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Theory of Electromagnetic Cyclotron Wave Growth in a Time-Varying Magnetoplasma

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The time-dependent growth rate for parallel propagating electromagnetic cyclotron waves is derived for a magnetoplasma which is characterized by a time dependent compressional perturbation superimposed on an equilibrium configuration. Such perturbations are commonly observed in the Earth's magnetosphere as a consequence of resonant field line oscillations, solar-wind disturbances, and other phenomena. The time dependencies of the magnetic field, thermal plasma density, energetic particle distribution function, and resonance condition are first related through a single dimensionless time parameter $b(t)$ using the ideal MHD assumption. For cases in which the particle distribution can be described by $F(\alpha, E) = f(E)\sin^{a(E)}\alpha$, the time dependent wave growth rate is then given by $\gamma \simeq \gamma_0(1+\Gamma)$ where γ_0 is the equilibrium growth rate and $\Gamma(b)$ is a function of the equilibrium parameters and the time parameter b. The term $|\Gamma|$ is generally small compared to 1, and the effect is a small modulation of the equilibrium growth rate by Γ. If the particle distribution is locally near marginal stability, however, $|\Gamma|$ is large compared to 1, and the growth rate modulation can be much larger than for a distribution which is not near marginal stability. The results suggest that particle populations which are near marginal stability may be strongly influenced by perturbations in the magnetic field and plasma. Marginally stable distributions may thus play an important role in magnetospheric dynamics as well as determination of radiation belt characteristics.

1. INTRODUCTION

Quasi-linear theories of wave-particle interactions have been widely applied to describe equilibrium wave and particle distributions throughout the magnetosphere [*Kennel and Engelmann*, 1966; *Kennel and Petschek*, 1966; *Lyons and Thorne*, 1973; *Etcheto et al.*, 1973; *Huang et al.*, 1983; *Church and Thorne*, 1983; *Cornilleau-Wehrlin et al.*, 1985; *Korth et al.*, 1985; *Schulz and Davidson*, 1988]. The equilibrium condition results from a balance between wave growth and particle scattering. An anisotropic particle distribution causes wave growth; the waves in turn scatter particles, reducing the anisotropy and the growth rate. The equilibrium description is valid as long as time scales for temporal changes in the magnetoplasma are long compared with the time scales of the interaction process. The limiting time scale is in general established by the particle lifetime, which is typically on the order of a few minutes [*Kennel and Petschek*, 1966; *Cornilleau-Wehrlin et al.*, 1985].

While equilibrium quasi-linear theories have been successfully applied to many problems, the magnetosphere is ultimately a dynamic medium. Magnetic field and plasma fluctuations, caused by phenomena such as solar wind driven disturbances, substorms, and local instabilities, are commonly observed on time scales ranging from less than one second to several minutes or longer. If the time scale of a given fluctuation is shorter than the time scales associated with local wave-particle interactions, the equilibrium description is not fully valid. Present quasi-linear theories thus may not accurately predict wave and particle distributions. This paper describes a modification to the electromagnetic cyclotron wave growth rate commonly used in quasi-linear theories to provide for inclusion of time-dependent perturbations in the magnetoplasma.

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Energetic particles which undergo resonance with waves of frequency ω satisfy the resonance condition

$$v_\parallel = \frac{\omega + s|\Omega_j|}{k} \qquad s = 0, \pm 1, \pm 2, \ldots \tag{1}$$

where $v_\parallel$ is the particle parallel velocity, k is the wave number, Ω_j is the gyrofrequency of species j, and s specifies the order of the resonance. For parallel propagating electron and ion cyclotron waves, only the $s = -1$ principal cyclotron resonance contributes to the interactions.

The growth rate for parallel electromagnetic cyclotron waves is given by [*Kennel and Petschek*, 1966; *Etcheto et al.*, 1973]

$$\gamma = \pi\Lambda(\omega) \sum_{\text{res } j} \frac{\omega_{pj}^2}{\omega^2} \eta_j(E_{Rj}) \left[A_j(E_{Rj}) - A_{j_c}\right] \tag{2}$$

where

$$\Lambda(\omega) = \left(\sum_{\text{all } j} \frac{\pm\Omega_j \omega_{pj}^2}{\omega^2(\omega \pm \Omega_j)^2} \right)^{-1} \tag{3}$$

is a multiplicative factor,

$$\eta_j(E_R) = 2\pi \left(\frac{\omega \pm \Omega_j}{k} \right)^3 \int_0^{\frac{\pi}{2}} \frac{\sin\alpha}{\cos^3\alpha} F_j(\alpha, E) d\alpha \tag{4}$$

is the fraction of the total particle distribution which satisfies the resonance condition,

$$A_j(E_{Rj}) = \int_0^{\frac{\pi}{2}} \frac{\sin^2\alpha}{\cos^4\alpha} \frac{\partial F_j(\alpha, E)}{\partial \alpha} d\alpha \Big/ 2 \int_0^{\frac{\pi}{2}} \frac{\sin\alpha}{\cos^3\alpha} F_j(\alpha, E) d\alpha \tag{5}$$

is the pitch angle anisotropy, and

$$A_{j_c} = \frac{-\omega}{\omega \pm \Omega_j} \tag{6}$$

is the critical anisotropy. The upper and lower signs correspond to the right (R) and left (L) wave modes respectively. The terms F_j and $\partial F_j/\partial\alpha$ are evaluated subject to $E_\parallel = E_{Rj}$, where E_{Rj} is the resonant energy, prior to integration. Note that the summation in equation (3) runs over all particle species while that in equation (2) includes only those species which satisfy the resonance condition for the particular wave mode. The distribution function F_j is specified in terms of the particle energy E and the pitch angle α for the particular species. For $\Omega_i \ll \omega < |\Omega_e|$, the R mode corresponds to electron cyclotron or whistler waves. For $\omega < \Omega_i$, the L mode corresponds to ion cyclotron waves. At low frequencies ($\omega \ll \Omega_i$), the R mode becomes the magnetosonic mode, and the L mode becomes the Alfvén mode.

For the two cyclotron modes, the plasma is locally unstable when the anisotropy A_j is greater than the critical anisotropy A_{j_c}, leading to wave growth [*Kennel and Petschek*, 1966]. Enhanced wave growth causes increased particle scattering and thus modification of the particle distribution; this feedback process is generally described through use of a diffusion equation. The coupling of the growth and diffusion processes constitutes the quasi-linear description of wave-particle interactions. In a bounded plasma such as the magnetosphere, parallel propagating waves will be unstable in a global sense only if the path-integrated gain is greater than the integrated loss along the path and at the reflection points.

