

HEATING AND IONIZATION OF THE LOWER IONOSPHERE BY LIGHTNING

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**Abstract.** Nighttime ionospheric electrons at 90-95 km altitude are found to be heated by a factor of 100-500 during the upward passage of short (<100  $\mu$ s) pulses of intense (5-20 V/m at 100 km distance) electromagnetic radiation from lightning. Heated electrons with average energy of 4-20 eV in turn produce secondary ionization, of up to 400  $\text{cm}^{-3}$  at  $\sim$ 95 km altitude in a single ionization cycle ( $\sim$ 3  $\mu$ s). With the time constant of heating being 5-10  $\mu$ s, a number of such ionization cycles can occur during a 50  $\mu$ s radiation pulse, leading to even higher density enhancements. This effect can account for previously reported observations of 'early' or 'fast' subionospheric VLF perturbations.

1. Introduction

New experimental evidence of direct upward coupling of lightning energy to the lower ionosphere has recently emerged in the form of 'early' or 'fast' perturbations of subionospheric VLF signals. The 'early' events occur within <50 ms following whistler-producing radio atmospherics [Armstrong, 1983] or cloud-to-ground lightning [Inan et al., 1988a] while the 'fast' events are unusually rapid (<50 ms) amplitude changes [Inan et al., 1988a,b]. The subionospheric VLF signatures of these events resemble those due to ionospheric disturbances produced in lightning-induced electron precipitation (LEP) events, which involve rapid (<2 s) onsets followed by slower (10-100 s) recoveries. However, the onsets of LEP events typically follow the causative lightning with a 0.3-1.5 s delay due to the wave and particle travel times to and from the high altitude interaction region, and exhibit an onset duration of 0.5-2.0 s representing the duration of the LEP burst. [Inan et al., 1990, and references therein].

Both the 'early' and the 'fast' aspects of the new class of events suggest a process in which the lightning energy is directly coupled to the lower ionosphere. Preliminary analysis indicates that the 'early' and 'fast' events occur less often (<20% of observed events) and that the event recoveries last somewhat longer than the typical LEP events. The signal amplitude changes observed in the two classes of events are similar, so that the ionospheric disturbances are likely to involve enhancements of ionization at D-region altitudes (85 km) of 10-100  $\text{cm}^{-3}$  as has recently been ascribed to LEP events [Inan and Carpenter, 1987].

The possibility of ionization of the lower ionosphere by lightning was considered earlier [Wilson, 1925; Bailey and Martyn, 1934] with generally inconclusive results principally because the peak electric fields were not well known [Healey, 1938]. Possible heating by VLF radiation from lightning was suggested in a recent observation of ionospheric heating by 28.5 kHz waves from a transmitter with 100 kW radiated

power. However, it was noted that an additional mechanism producing density enhancements would be needed to explain the observed 'early' or 'fast' effects [Inan, 1990]. In this paper we quantitatively study this possibility, using a generalization of the formulation of Inan [1990].

The electric field intensities in radiation from lightning discharges are well documented [Uman, 1987; p.111-120]. The mean electromagnetic power radiated by the first and subsequent strokes in a cloud-to-ground lightning flash is estimated to be respectively  $1-2 \times 10^{10}$ W and  $3-5 \times 10^9$ W at the times of observed peak radiation field intensities of 6-11 V/m and 4-6 V/m (normalized to 100 km distance) [Krider and Guo, 1983]. Typical first return stroke electric field waveforms can have durations of 50-100  $\mu$ s [Uman, 1987; p.113]. The energy in short radiation pulses from lightning is typically in the VLF range. The power spectral densities of return stroke waveforms are relatively flat in the range 2-6 kHz and decrease as  $f^{-1}$  above  $\sim$ 10 kHz [Uman, 1987; p.118]. In the analysis below, we use 50  $\mu$ s lightning pulses with peak normalized electric field intensities of 5-20 V/m.

