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Resonance Between Coherent Whistler Mode Waves and Electrons in the Topside Ionosphere

T. NEUBERT, T. F. BELL, AND L. R. O. STOREY

STAR Laboratory, Stanford University, Stanford, CA

Landau resonance and cyclotron resonance of coherent whistler mode waves and energetic electrons are explored for magnetoplasmas with appreciable gradients in the plasma density and magnetic field strength. It is shown that in the topside ionosphere of the earth near the ion transition height the gradients in plasma density and magnetic field strength along a magnetic field line may match in a way which enhances both Landau and cyclotron interactions between waves and electrons at the loss cone pitch angle. The pitch angle scattering induced by a signal from a ground-based VLF transmitter in the ionosphere above the transmitter has been estimated and compared to the pitch angle scattering induced by naturally occurring ELF hiss through cyclotron resonance. It is found that the expected scattering due to plasmaspheric hiss is an order of magnitude larger than that due to Landau resonance in the topside ionosphere. Pitch angle scattering due to cyclotron resonance in the topside ionosphere, however, may be larger by a factor of 2. We suggest that the "fast Trimpi" effect may be caused by a cyclotron resonance interaction in the topside ionosphere.

1. INTRODUCTION

Cyclotron resonance interactions between coherent VLF waves and electrons near the magnetic equator are thought to create enhanced electron precipitation and VLF emissions in the earth's magnetosphere. While substantial literature exists on the subject (see references of *Matsumoto* [1978] and *Bell* [1984]), it is only recently, with the advent of high energy and time resolution electron analyzers, that a direct one-to-one correlation between wave pulses and pulses of electron precipitation has been established [*Imhof et al.*, 1983; *Voss et al.*, 1985]. Electron energy spectra of precipitating electrons observed from low altitude satellites near ground-based VLF transmitters [*Imhof et al.*, 1981a, b] have characteristics that to a large extent are reproduced in model calculations of equatorial cyclotron wave-particle interactions [*Chang and Inan*, 1983; *Inan et al.*, 1984]. However, a number of assumptions generally enter when comparing measurements with theory. The waves are usually assumed to propagate in ducts along the magnetic field lines, and the plasma density is assumed to follow a diffusive equilibrium model. Calculations using such models only reproduce a subset of the observations as found by *Imhof et al.* [1983], who report that only 15–50% of the enhanced precipitating electron flux above a VLF transmitter was concentrated near the resonant energies for cyclotron resonance occurring close to the magnetic equator.

Full distribution test particle simulations of electrons in Landau resonance with whistler mode waves are reported by *Tkalcevic et al.* [1984]. They find that for typical magnetospheric parameters the electron precipitation fluxes induced for equatorial Landau interactions are much smaller than those induced in cyclotron interactions. The reason for this is that in cyclotron interactions, electrons are scattered mainly tangential to the particle velocity by the wave magnetic field [*Kennel and Petchek*, 1966], while in Landau resonance, electrons are mainly scattered in parallel velocity by the relatively small-wave electric field component parallel to the background magnetic field. As loss cone particles at the magnetic equator have small pitch angles, the resultant pitch angle scattering through Landau resonance is small.

The purpose of this paper is to discuss the importance of off-equatorial resonance interactions of whistler mode waves and electrons. We first introduce the concept of gradient matching and derive a general relationship between gradients in the electron gyrofrequency (magnetic field) and plasma frequency (density) for matching to occur. Then the theory is applied to the magnetosphere of the earth, and finally, some consequences are discussed.

2. THEORY

We consider a fully ionized magnetized plasma with a magnetic field $\mathbf{B}$ directed along the Z axis of a cartesian XYZ coordinate system:

$$\mathbf{B} = B\hat{\mathbf{z}}$$

The plasma is characterized by the electron plasma frequency f_p, and the electron gyrofrequency f_c, defined by:

$$f_p = \frac{1}{2\pi}\sqrt{\frac{en}{\epsilon_o m}}$$

$$f_c = \frac{1}{2\pi}\frac{eB}{m}$$

where e is the magnitude of the electron charge, n the thermal electron density, m the electron mass, and ϵ_o the electrical permittivity.