A few attempts have been made at applying the Kennel-Petschek quasi-linear theory to situations in which the ambient magnetic field amplitude has a time-varying component. *Coroniti and Kennel* [1970] addressed the role of hydromagnetic waves in controlling electron precipitation by introducing a small sinusoidal magnetic field fluctuation into the Kennel-Petschek theory. They found that such fluctuations can strongly modulate the whistler mode wave amplitudes, leading to observable precipitation pulsations. *Perona* [1972], using a similar approach, added a perturbation which was linear in time in an effort to explain observed precipitation enhancements during SC.

In this paper, the effect of a time-dependent perturbation in the magnetoplasma on the wave and particle populations is investigated using the Kennel-Petschek approach. Perturbations in the cold plasma density, energetic particle distribution, and resonance condition are calculated based on the ideal MHD assumption given an arbitrary compressional magnetic field perturbation. The necessary modifications for extending the work to include nonparallel propagation and changes in the magnetic field direction are also discussed. The result of the derivation, an analytical form of the time-dependent cyclotron growth rate for constant frequency waves, provides significant insight into how wave and particle populations respond to a time-dependent magnetoplasma.

2. APPROACH

Consider a general magnetoplasma which can be described in terms of a time-dependent perturbation superimposed on an equilibrium configuration. The magnetoplasma is characterized by a set of parameters (e.g., magnetic field strength, thermal plasma density) whose time dependence can be related through a single time-dependent variable given the ideal MHD assumption. Let the magnetic field amplitude be described by

$$B(t) = B_0(1 + b) \qquad (7)$$

where B_0 is the equilibrium amplitude and $b(t)$ is a dimensionless parameter containing the time dependence. The perturbation is assumed to be entirely compressional for this analysis; the effect of changes in field direction will be discussed in section 9. The thermal plasma and energetic plasma may also be described in terms of their characteristics in the equilibrium environment multiplied by functions of b. It is thus possible to write the time-dependent growth rate as $\gamma = \gamma_0(1 + \Gamma)$, where γ_0 is the equilibrium growth rate and $\Gamma(b)$ is a function of the plasma and magnetic field parameters in the equilibrium environment and the time parameter b. The goal of this work is to determine the function Γ.

Throughout the derivation, notation is critical for distinguishing between the equilibrium and time-dependent environments. The notation used is illustrated in Figure 1. The distribution function and corresponding variables associated with the equilibrium environment are denoted with a subscript 0 while those in the time-dependent environment have no subscript. Additional notation is required to distinguish between resonance conditions satisfied in the equilibrium and time-dependent environments. Variables corresponding to particles which satisfy the resonance condition in the equilibrium environment are denoted with overbars (e.g., $\overline{E_0}$); those which satisfy the resonance condition in the time-dependent environment are denoted with double overbars (e.g., $\overline{\overline{E_0}}$).

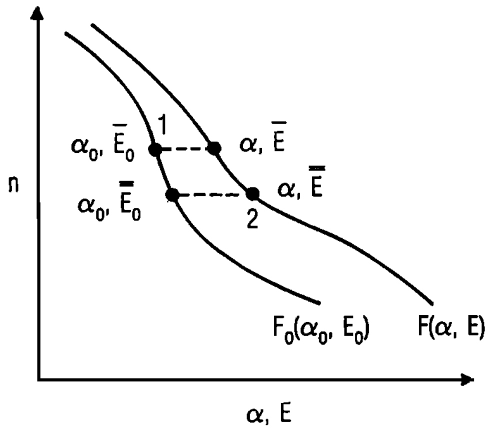

Fig. 1. Schematic representation of the variables used to describe the particle distribution functions. The curves represent the surfaces in (α, E) space which characterize the distributions in the equilibrium (F_0) and time-dependent (F) environments. The points labeled 1 and 2 correspond to the portions of the distributions which satisfy the resonance condition.

We first show that the time-dependent expressions for the particle pitch angle α, energy E, and distribution function $F_j(\alpha, E)$ can be written in terms of the equilibrium quantities α_0, E_0, $F_{j_0}(\alpha_0, E_0)$, and the time parameter b. These relations are then used to derive an expression for $F_j(\alpha, \overline{\overline{E}})$, the distribution function for resonant particles in the time-dependent formulation, in terms of the quantities $F_{j_0}(\alpha_0, \overline{E_0})$ and b (see Figure 1). By assuming that $F_{j_0}(\alpha_0, \overline{E_0}) \simeq F_{j_0}(\alpha_0, \overline{\overline{E_0}})$, $F_{j_0}(\alpha_0, \overline{\overline{E_0}})$ can be related to $F_{j_0}(\alpha_0, \overline{E_0})$ through a Taylor expansion. A general form of the distribution function is then assumed to facilitate the evaluation of several integrals; the parameters characterizing the function do not appear in the final solution. Finally, an expression for the time-dependent growth rate is obtained which is written as a function of equilibrium parameters and the time parameter b.

The calculation is done at constant wave frequency rather than using a fixed value of normalized frequency $\omega/|\Omega_j|$ in order to maintain the identity of a wave packet as Ω_j changes with time. For a given wave frequency ω, the resonance condition will in general be satisfied by physically different particles for different values of b.

3. TIME-DEPENDENT FORM OF PHYSICAL PARAMETERS

Using the time-dependent magnetic field, the gyrofrequency and the plasma frequency for each species are

$$\Omega_j = \Omega_{j0}\frac{B(t)}{B_0} = \Omega_{j0}(1+b) \tag{8}$$

$$\omega_{pj} = \omega_{pj0}(1+b)^{\frac{1}{2}} \tag{9}$$

where the frozen-in flux condition has been used based on the assumption of ideal MHD.

For this problem, we will consider only the contribution of betatron acceleration in calculating changes in the particle distribution (a rigorous discussion of the interaction between energetic particles and hydromagnetic waves has been given by *Tamao* [1984]). Assuming conservation of the first adiabatic invariant and ignoring the change in $v_\parallel$, the pitch angle and the total particle energy can be written

$$\tan^2\alpha = \frac{{v_\perp}^2}{{v_\parallel}^2} = \tan^2\alpha_0(1+b) \tag{10}$$

$$E = \frac{1}{2}m\left({v_{\parallel 0}}^2 + {v_{\perp 0}}^2(1+b)\right) = E_0(1+b\sin^2\alpha_0) \tag{11}$$