2. Ionospheric Heating by VLF Radiation From Lightning

For a Maxwellian distribution of electrons (maintained during heating due to the collision rates) and an effective electron collision frequency ( $\nu$ ) for elastic collisions proportional to the square of velocity, the temperature of thermal electrons is proportional to  $\nu$ , which is given by [Maslin, 1974]:

$$\frac{d\nu}{dt} + G\nu(\nu - \nu_0) = \frac{2\nu_0 U_p}{3N_0 k T_0} \quad (1)$$

where  $N_0$  is the ambient electron density,  $G$  is a constant commonly taken at D-region altitudes to be  $G=1.3 \times 10^{-3}$  [Budden, 1985; p.393-398],  $\nu_0$  is the unperturbed value of  $\nu$ ,  $k$  is Boltzmann's constant,  $U_p$  is the wave power absorbed in the medium and  $T_0$  is the ambient (unperturbed) electron temperature. The effects of inelastic collisional processes are not included in (1) but are briefly discussed below.

For a harmonic wave the local value of  $U_p$  is proportional to  $\omega\chi P_d$ , where  $P_d$  is the wave power density ( $\text{W/m}^2$ ),  $\omega$  is the angular wave frequency and  $\chi$  is the imaginary part of the refractive index  $n=\mu-i\chi$ . For frequencies of 1-100 kHz,  $n=\mu-i\chi$  must be computed using the general magnetoionic expression since  $\omega$  is of the same order as  $\nu_0$  at 70-90 km altitudes [Inan, 1990]. In general,  $U_p$  can be found by integrating  $\omega\chi P_d$  over the frequency spectrum of the lightning pulse. For a 50  $\mu$ s rectangular pulse considered here, the frequency spectrum behaves as  $(1/f) \sin 50\pi \times 10^{-6} f$ , containing significant energy up to the first spectral null at 20 kHz. However, for our parameters,  $\omega\chi$  depends only weakly on  $\omega$  for 1-100 kHz due to relatively intense heating that drives the plasma into a collision-dominated regime. Thus, for the purpose of simplicity, we estimate  $U_p$  directly from  $\omega\chi P_d$  for 5 kHz.

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The general solution of (1) for a given  $U_p$  is given by:

$$\Delta\nu(t) = \left[ \sqrt{\frac{\nu_o^2}{4} + \frac{2\nu_o U_p}{3GN_o kT_o} - \frac{\nu_o}{2}} \frac{(1 - e^{-\gamma t})}{(1 + e^{-\gamma t})} \right] \quad (2)$$

where  $\gamma = \sqrt{8G\nu_o U_p (3kN_o T_o)^{-1} + G^2 \nu_o^2}$ . We evaluate  $\Delta\nu$  from (2) for a 50  $\mu$ s pulse (i.e.,  $t=50 \mu$ s). For the  $P_d$  values used here,  $\gamma^{-1}$  in the peak heating region (90-95 km) is  $\sim 10 \mu$ s so that the steady state temperatures are reached well before the termination of a 50  $\mu$ s pulse. The time constant  $\gamma^{-1}$  sharply increases with altitude above the heated region ( $U_p$  and  $\Delta\nu$  are smaller), reaching to  $\sim 1$  ms at 102 km altitude.

At each altitude, (2) can be used to determine  $\Delta\nu$  for a given  $U_p$ , estimated from  $P_d$  and  $\chi$  evaluated at 5 kHz. As the wave energy propagates upward, the absorption of the wave causes an increase in  $T$  with a resultant increase in  $\nu$ , leading to a modification of  $\chi$ , the rate at which the wave is absorbed. The quantities  $U_p$ ,  $\Delta T$  (temperature enhancement) and  $\Delta\nu$  are computed at steps of 0.1 km starting at a low altitude (e.g., 20 km). At the next step in altitude  $\Delta\nu$  from the previous step is added to  $\nu_o$  and  $\nu=\nu_o+\Delta\nu$  is used for computing  $\chi$  (and hence  $U_p$ ,  $\Delta T$ ,  $\Delta\nu$ ). The spatial divergence of  $P_d$  with altitude (in accordance with  $h^{-2}$ ) is separately accounted for.