The magnetic field strength and the density are assumed to vary negligibly in the x and y directions over the scale of a gyroradius but, in general are assumed to vary significantly in the z direction.

We assume, furthermore, that monochromatic whistler mode waves propagate in the plasma with a frequency f

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and a normalized wave vector **k**, which has an angle θ to the magnetic field. We define a function $R_s(z)$, the resonance function, as:

$$R_s(z) = sf_c^*(z) + k_z(z)v_z(z) - f \quad (1)$$

and express Landau resonance ($s = 0$) and cyclotron resonance ($s = 1$) of whistler waves and electrons at a point $z = z_0$ by

$$R_s(z_0) = 0 \quad (2)$$

In (1), f_c^* is the relativistic electron gyrofrequency:

$$f_c^* = f_c\sqrt{1 - v^2/c^2}$$

where v is the electron velocity and c is the velocity of light in vacuum. The component of the electron velocity parallel to the magnetic field is v_z:

$$v_z = v\cos\alpha$$

where α is the electron pitch angle.

Similarly, for the component of **k** parallel to the magnetic field,

$$k_z = k\cos\theta$$

where k equals $1/\lambda$, λ being the wavelength.

From the quasi-longitudinal approximation of the dispersion relation for whistler mode waves [*Helliwell*, 1965] we find for k,

$$k = \frac{f_p}{c}\sqrt{\frac{f}{f_c\cos\theta - f}} \quad (3)$$

Note that the properties of the waves are expressed by the cold plasma dispersion relation (3), while the resonance conditions (1) and (2) can generally be satisfied only by the energetic electron component. For a given wave frequency and given plasma parameters, (2) defines the parallel velocity v_R of electrons in resonance with the wave. Equation (2) is often referred to as the first-order resonance condition.

For whistler mode waves, $f < f_c\cos\theta$, and except for extreme cases of cyclotron resonance with very energetic particles or wave frequencies close to the gyrofrequency we have $f < f_c^*$. In the following we assume that $f < f_c^*$, and noting that the electron energy E normalized by the electron rest mass (511 keV) is

$$\mathrm{E} = (1 - \frac{v^2}{c^2})^{-1/2} - 1$$

we find the corresponding condition on the electron energy

$$\mathrm{E} < \frac{f_c}{f} - 1$$

Thus for Landau resonance, electrons are propagating in the same direction along the magnetic field as the waves with a parallel velocity that equals the parallel phase velocity, while for cyclotron resonance, waves and particles are streaming in opposite directions.

The strength of the interaction between waves and particles is proportional to the time they stay in resonance. In a homogeneous medium, the interaction time is limited only by the scattering of particles out of resonance with the waves. In an inhomogeneous medium gradients in f_p and f_c in the direction along the magnetic field may seriously affect the time that waves and particles are in resonance. This leads to the concept of second-order resonance [*Matsumoto*, 1978; *Carlson et al.*, 1985] or gradient matching:

$$R_s'(z = z_o) = 0 \quad (4)$$

where the notation $dR_s(z)/dz = R_s'(z)$ has been used. With (2) and (4) we find that

$$\frac{k_z'}{k_z} + \frac{v_z'}{v_z} = \frac{s}{g_c^*}\frac{f_c'}{f_c} \quad (5)$$

$$g_c^* = 1 - f/f_c^*$$

Particles in second-order resonance will stay in resonance for an increased length of time, or an increased length along the magnetic field line. While first-order resonance determines the parallel velocity of the electrons, second order resonance imposes a condition on the pitch angle of the electrons through v_z'. In a homogeneous medium all derivatives equal zero, and no pitch angle is preferred. In an inhomogeneous medium the interaction depends critically on the pitch angle, as we will show in section 3. Thus at a fixed point in space, fewer particles will interact significantly in an inhomogeneous plasma. This concept is illustrated in Figure 1.

