintner, P. M., and S. P. Gorney, A search for the plasma proceses associated with perpendicular ion heating, *J. Geophys. Res., 2,* 937, 1984.

Kintner, P. M. and W. Scales, The plasma wave environment of a mid-altitude auroral arc, paper presented at Cambridge Workshop on Theoretical Geoplasma Physics, Mas. Inst. of Technol., , Cambridge, June 1989.

Koskinen, H. E. J., Lower hybrid parametric processes on auroral field lines in the topside ionosphere, *J. Geophys. Res., 90,* 8361, 1985.

Laaspere, T., W. C. Johnson, and L.C. Semprebon, Observations of auroral hiss, LHR noise, and other phenomena in the frequency range 20 Hz-540 kHz on OGO 6, *J. Geophys. Res., 76,* 4477, 1971.

Lee, Me. C., and S. P. Kuo, Production of lower hybrid waves and field-aligned plasma density striations by whistlers, *J. Geophys. Res., 89,* 10,873, 1984.

Lysak, R.L., Electron-wave interactions on auroral field lines, paper presented at Cambridge Workshop on Theoretical Geoplasma Physics, Mass. Inst. of Technol., Cambridge, June 1989.

Lysak, R. L., M. K. Hudson, and M. Temerin, Ion heating by strong electrostatic ion cyclotron turbulence, *J. Geophys. Res., 85,* 678, 1980.

McWilliams, R., Laboratory studies of ion heating and transport, paper presented at Cambridge Workshop on Theoretical Geoplasma Physics, Mass. Inst. of Technol., Cambridge, June 1989.

Mellott, M. M. and E. W. Greenstadt, Plasma waves in the range of the lower hybrid frequency: ISEE 1 and 2 observations at the Earth's low shock, *J. Geophys. Res., 93,* 9695, 1988.

Papadopoulos, K., J. D. Gaffey, Jr., and P. J. Palmadesso, Stochastic acceleration of large M/Q ions by hydrogen cyclotron waves in the magnetosphere, *Geophys. Res. Lett., 7,* 1014, 1980.

Peterson, W. K., E. G. Shelley, S. A. Boardsen, D. A. Gurnett, G. G. Lidley, M. Suginra, T. E. Moore, and J. H. Waite, Transverse ion energization and low frequency plasma waves in the mid-altitude auroral zone: A case study, *J. Geophys. Res., 93,* 11,405, 1988.

Riggin, D., and M. C. Kelley, The possible production of lower hybrid-parametric instabilities by VLF ground transmitters and by material emission, *J. Geophys. Res., 87,* 2545, 1982.

Retterer, J. M., T. Chang and J.R. Jasperse, Ion acceleration by lower hybrid waves in the suprauroral region, *J. Geophys. Res., 91,* 1609, 1986.

Retterer, J. M., T. Chang, and J. R. Jasperse, Particle acceleration by intense auroral VLF turbulence, paper presented at Cambridge Workshop on Theoretical Geoplasma Physics, Mass. Inst. of Technol., Cambridge, June, 1989.

Sentman, D. D., Thermal fluctuations and the diffuse electrostatic emissions, *J. Geophys. Res., 87,* 1455, 1982.

Singh, N., and R. W. Schunk, Energization of ions in the auroral plasma by broadband waves: Generation of ion conics, *J. Geophys. Res., 89,* 5538, 1984.

Stix, T. H., *The Theory of Plasma Waves*, McGraw-Hill, New York, 1962.

Tanaka, Y., M. Hayakawa, D. Lagoutte, and F. Lefeuvre, Study of the middle latitude broadening of the spectrum of the waves emitted from ground, paper presented at International Conference on Results of the ARCAD-3 Project and of Recent Programmes in the Magnetospheric and Ionospheric Physics, Cent. Natl. d'Etud. Spatiales, Toulouse, France, May 1984.

Tanaka, Y., D. Lagoutte, M. Hayakawa, F. Lefeuvre, and S. Tajima, Spectral broadening of VLF transmitter signals and sideband structure observed on auroral 3 satellite at middle latitudes, *J. Geophys. Res., 92,* 7551, 1987.

Titova, E. E., V. I. Di, V. E. Yurov, O. M. Raspopov, V. Yu Trakhtengertz, F. Jiricek, and P. Triska, Interaction between VLF waves and turbulent ionosphere, *Geophys. Res. Lett., 11,* 323, 1984.

Tripathi, V. K., C. Grebogi, and C. S. Liu, Unified formalism of lower hybrid parametric instabilities in plasmas, *Phys. Fluids, 20,* 1525, 1977.

Wong, A. Y., G. J. Morales, D. Eggleston, J. Santoru, and R. Behnke, Rapid conversion of electromagnetic to electrostatic waves in the ionosphere, *Phys. Rev. Lett., 47,* 1340, 1981.

T. F. Bell and R. A. helliwell, STAR Laboratory, Department of Electrical Engineering/SEL, Stanford University, Stanford, CA 94305.

M. K. Hudson, Department of Physics and Astronomy, Dartmouth College, Hanover, NH 03755.

(Received June 14, 1990;
revised February 7, 1991;
accepted February 8, 1991.)