The two magnetoionic components (i.e., ordinary and extraordinary) of the wave are treated separately in the evaluations of  $n=\mu-i\chi$  and in the evolution of  $P_d$  with altitude. However,  $U_p$  from each mode are added together in estimating  $\Delta T$  and the total  $\Delta\nu$ . Although the interference between the two waves would lead to a modulation of heating with altitude, more general modeling shows that this approximation is justified [Maslin, 1974]. The distinction between the two modes is even less important for the case of intense heating, leading to a collision-dominated plasma.

The formulation described above is appropriate as long as  $\nu$  is proportional to the average electron kinetic energy  $Q$ . However when  $Q$  exceeds the ionization potential of  $N_2$  (15.6 eV), the elastic collision cross section exhibits a broad maximum and a subsequent decrease with increasing  $Q$  [Rees, 1989; p.109-119, 271-274]. Thus  $\nu$  has a maximum value given by  $\nu_{max} \simeq (15.6 \text{ eV}) \nu_o / Q_o$ , where  $Q_o \simeq 0.04 \text{ eV}$  is the ambient value corresponding to  $T_o=300^\circ \text{ K}$ . To account for this effect, we assume that once  $\nu$  reaches  $\nu_{max}$  the heating of the plasma is governed by an equation similar to (1), but written in terms of  $Q$ . In this regime, with  $\nu=\nu_{max}$ , the additional heating is given by  $\Delta Q \simeq U_p (G\nu_{max} N_o)^{-1} + c$ , where  $c$  is a constant adjusted to ensure continuity of  $T$  with altitude.

In estimating the heating, we do not explicitly account for energy loss due to the excitation of rotational and vibrational states or the electronic excitation of  $N_2$ . The cross section for excitation of rotational and vibrational states is large only over the electron energy range  $1.5 \leq \mathcal{E} \leq 3.5 \text{ eV}$  [Rees, 1989; p.272], and the energy transfer rate over this range is small compared to the available wave power for wave intensities considered here. The cross section for electronic excitation is significant only for  $\mathcal{E} \geq 10 \text{ eV}$ , but over this range the electronic energy transfer rate is much larger than that due to elastic collisions. Thus a complete model of electron heating should include the effects of electronic excitation for  $\mathcal{E} \geq 10 \text{ eV}$ . However, due to the multiplicity of possible transitions their inclusion is well beyond the scope of the present paper. Since we neglect this effect our results should be viewed as an upper bound for the heating due to lightning.

### 3. Secondary Ionization by the Heated Electrons

The secondary ionization  $\Delta N$  produced by the heated electrons is estimated by assuming that a steady state is reached in which  $\nu$  (or  $Q$  or  $T$ ) is given by (2) and the electron density is given approximately by the ambient value.

The assumed isotropic Maxwellian velocity space distribution is completely specified by  $T$  and  $N_o$  at each altitude. If  $\mathcal{E}_i$  is the minimum kinetic energy necessary for ionization ( $\sim 15.6 \text{ eV}$  for  $N_2$ ), then all particles with  $\mathcal{E} > \mathcal{E}_i$  have a finite probability of producing one or more secondary electrons over their effective range. However, the average primary electron energy necessary to produce a single secondary electron is  $\sim 35 \text{ eV}$  and the electron ionization cross section of  $N_2$  drops sharply below 30 eV [Rees, 1989, p.271-274]. Thus for simplicity we assume that secondary production is probable only for  $\mathcal{E} > 35 \text{ eV}$ . In this case  $\Delta N$  is roughly proportional to the total energy of the heated primary electrons with  $\mathcal{E} > 35 \text{ eV}$  divided by 35 eV, i.e.,:

$$\frac{\Delta N}{N_o} = \frac{4kT}{(35\text{eV})\sqrt{\pi}} \int_{x_i}^{\infty} x^4 e^{-x^2} dx \quad (3)$$

where  $x^2 = (1/2)mV^2/kT$  and  $x_i = [(35 \text{ eV})/kT]^{1/2}$ .