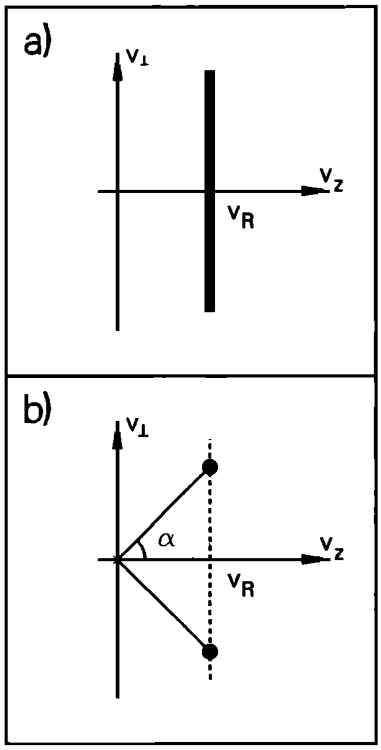

Fig. 1. Electrons in phase space in resonance with a monochromatic wave (solid regions). (*a*) A homogeneous plasma (*b*) An inhomogeneous plasma.

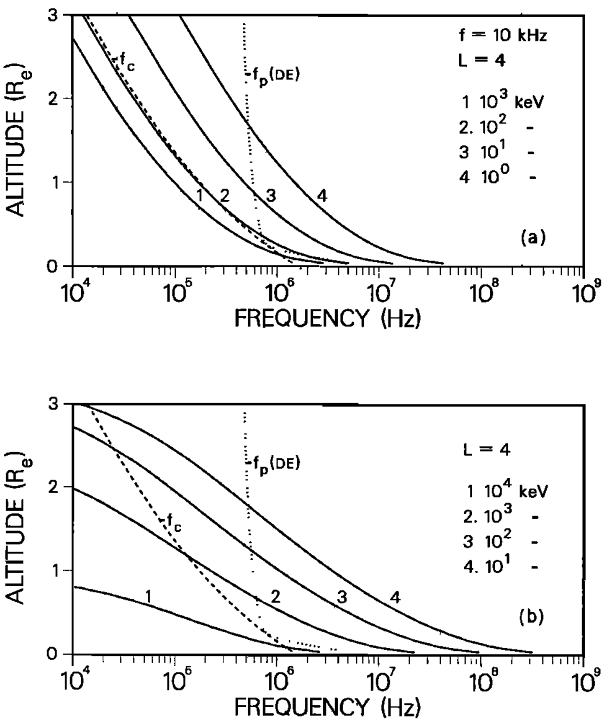

Fig. 2. Plasma frequency as function of altitude in units of R_E for electrons at the loss cone pitch angle necessary for resonance with a 10 kHz wave everywhere along the dipole field line $L = 4$. The plasma frequency profiles for four electron energies are shown along with the plasma frequency determined from a DE model (dotted curve) and the gyrofrequency (dashed curve). (*a*) Landau resonance. (*b*) Cyclotron resonance.

Assuming the magnetic moment of the electrons to be conserved, we find that

$$\frac{v_z'}{v_z} = -\frac{1}{2}tg^2\alpha\frac{f_c'}{f_c} \qquad (6)$$

which with (5) gives

$$\frac{k_z'}{k_z} = (\frac{s}{g_c^*} + \frac{1}{2}tg^2\alpha)\frac{f_c'}{f_c} \qquad (7)$$

For whistler mode waves the direction of the phase velocity differs from the direction of the group velocity, except at certain wave normal angles. To simplify the problem, we consider waves propagating along the magnetic field. This occurs for $\theta = 0°$ and, for $f < f_c/2$, for $\theta = \theta_g$, the so-called Gendrin angle. In the following we consider two cases. For cyclotron resonance we choose $\theta = 0°$, at which angle strong cyclotron interaction is possible. The Landau resonance interaction is dependent on a component E_z of the wave electric field parallel to the background magnetic field. As $E_z = 0$ for $\theta = 0°$ and E_z is finite for $\theta = \theta_g$, we investigate the Landau resonance for waves propagating with the wave vectors at the Gendrin angle. The parallel component of the wave vector k_g at this angle is

$$k_g = 2\frac{f_p f}{c f_c} \qquad (8)$$

Combining (7) and (8), we then find for the case of Landau resonance between electrons and waves propagating at the Gendrin angle:

$$\frac{f_p'}{f_p} = (1 + \frac{1}{2}tg^2\alpha)\frac{f_c'}{f_c} \qquad (9)$$

Combining (3) and (7), we find for cyclotron resonance with parallel propagating waves ($\theta = 0$):

$$\frac{f_p'}{f_p} = (\frac{1}{g_c^*} + \frac{1}{2g_c} + \frac{1}{2}tg^2\alpha)\frac{f_c'}{f_c} \qquad (10)$$

$$g_c = 1 - f/f_c$$

In the remainder of this paper we use the term "gradients" when referring to f_p'/f_p and f_c'/f_c, although they truly are "normalized" gradients or gradients in $ln(f_{p,c})$.