The refractive index for parallel propagating cyclotron waves in a cold multicomponent magnetoplasma is given by [*Stix*, 1962]

$$n^2 = 1 - \sum_{\text{all } j}\frac{\omega_{pj}^2}{\omega(\omega \pm \Omega_j)} = {n_0}^2(1+N) \tag{12}$$

where

$$N(b) = \left(\sum_{\text{all } j}\frac{{\omega_{pj0}}^2}{\omega\Omega_{j0}}\lambda_j\frac{(\pm\lambda_j - 1)b}{1 \pm \lambda_j b}\right)\left(1 - \sum_{\text{all } j}\frac{{\omega_{pj0}}^2}{\omega\Omega_{j0}}\lambda_j\right)^{-1} \tag{13}$$

and we have defined

$$\lambda_j = \frac{\Omega_{j0}}{\omega \pm \Omega_{j0}} \tag{14}$$

Using this form of the refractive index, the wave number can be written

$$k = \frac{\omega n}{c} = k_0(1+N)^{\frac{1}{2}} \tag{15}$$

and the resonant energy is

$$\overline{\overline{E_{Rj}}} = \overline{E_{Rj0}}(1+V_j)^2 \tag{16}$$

where

$$V_j(b) = \frac{1 - (1+N)^{\frac{1}{2}} \pm b\lambda_j}{(1+N)^{\frac{1}{2}}} \tag{17}$$

The distribution functions are related through the Liouville theorem by

$$F_j(\alpha, E) = F_{j0}(\alpha_0, E_0) \tag{18}$$

where α and E are calculated from α_0 and E_0 using equations (10) and (11).

4. The Time-Dependent Distribution for Resonant Particles

For resonant particles, the time-dependent distribution function for each species can be written in terms of the equilibrium distribution as

$$F_j(\alpha, \overline{\overline{E}}) = F_{j0}(\alpha_0, \overline{\overline{E_0}}) \tag{19}$$

Expanding F_{j0} in a Taylor series about $E_0 = \overline{E_0}$ gives

$$F_{j0}(\alpha_0, E_0) \simeq F_{j0}(\alpha_0, \overline{E_0}) + (E_0 - \overline{E_0})\left.\frac{\partial F_{j0}}{\partial E_0}\right|_{E_0=\overline{E_0}} \tag{20}$$

to first order in $(E_0 - \overline{E_0})$. The convergence and accuracy requirements for this approximation are discussed in section 6. The derivative $\partial F_j/\partial\alpha$ can be written in terms of equilibrium quantities by expanding in the form

$$\frac{\partial F_j}{\partial\alpha} = \frac{\partial F_{j0}}{\partial\alpha_0}\frac{\partial\alpha_0}{\partial\alpha} + \frac{\partial F_{j0}}{\partial E_0}\frac{\partial E_0}{\partial\alpha} \tag{21}$$

Rewriting time-dependent quantities in terms of equilibrium equivalents and introducing the Taylor expansion form of F_{j0}, we have

$$\frac{\partial F_j}{\partial\alpha} \simeq \frac{1+b\sin^2\alpha_0}{(1+b)^{\frac{1}{2}}}\left[\frac{\partial F_{j0}}{\partial\alpha_0} + (E_0 - \overline{E_0})\frac{\partial^2 F_{j0}}{\partial\alpha_0\partial E_0}\right] - \frac{2bE_0\sin\alpha_0\cos\alpha_0}{(1+b)^{\frac{1}{2}}}\left[\frac{\partial F_{j0}}{\partial E_0} + (E_0 - \overline{E_0})\frac{\partial^2 F_{j0}}{\partial {E_0}^2}\right] \tag{22}$$

where the derivatives are evaluated at $E_0 = \overline{E_0}$.

For resonant particles in the time-dependent formulation, we evaluate F and $\partial F_j/\partial\alpha$ at $E_0 = \overline{\overline{E_0}}$ using

$$(E_0 - \overline{E_0}) \Rightarrow (\overline{\overline{E_0}} - \overline{E_0}) \simeq K_j\frac{\overline{E_{Rj0}}}{\cos^2\alpha_0} \tag{23}$$

where

$$K_j = (1+V_j)^2 - 1 \tag{24}$$

We thus have the desired forms of the time-dependent distribution function and its derivative,

$$\overline{\overline{F_j}} \simeq \overline{F_{j0}} + K_j\frac{\overline{E_{Rj0}}}{\cos^2\alpha_0}\overline{\frac{\partial F_{j0}}{\partial E_0}} \tag{25}$$

$$\overline{\overline{\frac{\partial F_j}{\partial\alpha}}} \simeq \frac{1+b\sin^2\alpha_0}{(1+b)^{\frac{1}{2}}}\left[\overline{\frac{\partial F_{j0}}{\partial\alpha_0}} + K_j\frac{\overline{E_{Rj0}}}{\cos^2\alpha_0}\overline{\frac{\partial^2 F_{j0}}{\partial\alpha_0\partial E_0}}\right] - \frac{2b\overline{E_{Rj0}}}{(1+b)^{\frac{1}{2}}}\frac{\sin\alpha_0}{\cos\alpha_0}\left[\overline{\frac{\partial F_{j0}}{\partial E_0}} + K_j\frac{\overline{E_{Rj0}}}{\cos^2\alpha_0}\overline{\frac{\partial^2 F_{j0}}{\partial {E_0}^2}}\right] \tag{26}$$

5. The Time Dependent Growth Rate

The anisotropy function (equation (5)) for particles with parallel energies given by $E_{Rj} = \overline{\overline{E_{Rj}}}$ can now be written as

$$\overline{\overline{A_j}} \simeq \int_0^{\frac{\pi}{2}}\left\{\left[\frac{\sin^2\alpha_0}{\cos^4\alpha_0} + b\frac{\sin^4\alpha_0}{\cos^4\alpha_0}\right]\overline{\frac{\partial F_{j0}}{\partial\alpha_0}} - \left[2b\overline{E_{Rj0}}\frac{\sin^3\alpha_0}{\cos^5\alpha_0}\right]\overline{\frac{\partial F_{j0}}{\partial\alpha_0}} + \left[K_j\overline{E_{Rj0}}\frac{\sin^2\alpha_0}{\cos^6\alpha_0} + bK_j\overline{E_{Rj0}}\frac{\sin^4\alpha_0}{\cos^6\alpha_0}\right]\overline{\frac{\partial^2 F_{j0}}{\partial\alpha_0 E_0}} - \left[2bK_j\overline{E_{Rj0}}^2\frac{\sin^3\alpha_0}{\cos^7\alpha_0}\right]\overline{\frac{\partial^2 F_{j0}}{\partial E_0^2}}\right\}d\alpha_0$$

$$\div 2\int_0^{\frac{\pi}{2}}\left\{\frac{\sin\alpha_0}{\cos^3\alpha_0}\overline{F_{j0}} + K_j\overline{E_{Rj0}}\frac{\sin\alpha_0}{\cos^5\alpha_0}\overline{\frac{\partial F_{j0}}{\partial E_0}}\right\}d\alpha_0 \tag{27}$$