At 90 km altitude the stopping distance of the primaries is  $\sim 10 \text{ m}$  and their velocity is  $> 3 \times 10^6 \text{ m/s}$ . Thus, the secondaries will be produced within 3  $\mu$ s after the steady state temperature is reached, and the  $\Delta N$  given by (3) represents only the secondaries produced during these 3  $\mu$ s. Since steady state is reached in  $\sim 10 \mu$ s at 90 km, a pulse of 50  $\mu$ s duration can potentially produce  $\sim 13$  ionization cycles of 3  $\mu$ s duration after steady state heating conditions are achieved. Noting that recombination times are many seconds at 90 km altitude, the final values of  $\Delta N$  (at 90 km) could in principle be an order of magnitude greater than that given by (3) as long as  $\Delta N < N_o$ . To properly treat the case where  $\Delta N > N_o$ , (1) must be modified to include ionizing collisions and the electron density in (1) must be taken to be  $N=N_o + \Delta N$ .

### 4. Results

Figures 1 and 2a show results for a 50  $\mu$ s lightning radiation pulse of intensity 10 V/m (range-normalized to 100 km), where we have considered the integrated effects of heating starting at 20 km for a lightning source at 10 km altitude. The real part of  $n$  ( $\mu$ ) and  $\alpha=\omega\chi/c$  computed for 5 kHz are given in Figure 1 for both the ordinary and extraordinary wave modes whereas the resultant  $\nu$ ,  $T$  and  $\Delta N$  are shown in Figure 2a. For the parameters considered, both  $\mu$  and  $\alpha$  depend weakly on  $\omega$  so that the 5 kHz values are representative of those for the 1-100 kHz components of the lightning pulse and are used here to estimate  $U_p$ . Typical exponential nighttime profiles ( $N_o$  proportional to  $e^{h/H}$ ) were assumed for  $N_o$  and  $\nu_o$  for altitudes  $< 100 \text{ km}$  with  $N_o \simeq 60 \text{ cm}^{-3}$  and  $\nu_o = 5.82 \times 10^5 \text{ s}^{-1}$  (corresponding to  $T_o \simeq 300^\circ \text{ K}$ ) at 85 km [Galejs, 1972, p.28-30]. The magnetic dip angle was assumed to be  $45^\circ$ , with the wave propagating vertically upward. The scale height  $H$  was taken to be  $H=2.86 \text{ km}$  for 60-100 km and  $H=8.21 \text{ km}$  for  $< 60 \text{ km}$ , with  $N_o=5000 \text{ cm}^{-3}$  between 100-120 km.

The effects of the geomagnetic field on the wave propagation are negligible up to 90 km altitude since  $\nu \gg \omega$  and  $\omega_H$  (electron gyrofrequency). Nevertheless, the exact solution for  $n=\mu-i\chi$  is used and the development of the two magneto-ionic modes with altitude is shown in Figure 1. For the ordinary

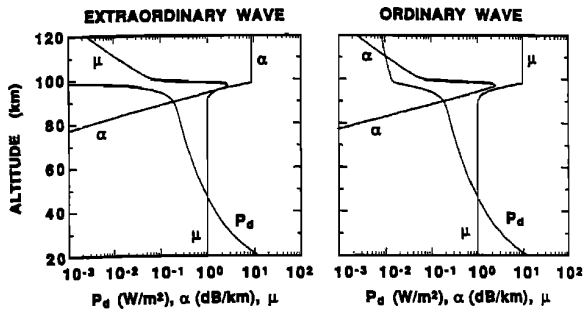


Fig. 1. Computed results for typical ambient conditions and a 50  $\mu$ s lightning pulse with 100-km range normalized intensity of 10 V/m. For both the whistler mode (ordinary) shown on the right hand panel and the extraordinary wave (left hand panel), the wave power density  $P_d(h)$ , attenuation constant  $\alpha$ , and refractive index  $\mu$  are shown as a function of altitude. The values shown are for 5 kHz but results are only weakly dependent on frequency over the range of 1-100 kHz. The steady decrease in  $P_d$  between 20 and 70 km altitude is mainly due to  $h^{-2}$  divergence based on an assumed source located at 10 km.