From (9) and (10) we draw the following conclusions:

1. The gradients in the density and the magnetic field strength must have the same sign for second-order resonance to be possible. If the magnetic field strength is increasing, so must the density. This is the reverse of what is found in an isothermal plasma in equilibrium with no external forces acting. However, if an additional force exists, like the gravitational force in planetary magnetospheres, this is exactly the type of variation that is found.

2. The larger the gradient in the plasma frequency for a given magnetic field configuration, the larger the pitch angle of the electrons in second-order resonance.

3. Since $0 < g_c^*, g_c < 1$, the gradient in the plasma frequency must be larger than that of the gyrofrequency. Therefore, the gradients in the plasma frequency must be larger for cyclotron resonance than for Landau resonance.

4. The gradient matching relation (5) for Landau resonance is independent of the wave frequency. This is a consequence of the assumption that the waves are propagating with their wave normals at the Gendrin angle.

3. APPLICATIONS TO THE EARTH'S MAGNETOSPHERE

Cyclotron resonance and Landau resonance of whistler waves and electrons in the earth's magnetosphere have recently been studied by computer code simulations [*Chang et al.*, 1983; *Tkalcevic et al.*, 1984]. In these studies the magnetic field is described by a dipole field, and the plasma density by a diffusive equilibrium (DE) model [*Bell*, 1985]. It was found that the strongest interaction takes place in the equatorial region, where the gradients in the plasma parameters along the magnetic field minimize. Some experimental evidence exists, however, that significant off-equatorial resonance interactions also occur [*Inan et al.*, 1977]. We will first show why calculations using a DE model, considered a good description of the magnetospheric plasma inside the plasmasphere for magnetically quiet periods, is unlikely to give any significant off-equatorial resonance interaction for fixed-frequency waves.

We consider the resonances along a dipole field line inside the plasma pause, from the topside ionosphere and to the magnetic equator. The expression for a field line is

$$r = R_E L \sin^2 \phi \tag{11}$$

where r is the radial distance from the center of the earth, R_E is radius of the earth, L the maximum distance from the center of the earth measured in earth radii, and ϕ the colatitude. The gyrofrequency is given by

$$f_c = \frac{A}{r^3}\sqrt{4 - 3\frac{r}{R_E L}}$$

where A is a constant. We now let z denote the distance along the field line. We then find for the derivative f_c' with respect to z:

$$\frac{f_c'}{f_c} = -3H_c(r)\frac{r'}{r} \tag{12}$$

$$H_c(r) = \frac{8 - 5r/R_E L}{8 - 6r/R_E L}$$

In the region of interest the variation of the plasma density n along a field line in a diffusive equilibrium model is given by

$$n_{DE} = n_o[\sum_i \delta_i e^{-G(r)/S_i}]^{1/2} \tag{13}$$

$$G(r) = (1 - r_b/r)$$

$$S_i = 0.282\frac{\beta r_b}{m_i R_E}$$

$$\beta = \frac{T}{T_o}$$

$$T_o = 1594.5^\circ K$$

In (13), n_o and r_b are constants, T the temperature, m_i the ion mass measured in proton mass units, and S_i a measure of the scale height of the ion density. The summation is done over the number of ion species, and δ_i is the relative density of ion species at $r = r_b$, which is often taken at $R_E + 1500$ km, which is close to the ion transition height.