In order to evaluate the integrals, we introduce a distribution function of the form

$$F_{j0}(\alpha_0, E_0) = f_{j0}(E_0)\sin^{a_{j0}(E_0)}\alpha_0 \tag{28}$$

This distribution is sufficiently general to allow satisfactory modeling of distributions such as those measured by *Cornilleau-Wehrlin et al.* [1985] while providing the explicit pitch angle dependence which is needed to obtain an analytical result. The explicit pitch angle dependence allows the partial derivatives of f_{j0} and a_0 with respect to E_0 and α_0 to be changed to full derivatives with respect to α_0 using the chain rule. The integrals can then be evaluated using integration by parts. After somewhat lengthy algebra, the anisotropy $A_j(\overline{\overline{E_{Rj}}})$ (equation (5)) becomes

$$\overline{\overline{A_j}} \simeq \overline{A_j}\left[1 + \frac{(2b + K_j(2\overline{A_{j1}} - 3b - 1))\overline{A_j}}{2(1 - K_j - K_j\overline{A_{j1}})A_j} + \frac{-K_j(1+2b)\overline{A_{j1}} - K_j(1+b)\overline{A_{j2}} + b(2+K_j)}{2(1 - K_j - K_j\overline{A_{j1}})A_j}\right] \tag{29}$$

the resonant flux $\eta_j(\overline{\overline{E_{Rj}}})$ (equation (4)) is

$$\overline{\overline{\eta_j}} \simeq \overline{\eta_j}(1+b)(1+K_j)^{\frac{3}{2}}\left[1 - K_j - K_j\overline{A_{j1}}\right] \tag{30}$$

and the critical anisotropy A_{jc} (equation (6)) is

$$A_{jc} = \frac{-\omega}{\omega \pm \Omega_{j0}(1+b)} \simeq A_{jc0}(1 - L_j) \tag{31}$$

where the anisotropy terms A_{j1} and A_{j2} are defined as

$$A_{j1} = \int_0^{\frac{\pi}{2}} \frac{1}{\cos^2\alpha}\frac{\partial F_j}{\partial \alpha}d\alpha_0 \Big/ 2\int_0^{\frac{\pi}{2}} \frac{\sin\alpha}{\cos^3\alpha}F_j\,d\alpha \tag{32}$$

$$A_{j2} = \int_0^{\frac{\pi}{2}} \frac{\sin\alpha}{\cos^3\alpha}\frac{\partial^2 F_j}{\partial \alpha^2}d\alpha \Big/ 2\int_0^{\frac{\pi}{2}} \frac{\sin\alpha}{\cos^3\alpha}F_j\,d\alpha \tag{33}$$

and

$$L_j = b\frac{\pm\lambda_j}{1 \pm b\lambda_j} \tag{34}$$

The multiplicative factor $\Lambda(\omega)$ (equation (3)) is

$$\Lambda = \Lambda_0(1 + R) \tag{35}$$

where

$$R = \left(\sum_{\text{all } j} \frac{\pm\Omega_{j0}\omega_{pj0}^2}{\omega^2(\omega \pm \Omega_{j0})^2}\frac{(1 \pm \lambda_j b)^2 - (1+b)^2}{(1 \pm \lambda_j b)^2}\right) \bullet \left(\sum_{\text{all } j} \frac{\pm\Omega_{j0}\omega_{pj0}^2}{\omega^2(\omega \pm \Omega_{j0})^2}\frac{(1+b)^2}{(1 \pm \lambda_j b)^2}\right)^{-1} \tag{36}$$

The growth rate (equation (2)) can now be written

$$\gamma \simeq \gamma_0(1 + \Gamma) \tag{37}$$

where the modulation index $\Gamma(b)$ is

$$\Gamma(b) = \sum_{\text{res } j} (1+b)^2(1+R)(1+K_j)^{\frac{3}{2}}(1+G_j) - 1 \tag{38}$$

and

$$G_j = \frac{\frac{1}{2}(-3K_j + 2b - 3bK_j)\overline{A_j} + (L_j + K_j - K_jL_j)A_{jc0}}{\overline{A_j} - A_{jc0}} + \frac{-\frac{1}{2}K_j(1+b)(\overline{A_{j1}} + \overline{A_{j2}})}{\overline{A_j} - A_{jc0}} + \frac{-\frac{1}{2}bK_j\overline{A_{j1}} - K_j(L_j - 1)\overline{A_{j1}}A_{jc0} + \frac{1}{2}b(2 + K_j)}{\overline{A_j} - A_{jc0}} \tag{39}$$

Note that f_{j0} and $a_{j0}(E)$ do not appear explicitly in these equations. Moreover, since the growth rate equation is linear in F_j, any linear combination of functions of the type used in equation (28) is acceptable for describing the particle distribution. No small amplitude assumptions have been used other than those directly implied by the Taylor series convergence and accuracy conditions; the distribution function energy dependence must be specified in order to identify these assumptions.

In general, $|\Gamma|$ is small compared with 1, and the resulting growth rate is the equilibrium growth rate modulated by the term Γ. If, however, $|\Gamma|$ is large compared with 1, the growth rate can be approximated by

$$\gamma \simeq \gamma_0\Gamma \tag{40}$$

and is directly proportional to b. This occurs when $R, K_j \gg 1$ or when $G_j \gg 1$. The first condition corresponds to the appearance of finite growth rates at the equilibrium cyclotron ($\omega = \Omega_j$) and lower hybrid ($\omega = \Omega_{LH}$) frequencies for which the equilibrium growth rates are zero. The second condition, which is a consequence of $\overline{A_j} \simeq A_{jc0}$, implies that the time-dependent growth rate may be much larger than the equilibrium growth rate at a fixed frequency when the anisotropy is nearly equal to the critical anisotropy. In the limit $|\overline{A_j} - A_{jc0}| \to 0$, however, the growth rate remains finite because the term $\overline{A_j} - A_{jc0}$ in γ_0 cancels the denominator of G_j. This result indicates that the plasma can become unstable even when the equilibrium anisotropy $\overline{A_j}$ is less than the critical anisotropy A_{jc0}.

6. Validity And Accuracy

The validity of the analysis and the accuracy of the result are determined by the accuracy of the Taylor series approximation. The truncation error for the Taylor series for species j is

$$R_n = \frac{1}{n!}\int_{E_0}^{E} F^{(n+1)}(x)(E - x)^n\,dx \tag{41}$$

where n is the order of the last term retained. For the $n = 1$ approximation used here, we require

$$|R_1| \ll \left|\left.\frac{\partial F(x)}{\partial E}\right|_{E=E_0}(E - E_0)\right| \tag{42}$$

in order to get an accurate result.