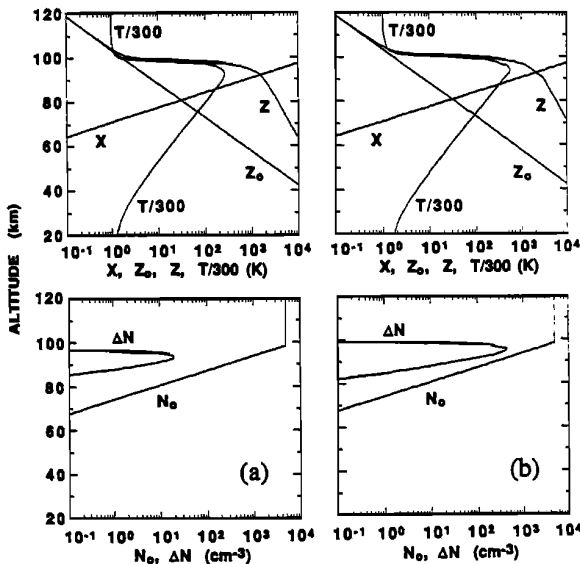


Fig. 2. (a) Results for 10 V/m 100-km range normalized intensity. The upper panel shows  $X(h)$ ,  $Z_o(h)$ , ( $X = \omega_p^2/\omega^2$ , with  $\omega_p$  and  $\omega$  being the electron plasma and wave frequencies respectively, and  $Z_o = \nu_o/\omega$ ), the electron temperature  $T$ , normalized to 300°K, and  $Z(h) = \nu/\omega$ . The lower panel shows the ambient electron density  $N_o$  and the  $\Delta N$  that would result from a single ionization cycle. (b) Results for 20 V/m 100-km range normalized intensity.

wave,  $\alpha$  peaks around 95 km, and  $\mu$  increases as the wave enters the ionosphere. The total reduction in  $P_d$  between 20 to 120 km is  $\sim 30$  dB primarily due to the  $h^{-2}$  spreading factor but also due to  $\sim 11$  dB of net absorption loss between 90-100 km. The extraordinary mode is evanescent ( $\mu \ll 1$  and  $\alpha \gg 1$ ) above 100 km.

Figure 2a shows that  $T$  increases with altitude up to  $\sim 92$  km, where  $\Delta T \simeq 280T_o$  and  $Q$  is  $\sim 11$  eV. The secondary

ionization (produced during the first 3  $\mu$ s after steady state), given by (3) shows (lower panel) a peak  $\Delta N = 20 \text{ cm}^{-3}$  at  $\sim 93$  km.

The resultant  $\nu$ ,  $T$  and  $\Delta N$  for a 100-km range normalized intensity of 20 V/m are shown in Figure 2b. In the range 90-96 km,  $Q$  is large enough so that  $\nu = \nu_{max}$  and the additional  $\Delta T$  is evaluated as discussed above. The peak  $\Delta T$  at  $\sim 93$  km is  $\sim 550T_o$ , and  $Q \simeq 22$  eV leading to a peak  $\Delta N = 400 \text{ cm}^{-3}$ .

Similar calculations (not shown) for 100-km intensity of 5 V/m give  $\Delta T \simeq 100T_o$  and  $\Delta N \sim 0.1 \text{ cm}^{-3}$  at  $\sim 90$  km.

### 5. Summary and Discussion

Substantial heating (by factors of 100-500) and density enhancements of up to  $400 \text{ cm}^{-3}$  may be generated at 90-95 km altitude by individual lightning discharges with 100-km range normalized intensities of 5-20 V/m. The heating and the subsequent ionization occur respectively in  $\sim 10 \mu$ s and  $\sim 3 \mu$ s, so that a number of ionization cycles could occur during a typical 50- $\mu$ s lightning pulse. While  $\Delta N$  for each lightning event can thus be considerably higher than that given by (3), a more general formulation is needed for quantitative evaluation. The heated region cools in  $\sim 10 \mu$ s after the termination of the lightning pulse, while the density enhancements decay over time scales of 10-100 s.