Differentiation of (13) yields

$$\frac{f_p'}{f_p} = -H_p(r)\frac{R_E r'}{r^2} \tag{14}$$

$$H_p(r) = \frac{\tilde{m}_i(r)}{1.128\beta}$$

where $\tilde{m}_i(r)$ is a measure of the average ion mass:

$$\tilde{m}_i(r) = \frac{\sum_i \delta_i m_i e^{-G(r)/S_i}}{\sum_i \delta_i e^{-G(r)/S_i}}$$

Combining (12) and (14), we find that

$$\frac{f_p'}{f_p} = \frac{H_p(r)}{3H_c(r)}\frac{R_E}{r}\frac{f_c'}{f_c} \tag{15}$$

For r varying from R_E to LR_E, $H_c(r)$ varies from ~1 to 1.5. For typical magnetospheric conditions, $\beta \simeq 1$. Comparing (9), (10), and (15), we find that the possibility of second-order resonance depends on the average ion mass. Assuming, for instance, a proton plasma ($\tilde{m}_i = 1$), we find that in a DE model the gradient in the plasma frequency along a dipole field line inside the plasma pause is smaller than the gradient in the gyrofrequency. This is the reverse of what is needed for second-order resonance. However, for higher average ion mass, second-order resonance is possible. In the ionosphere the major constituent is O^+, while at higher altitudes H^+ is dominant. Thus we see from (9), (10), and (15) that the ion transition height plays a crucial role for second-order resonance.

However, since the plasma density and composition of the topside ionosphere are highly variable [*Titheridge*, 1976; *Rawer et al.*, 1978], the DE model is not very accurate in these regions. Consequently, model calculations of whistler wave-electron interactions in the magnetosphere typically assume a lower boundary well above the region where second-order resonance is possible.

The density profiles encountered in the magnetosphere depend on the geomagnetic activity and on latitude. Outside the plasma pause, for instance, the gradient in the plasma density along a field line is, in general, larger than inside. During disturbed periods the plasmapause moves to lower L shells. The wide range of equatorial plasma densities that are encountered is illustrated by *Carpenter et al.* [1981]. We have then found it of interest to determine the plasma frequency profiles needed for waves to stay in resonance with the same group of particles all along a field line. We mentioned in section 2 that at a given location, only electrons at a particular pitch angle can be in second-order resonance with a wave at a particular frequency. For the dynamics of the energetic electron distribution in the magnetosphere the loss cone is of particular interest, as particles at the loss cone interacting with waves may be scattered into the loss cone and precipitate into the atmosphere. We have thus chosen to determine the profiles of the plasma frequency needed for a wave to stay in resonance with loss cone particles. Assuming again the magnetic moment to be conserved, we find with (2) and (8) for waves propagating at the Gendrin angle in Landau resonance with electrons:

$$f_p = \frac{c}{v}\frac{f_c}{2}g_m^{-1/2} \tag{16}$$

$$g_m = 1 - f_c/f_m$$

where f_m is the gyrofrequency at the altitude of the mirror point.

With (2) and (3) we find, similarily, for parallel propagating waves in cyclotron resonance

$$f_p = \frac{c}{v}(f_c^* - f)(\frac{f_c g_c}{f g_m})^{1/2} \tag{17}$$

The results are shown in Figure 2*a* for Landau resonance and Figure 2*b* for cyclotron resonance. The calculations are done along the dipole field line $L = 4$ for a 10-kHz wave, assuming the loss cone mirror point to be at 100-km altitude. Shown in the figures are the electron gyrofrequency,