For a single-species particle distribution with a power law energy dependence given by E^{-m}, the accuracy condition has the particularly simple form

$$P = \left|\frac{1 - mb - (1+b)^{-m}}{m(1+b)}\right| \ll 1 \tag{43}$$

A plot of the function P for different values of m is shown in Figure 2. The plot indicates that accurate approximations can be

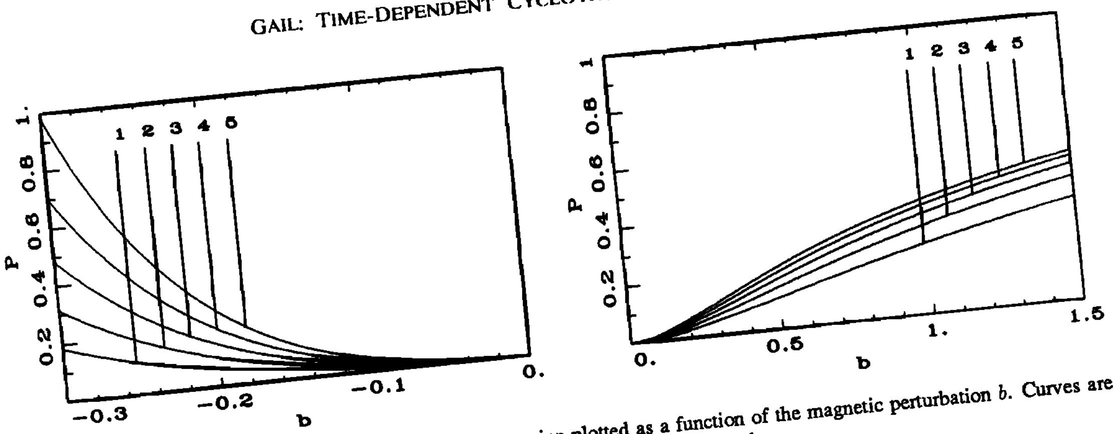

Fig. 2. The accuracy criterion for the Taylor series expansion plotted as a function of the magnetic perturbation b. Curves are given for five values of the spectral index m. The accuracy criterion requires $P \ll 1$.

made for relatively large values of $|b|$. For a $m = 2$ distribution, for example, P is less than 0.2 over the range $-0.3 < b < 0.6$. The theory is thus not strictly limited to small magnetoplasma perturbations.

7. Small Value Approximation

The growth rate simplifies considerably under the approximation $|\lambda_j b| \ll 1$. This condition is not very restrictive since most perturbations of interest have magnitudes much smaller than the equilibrium field. Nevertheless, the unsimplified result given by equation (37) can be used when the small value condition is not satisfied. Figure 3 shows $|\lambda_j b|$ plotted versus b for five values of the normalized wave frequency. At a frequency of $\omega/|\Omega_{j0}| = 0.5$, for example, a reasonable approximation can be made for magnetic field perturbations of several tens of percent of the total field assuming that the energy dependence of the distribution function is such that the Taylor series condition is also satisfied.

Under these conditions, the modulation index $\Gamma(b)$ can be written as

$$\Gamma(b) \simeq \beta b \tag{44}$$

where

$$\beta = \sum_{\text{res } j} \frac{(2\lambda_j + 1)\overline{A_j} - \frac{1}{2}(5\lambda_j - 1)A_{j\,c0}}{\overline{A_j} - A_{j\,c0}} + \frac{-\frac{1}{2}(3\lambda_j - 1)(\overline{A_{j\,1}} + \overline{A_{j\,2}} - 2A_{j\,c0}\overline{A_{j\,1}}) + 1}{\overline{A_j} - A_{j\,c0}} \tag{45}$$

for R mode waves and

$$\beta = \sum_{\text{res } j} \frac{\left(3 - \frac{\lambda_j^2(1+\lambda_j)}{(1+\lambda_j^2)}\right)\overline{A_j} - \left(2 + \frac{\lambda_j^2(1+\lambda_j)}{(1+\lambda_j^2)}\right)A_{j\,c0}}{\overline{A_j} - A_{j\,c0}} + \frac{-b\lambda_j\,(\overline{A_{j\,1}} + \overline{A_{j\,2}} - 2A_{j\,c0}\overline{A_{j\,1}}) + 1}{\overline{A_j} - A_{j\,c0}} \tag{46}$$

for L mode waves. At low frequencies ($\omega \ll \Omega_j$), $\lambda_j \simeq 1$ and β can be simplified further. Note that β is a function of the equilibrium variables only and contains no time dependence.

8. Numerical Example: A Simple Distribution Function

The properties of the time-dependent growth rate can be illustrated by choosing a simple form for the distribution function and calculating the growth rate. Consider a distribution of electrons specified by

$$F_{j\,0}(\alpha_0, E_0) = E_0^{-m} \sin^a \alpha_0 \tag{47}$$

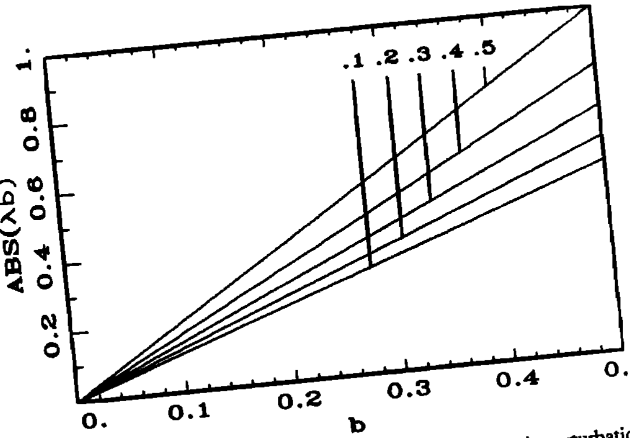

Fig. 3. The term $|\lambda_j b|$ plotted as a function of the magnetic perturbation b. Curves are given for five values of the normalized frequency $\omega/|\Omega_{j0}|$. The validity of the small value approximation requires $|\lambda_j b| \ll 1$.

The accuracy condition for the Taylor series approximation corresponding to this distribution is discussed in section 6. The parameter a is chosen to be a constant for this simple example, but it should be remembered that doing so implies that the wave spectrum has no frequency dependence. For this distribution, β is given by

$$\beta = \frac{1}{2}(13\lambda_j - 1) - (3\lambda_j - 1)m + \frac{(\lambda_j + 1)A_{j\,c0} + 1}{\overline{A_j} - A_{j\,c0}} \tag{48}$$

Figure 4 shows plots of β versus $\overline{A_j} - A_{j\,c0}$ for four values of the spectral index m with curves parametric in normalized frequency $\omega/|\Omega_{j0}|$. The curves for different values of m are qualitatively similar. For large values of $|\overline{A_j} - A_{j\,c0}|$, β is approximately constant with an asymptotic limit of $\beta = \frac{1}{2}(13\lambda_j - 1) - (3\lambda_j - 1)m$. For $\overline{A_j} \simeq A_{j\,c0}$, however, $|\beta|$ becomes very large due to the singularity at $\overline{A_j} = A_{j\,c0}$.