We note here that 5 V/m (at 100 km) represents the mean electric field intensity of lightning return stroke waveforms [Uman, 1987, p.115], so that  $\Delta T \simeq 100T_o$  can be expected to commonly occur above typical thunderstorm centers. However, the estimated ionization enhancement for 5 V/m is very small, so that subionospheric VLF 'fast' and 'early' perturbations would not be expected to be very common, consistent with experimental data. The 100-km intensities of  $> 10$  V/m and  $> 20$  V/m were found to represent 40% and 10%, respectively, of the observed flashes in 1980 data [Krider and Guo, 1983], so that perturbations as shown in Figure 2 should be less common. In this connection, we note that 'early' and 'fast' VLF events are found to occur less often than typical LEP event signatures.

The process described here accounts for most aspects of the reported 'early' and 'fast' subionospheric VLF responses [Inan et al., 1988a,b]. A computer-based model of earth-ionosphere waveguide propagation [Poulsen et al., 1990] gives amplitude and phase changes of  $\sim 0.04$  dB and  $\sim 0.5^\circ$  on a 25 kHz signal for a 150-km radius ionization patch and over a 4000 km path, for the profile in Figure 2b, and assuming no mode conversion within the disturbed region. Such amplitude and phase changes are within the range of observed values [Inan and Carpenter, 1987; Inan et al., 1988a,b]. That the peak  $\Delta N$  are estimated to be produced at higher (90-95 km instead of  $\sim 85$  km) altitudes is consistent with somewhat longer recovery times in 'early' and/or 'fast' events than in typical LEP events.

In addition to the effects due to individual lightning discharges that lead to the observed transient events, ionization enhancements may build up over thunderstorm centers due to successive flashes. Since the typical time between flashes can be of order of seconds [Uman, 1987, p.48-49], whereas the decay time of the ionization at 85-100 km is of order 10-100 seconds, substantial ionization 'bubbles' may be produced. The transverse extent of these bubbles would be at least as large as the transverse extent of the storm centers.

The net total absorption of the electromagnetic wave during its transionospheric propagation is shown here to be a nonlinear function of input field intensity. While the typical nighttime absorption for relatively low power densities (such as those from VLF transmitters) at 5-20 kHz is only ~2-4 dB [Helliwell, 1965, p.71], the net absorption for the case shown in Figure 1 is ~11 dB. Thus, in essence, the magnetosphere is 'screened' from the more intense component of the VLF energy from lightning. We note that the field intensities considered are comparable to values of up to 50 mV/m that have recently been measured with rockets at ~300 km altitude [Kelley et al., 1990]. For example, for the case of Figure 1, and assuming a typical refractive index of ~40 at 300 km, and the  $r^{-1}$  divergence of electric field, we estimate the field amplitude at 300 km to be ~100 mV/m. The corresponding value for the more common 5 V/m 100-km range normalized intensity (net absorption ~8.7 dB) is ~70 mV/m.

The intense heating produced by the lightning pulse may cause other effects that have yet to be evaluated. Factors of 100-500 increases in the electron temperature imply similar increases in the conductivity of the lower ionosphere. Modulation of the conductivity and thus existing currents in the medium by successive occurrence of lightning flashes may generate ULF waves with periods of many seconds. Furthermore, the electronic excitation of N<sub>2</sub> in inelastic collisions could possibly lead to airglow.

Other means of ionization of the ionosphere by lightning may be possible and need to be considered. For example, electrons accelerated upwards from thunderclouds by the large electric fields associated with lightning [Wilson, 1925] may produce ionization through bremsstrahlung X-rays [Cole et al., 1966]. Energetic X-ray fluxes (5 to > 110 keV) have been observed in thunderstorms [McCarthy and Parks, 1985].

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