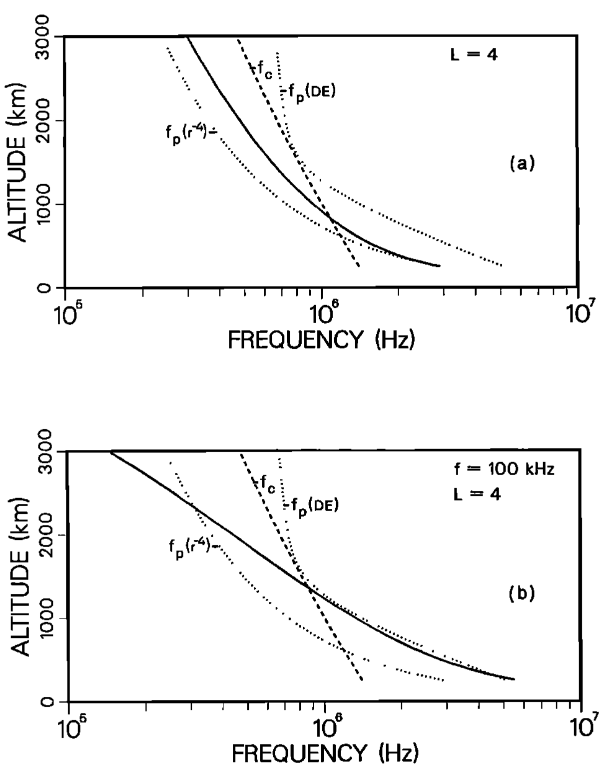

Fig. 3. A close-up of the plasma frequency as a function of altitude in the topside ionosphere for 1-MeV electrons at the loss cone pitch angle necessary for resonance with a wave along $L = 4$. Also shown are the plasma frequency determined from an r^{-4} model and a DE model. (*a*) Landau resonance. (*b*) Cyclotron resonance.

the plasma frequency from a DE model, and the plasma frequency necessary for interaction with loss cone electrons with energies in the range 1 keV to 1 MeV (Landau) and 10 keV to 10 MeV (cyclotron). The altitude on Figures 2*a* and 2*b* is the radial altitude in units of R_E.

As expected from the calculations in section 2, larger gradients in f_p are needed for cyclotron resonance than for Landau resonance, and both are larger than predicted from a diffusive equilibrium model except at the topside ionosphere. Note that the plasma frequency profiles for the Landau resonance interaction (16) is independent of the wave frequency and that relativistic effects cause the profiles for high-energy particles to be closer spaced.

Figure 3 shows the profiles needed in the topside ionosphere in more detail. In addition to the gyrofrequency and the plasma frequency determined from a DE model we have also shown the plasma frequency predicted by the r^{-4} model:

$$n_{r4}(z) = n_{eq}\left(\frac{1}{\cos^2\psi}\right)^4 \tag{18}$$

Where $n_{r4}(z)$ is the density along a dipole field line, ψ the magnetic latitude, and n_{eq} the equatorial density at the location of that field line.

The results for Landau resonance with 1-MeV electrons at the loss cone are shown in Figure 3*a*, and the results for a 100-kHz wave in cyclotron resonance with 1-MeV electrons are shown in Figure 3*b*. Plots like the ones in Figures 2 and 3 are very useful to graphically determine the resonant energies and heights for which off-equatorial resonances are likely to happen for a given density profile, and they represent an inviting avenue for evaluating resonant interactions in other planetary magnetospheres.

We now leave the analytical treatment of the problem and turn to computer calculations of the interaction length l_i and the interaction time t_i expected in the topside ionosphere. We follow electrons at a given pitch angle specified at $z = z_o$ and characterize the strength of the interaction through the interaction time:

$$t_i = l_i / v_z(z_o) \tag{19}$$

where $v_z(z_o)$ is the parallel velocity of the electrons considered at $z = z_o$ and l_i is the interaction length along the field line defined implicitly through the relation:

$$\left| \int_{z_o}^{z_o+l_i} \frac{R_s(z)}{v_z(z)} dz \right| = \frac{1}{4} \tag{20}$$

It is assumed in (19) and (20) that the interaction is switched off when the variation in phase between waves and particles reaches $\pi/2$. This approach is similar to the one used by *Neubert* [1982] who studied the off-equatorial parametric interaction of two whistler waves and an ion acoustic

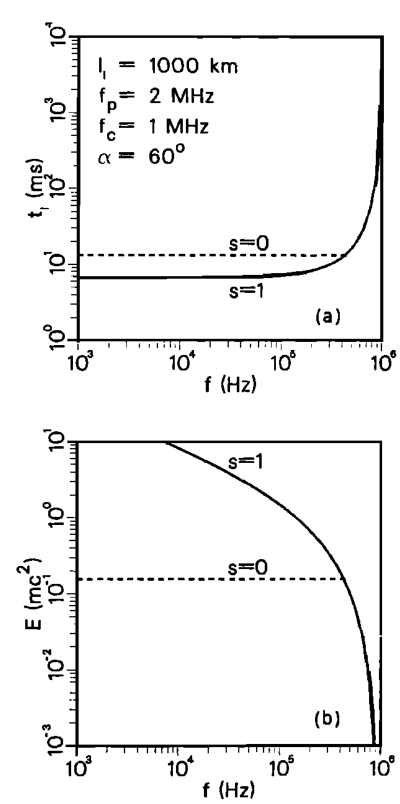

Fig. 4. (*a*) Interaction time t_i and (*b*) corresponding energy as function of wave frequency for Landau resonance ($s = 0$) and cyclotron resonance ($s = 1$) in the topside ionosphere. It is assumed that the interaction length $l_i = 1000$ km.