The effect of this singularity on the growth rate is illustrated in Figure 5 for $m = 2$. The figure shows the time-dependent growth

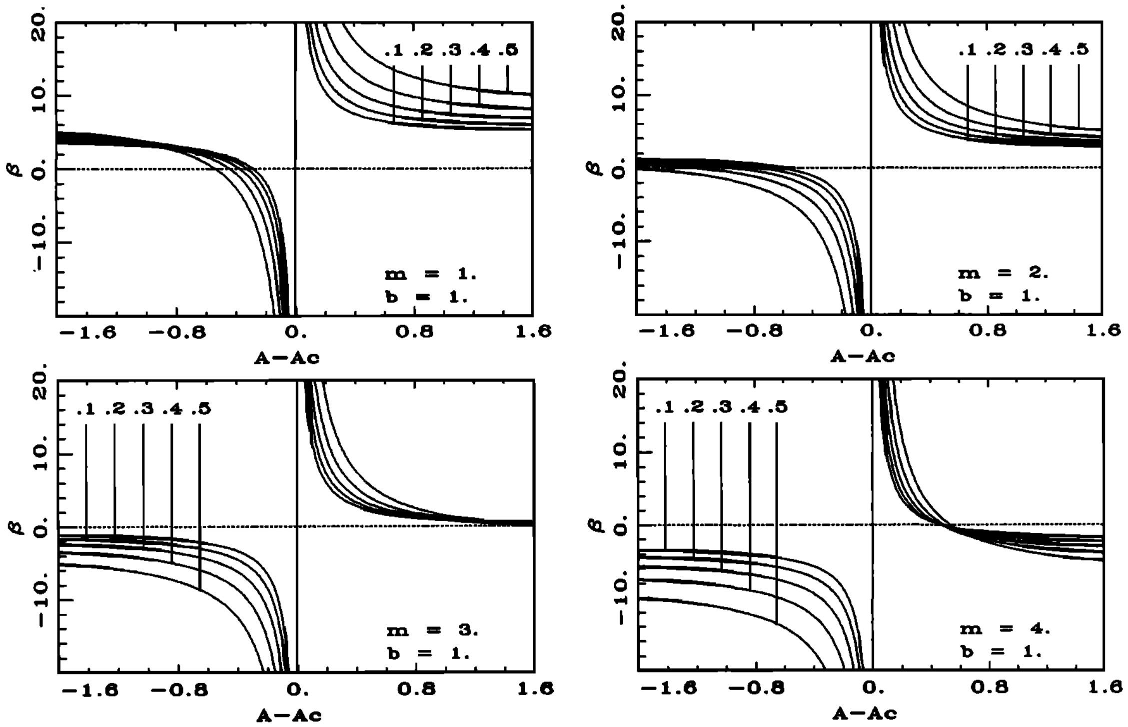

Fig. 4. The modulation coefficient β plotted as a function of the difference between the anisotropy $\overline{A_j}$ and the critical anisotropy $A_{j_{c0}}$ for four values of m. The curves are parametric in normalized frequency $\omega/|\Omega_{j0}|$.

rate γ (given in units of $|\Omega_{j_0}|\overline{\eta_j}$; these units were chosen because they are independent of frequency and equilibrium growth rate and are thus the same for each of the plots) and the growth rate modulation index βb plotted versus $\overline{A_j} - A_{j_{c0}}$ for magnetic field perturbations given by b =0.1, 0.01, −0.01, and −0.1. The curves in each plot are parametric in normalized frequency $\omega/|\Omega_{j_0}|$. The dashed lines correspond to the equilibrium growth rates.

A number of differences between the time-dependent and equilibrium growth processes can be ascertained from these figures. Most notably, positive growth rates now occur for equilibrium anisotropies which are less than the critical anisotropy. This effect is more pronounced for larger values of m and larger perturbations b. For large values of $|\overline{A_j} - A_{j_{c0}}|$, the modulation index is approximately 0, corresponding to a small modulation of the equilibrium growth rate. As the difference between $\overline{A_j}$ and $A_{j_{c0}}$ is reduced to 0, the modulation index approaches infinity. For example, for $m = 2$, $b = 0.1$, and $\omega/|\Omega_{j_0}| = 0.5$, the modulation index for $\overline{A_j} - A_{j_{c0}} = 0.01$ is 8 times that for $\overline{A_j} - A_{j_{c0}} = 0.1$. For fixed values of the equilibrium growth rate, much larger fluctuations in growth rate can occur for a marginally stable distribution than for an unstable distribution.

9. Discussion

9.1 *Frequency and Time Scale Limitations*

The validity of this analysis is restricted by limitations on the perturbation frequency spectrum and duration. The frequency limitation, specified by $\omega_b \ll |\Omega_j|$, is a consequence of the ideal MHD assumption. In a practical sense, this limitation is not very restrictive since higher frequencies correspond to ion and electron cyclotron waves which have amplitudes that are generally negligible compared with the ambient field. The duration limitation is implied by the assumption that modification of the distribution function by the enhanced wave scattering is negligible. To properly calculate the time evolution of the wave amplitude and diffusion coefficient over time scales long enough for the distribution to be modified, the time-dependent growth rate (equation (37)) must be solved simultaneously with a similarly modified form of the diffusion equation as specified in the quasi-linear formalism. This approach is necessary to provide a complete solution of the problem.

9.2 *Nonparallel Propagation and Field Direction Changes*

The limitation to parallel propagating waves is very helpful because it allows for an analytical representation of the growth rate. Parallel waves are also of particular interest in magnetospheric physics. Nevertheless, modification of the analysis to include nonparallel propagation is straightforward. For a wave normal angle θ, the refractive index (equation (12)) is calculated using the wave normal dependent form of the refractive index equation [e.g., *Stix*, 1962], and the resonant energy (equation (16)) is multiplied by $\sin^2|\theta|$. The introduction of nonparallel propagation also requires that the growth rate equation be modified to include the $s \neq -1$ resonances, although in many cases of interest the $s = -1$ term still dominates. The effect of a magnetic perturbation which includes a field direction change can be interpreted as a compressional perturbation accompanied by a change in the wave normal angle θ.

The introduction of a nonzero wave normal angle, however, implies that the wave will not remain associated with a particular field line unless it is guided by ducting. In order to determine the wave normal angle as well as the other quantities in the growth rate equation as a function of time, explicit knowledge of the spatial dependence of the magnetic field and plasma is required. The

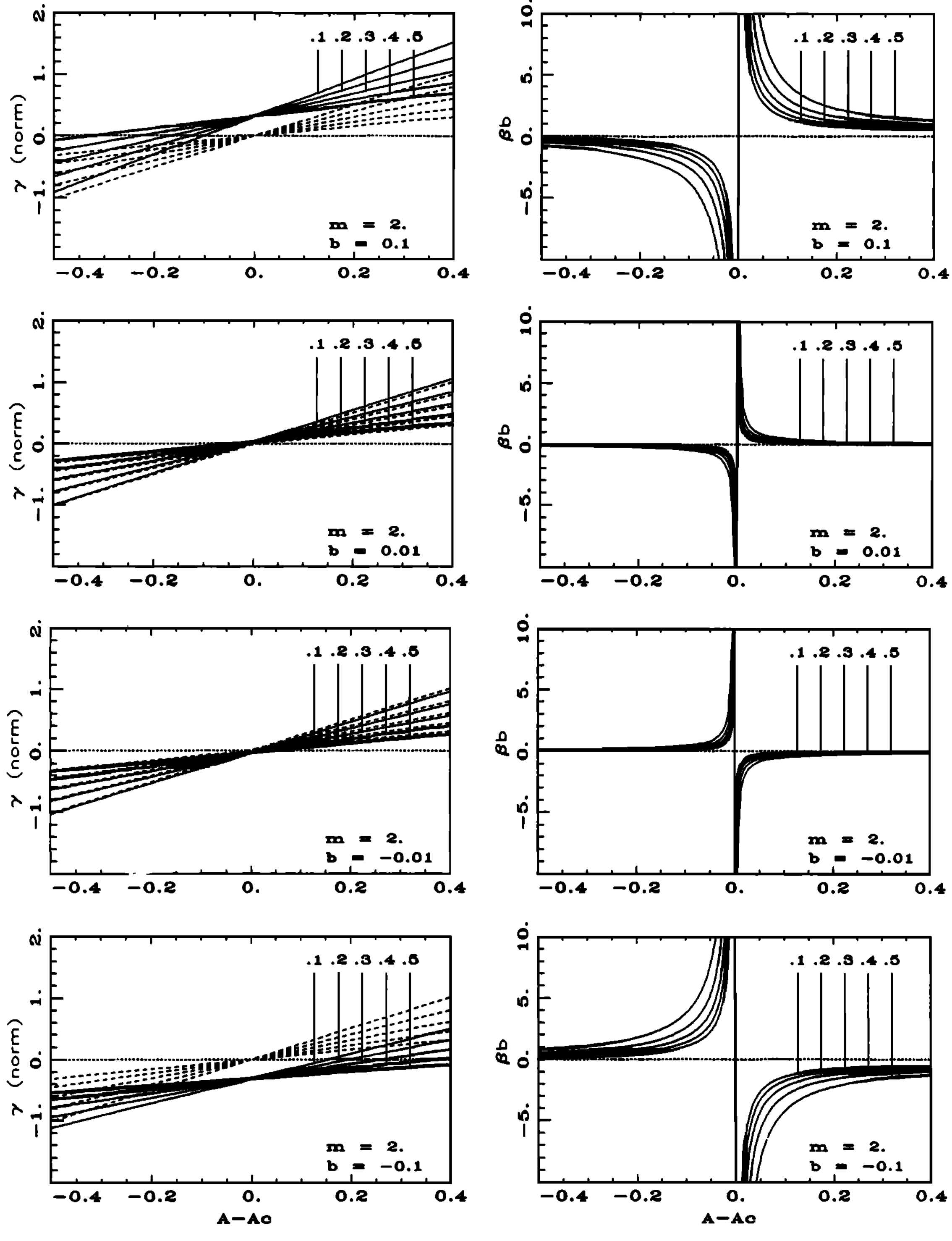

Fig. 5. The growth rate γ and growth rate modulation index βb for an $m = 2$ distribution function. The curves are plotted as a function of the difference between the anisotropy $\overline{A_j}$ and the critical anisotropy A_{jc0} and are parametric in normalized frequency $\omega/|\Omega_{j0}|$. The dashed lines correspond to the equilibrium growth rates. Plots are shown for magnetic perturbations specified by b =0.1, 0.01, −0.01, and −0.1. The growth rate is given in units of $|\Omega_{j0}|\overline{\eta_j}$.

growth rate along a given ray path can then be determined using ray tracing techniques and evaluating the equilibrium parameters separately at each point in space.

In the particular case of ducted propagation, the waves are guided approximately along field lines by field-aligned density enhancements or depletions, and there is an upper limit on the wave normal angle [*Helliwell*, 1965]. During a general magnetic field perturbation, the waves remain associated with the same field line, and the limiting wave normal angle is unchanged. If the density properties of the duct are known, the exact change in wave normal angle may be determined using ray tracing techniques; alternatively, the limiting wave normal angle may be used to provide an upper limit. In most cases of interest, the assumption of strictly parallel propagation provides a good approximation since wave normal angles are small.

9.3 *Properties of the Growth Rate*

The form of the time-dependent growth rate (equation (37)) is not a surprising result. Indeed, previous time-dependent analyses of quasi-linear theory [*Coroniti and Kennel*, 1970; *Perona*, 1972] have assumed a similar relationship and proceeded to solve the coupled set of growth and diffusion equations. This derivation provides a theoretical justification for the relationship and establishes the analytical form of Γ.

The existence of a special case for which $|\Gamma| \gg 1$ is less obvious, however. Situations which fulfill the required condition $\overline{A_j} \simeq A_{j\,c0}$ can be ascertained from the form of the equation for the equilibrium growth rate (equation (2)). In equilibrium, the wave growth rate is determined strictly by the requirement that the path-integrated growth rate equals the loss rate once the conditions for instability have been satisfied. The loss rate is established by processes external to the interaction, including ionospheric reflection, propagation, and Landau damping. The growth rate is matched to the loss rate through the particle scattering process which can modify both the trapped particle flux and the anisotropy. One scenario in which the condition $\overline{A_j} \simeq A_{j\,c0}$ is satisfied occurs when the diffusion process, in driving the growth rate toward the marginal stability condition, forces the anisotropy close to the critical value. *Cornilleau-Wehrlin et al.* [1985] have presented GEOS 1 and GEOS 2 measurements of wave and particle distributions which indicate that the anisotropy can be controlled by wave-induced diffusion when the instability condition is satisfied. In the cases they examined, the frequency dependence of the anisotropy $\overline{A_j}$ closely matched that of the critical anisotropy $A_{j\,c0}$, and the difference between $\overline{A_j}$ and $A_{j\,c0}$ decreased as the particle flux increased. Large trapped fluxes may thus provide the conditions under which $\overline{A_j} \simeq A_{j\,c0}$ is most likely. In this scenario, the bandwidth of the modulation, corresponding to the wave frequencies for which the plasma is near marginal stability, can be large. An alternate scenario which also satisfies $\overline{A_j} \simeq A_{j\,c0}$ is a band-limited wave spectrum for which the plasma is unstable only up to some fraction of the relevant particle gyrofrequency. A wave spectrum of this type occurs if the equilibrium anisotropy is independent of frequency. In such cases, the condition $\overline{A_j} \simeq A_{j\,c0}$ is necessarily obtained over a small bandwidth around the upper cutoff frequency of the wave spectrum at which $\overline{A_j} = A_{j\,c0}$.