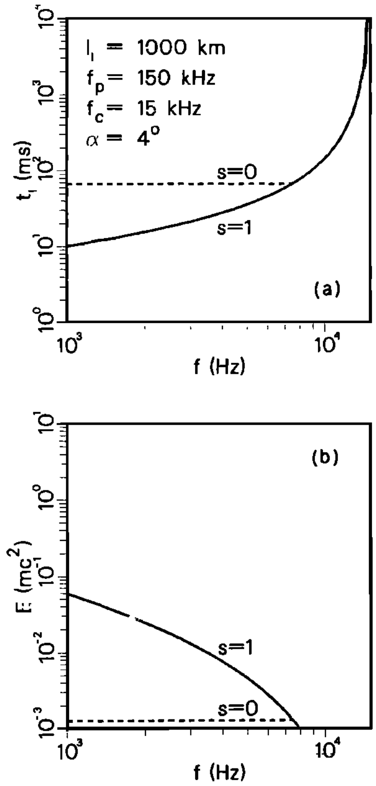

Fig. 5. (*a*) Interaction time and (*b*) corresponding energy as function of wave frequency in the equatorial region at $L = 4$. It is assumed that the interaction length $l_i = 1000$ km.

wave. To validate this criterion further, we mention that when it is applied to the equatorial region along the field line $L = 4$, with the assumption that the electron density follows a DE model with a 140-kHz equatorial plasma frequency, then a 3-kHz wave and loss cone electrons have $l_i =$ 1238 km for Landau resonance and $l_i = 926$ km for cyclotron resonance. The interaction length in the equatorial region estimated by *Matsumoto* [1978] is of the order of 800–4000 km, depending on the wave amplitude. Equation (20) thus represents a somewhat conservative estimate of l_i.

We first assume that $l_i = 1000$ km for both the topside ionosphere and the equatorial region and will later return to the estimation of l_i. The interaction time as a function of wave frequency for the topside ionosphere is shown in Figure 4*a* and the corresponding electron kinetic energy normalized to the electron rest mass is shown in Figure 4*b*. The loss cone pitch angle is assumed to be 60°, the plasma frequency 2 MHz, and the gyrofrequency 1 MHz.

Since the Landau resonance has been studied for waves propagating at the Gendrin angle, the resonant parallel electron velocity is independent of the wave frequency. Thus the interaction time is constant and equals 12 ms for the conditions chosen. Note, however, that the interaction time for cyclotron resonance, although frequency dependent, is also nearly constant up to ~100 kHz in spite of the large variation with frequency of the resonant energy, as shown in Figure 4*b*. Since very large energies are needed for cyclotron resonance at low frequencies, the constant interaction time below ~100 kHz is a consequence of the decrease in gyrofrequency for increasing electron energy. In the limit of very low frequencies, $f \ll f_c$, and for $f_p \simeq f_c$ the parallel velocity for cyclotron resonance as found from (2) is simply $v_R \simeq c\cos\alpha$.

Similar plots for the equatorial region are shown for comparison in Figure 5. The loss cone pitch angle is now estimated to be 4°, and again the interaction length equals 1000 km. We have furthermore assumed that $f_p = 150$ kHz and $f_c = 15$ kHz.

In general, the electron energy needed for resonance is smaller in the equatorial region than in the topside ionosphere. This is partly due to the smaller pitch angle considered in the equatorial region and partly due to the difference in the ratio f_p/f_c as the resonant velocity increases for decreasing f_p/f_c ratio.

With the current assumptions of plasma parameters and equal interaction length in the two regions the interaction time for Landau resonance is typically 5 times larger in the equatorial region, while for cyclotron resonance the interaction time of a 3-kHz wave in the equatorial region is ~3 times larger than that for waves below ~ 100 kHz in the topside ionosphere.