9.4 *Impact on Wave and Particle Populations*

In the magnetosphere, particle populations for which $\overline{A_j} \simeq A_{j\,c0}$ is satisfied may be strongly influenced by perturbations in the magnetic field and plasma. The large variations in growth rate could significantly change the wave amplitude, resulting in modification of the particle precipitation rate and consequently the trapped radiation flux. If perturbations occur commonly over time scales short compared with the time required for the distribution to reestablish an equilibrium, the equilibrium state would exist only infrequently. Marginally stable distributions could thus play an important role in magnetospheric dynamics as well as determination of radiation belt characteristics.

9.5 *Experimental Evidence*

There is experimental evidence that wave growth changes of the type described in this paper occur as a result of the magnetospheric compressions observed during sudden commencements (SC). At geosynchronous altitude, the SC compressional perturbation is typically $10-30\%$ of the total field and can be somewhat larger in particular cases. Changes in the characteristics of both ULF [*Tepley and Wentworth*, 1962; *Oguti and Kokubun*, 1969] and ELF/VLF [*Morozumi*, 1965; *Gail et al.*, 1990*b*; *Gail and Inan*, 1990] wave emissions are commonly observed during SC. In an effort to explain the precipitation enhancements that usually accompany this wave growth, *Perona* [1972] developed a simple time-dependent quasi-linear theory for SC using an approach similar to that of *Coroniti and Kennel* [1970]. A time-dependent quasi-linear mechanism involving ion gyroresonance has also been proposed to explain ULF emissions during SC [*Olson and Lee*, 1983]. In Perona's theory, the SC compression was modeled by adding a small linear magnetic perturbation of the form $B = B_0(1+\delta t)$, where δ is chosen such that B corresponds to the new equilibrium value at $t = T$ (T was chosen to be 100 s near noon and 200 s near midnight). The growth rate was determined to have the form $\gamma = \gamma_0(1+\delta t)A/A_0$, a result which is consistent with the growth rate obtained in section 7 for particular but reasonable combinations of m, $\overline{A_j}$, and $A_{j\,c0}$. Using this time-dependent growth rate and the diffusion equation, the electron diffusion coefficient was then calculated as a function of time. Despite the simplifying assumptions used in the theory, the predicted wave amplitude has growth characteristics which are remarkably similar to those of observed ELF/VLF emissions during SC [*Gail et al.*, 1990*a*]. In particular, the predicted growth rate of 1.5 dB/s, total growth of 30 dB, and growth duration of 20 s were found to be comparable to the observed values of 0.3-2.7 dB/s, 12-29 dB, and 10-20 s. In addition, the Perona theory predicts the occurrence of 3-4 cycles of damped oscillations with period 35-40 s as the wave amplitude reaches its maximum value, similar to the observed 3-4 cycles of 60-90 s period oscillations.

9.6 *Magnetospheric Applications*

A time-dependent quasi-linear theory has potential applications to many commonly observed magnetospheric phenomena which are associated with dynamic processes. In addition to the SC discussed in section 9.5, potential applications include quasi-periodic (QP) ELF/VLF emissions and triggering of chorus emissions.

QP emissions may be divided into several types depending on their spectral structure [*Sato and Fukunishi*, 1981], but it is generally believed that at least some QP types result from modulation of the wave-particle interaction by micropulsations [e.g., *Sato and Fukunishi*, 1981; *Lanzerotti et al.*, 1986; *Tixier and Cornilleau-Wehrlin*, 1986]. The compressional component of micropulsations is typically $1-3\%$ of the total field, but strongly compressional pulsations can also occur with amplitudes as large as $10-30\%$ of the total field (M. J. Engebretson, private communication, 1989). To explain the corresponding precipitation pulsations, *Coroniti and Kennel* [1970] developed a theory in which the micropulsations were modeled by a small sinusoidal compressional perturbation of the magnetic field. The work presented here is qualitatively consistent with the Coroniti and Kennel theory, but also indicates that the growth rate could be directly proportional to the micropulsation amplitude when the plasma is near marginal stability. Such a result could lead to much larger modulation effects in the wave amplitude and precipitation flux than were obtained by Coroniti and Kennel using the normal growth rate dependence.

The possibility that very large changes in growth rate can result from small changes in the magnetic field provides a potential explanation for triggering of spontaneous emissions or chorus. Spontaneous emissions are commonly believed to result from the coherent whistler instability [*Helliwell*, 1967] in which electrons are phase bunched by the wave field and radiate coherently. Both theoretical [*Helliwell and Inan*, 1982] and experimental [*Helliwell et al.*, 1980] work indicates that a minimum wave amplitude (triggering threshold) is required of an input signal in order to initiate the coherent instability. Occasional large, short duration increases in the wave growth rate resulting from small fluctuations in the magnetoplasma could increase the amplitude of incoherent waves above the triggering threshold, although additional requirements (such as coherence bandwidth [*Raghuram et al.*, 1977; *Chang et al.*, 1980]) may need to be met in order to initiate the coherent instability. The mechanism provides a deterministic alternative to stochastic fluctuations [*Koons*, 1981] or power line radiation [*Helliwell et al.*, 1975; *Luette et al.*, 1977; *Luette et al.*, 1979] as a trigger for spontaneous chorus emissions.

10. Summary and Conclusions

An equation has been derived describing the time-dependent growth rate for parallel propagating electromagnetic cyclotron waves in a magnetoplasma which is characterized by a time-dependent compressional perturbation superimposed on an equilibrium configuration. For the general class of distributions described by $F(\alpha, E) = f(E)\sin^{a(E)}\alpha$, the growth rate is given by $\gamma \simeq \gamma_0(1+\Gamma)$, where γ_0 is the equilibrium growth rate and $\Gamma(b)$ is a function of the equilibrium parameters and a dimensionless time parameter $b(t)$. The term $|\Gamma|$ is generally small compared to 1, and the effect is a modulation of the equilibrium growth rate by the term Γ. However, when the particle distribution is locally near marginal stability, $|\Gamma|$ is large compared to 1, and the growth rate is directly proportional to Γ. The resulting growth rate modulation can be much larger than for a distribution which is not near marginal stability, implying that marginally stable distributions may play an important role in magnetospheric dynamics as well as determination of radiation belt characteristics.

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