We now explore the resonances further by calculating the interaction length defined by (20) as a function of altitude above the earth's surface from 200 km to 3000 km along the field line $L = 4$. The interaction length assigned to a location $z = z_o$ on the field line depends on the model of the ionosphere $(f_p(z), f_c(z))$, the wave frequency, and the pitch angle and energy of the electrons for which the interaction length is calculated. As the Landau resonance requires smaller gradients in the plasma density than the cyclotron resonance, we have decided on two models of the ionosphere.

The plasmafrequency and gyrofrequency profiles for the model used for the Landau resonance are shown in Figure 6. They are characterized by a relatively low peak plasma frequency, ~3 MHz and a gentle transition from the O^+-dominated region below 1000 km and the H^+-dominated region above 2000 km.

The interaction length for electrons in first-order Landau resonance with a 10-kHz wave is shown as a function of altitude in Figure 7. It is determined for electrons at the pitch angle α_{max}, which gives the maximum interaction length and corresponds to the pitch angle at which electrons are also in second-order resonance with the wave, and for electrons at the loss cone pitch angle. The top panel shows α_{max}

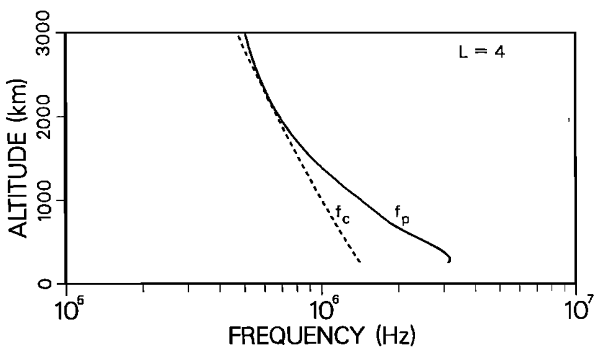

Fig. 6. Model ionosphere used for calculating the Landau resonance parameters shown in Figures 7 and 8.

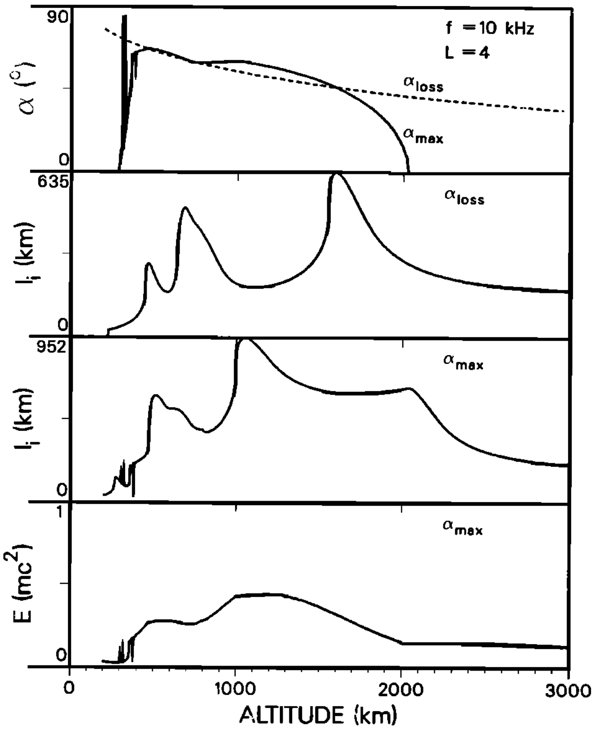

Fig. 7. Landau resonance in the model ionosphere shown in Figure 6. From top to bottom are the pitch angle $\alpha_{\max}$, for which the interaction length l_i maximizes (second-order resonance), and the loss cone pitch angle α_{loss}. The interaction length of loss cone electrons in first-order resonance; the maximum interaction length for electrons at $\alpha_{\max}$; and the corresponding electron energy.

and α_{loss}, where the loss cone mirror point is assumed to be at 100 km altitude. The next two panels show the interaction length found for electrons at α_{loss} and $\alpha_{\max}$, respectively, and the bottom panel shows the energy of electrons in resonance at $\alpha_{\max}$.

The altitude range where second-order resonance is possible (gradient matching) corresponds to the region where $\alpha_{\max} > 0$. While second order resonance is found from 300 to 2100 km, the region of interest is 500–1600 km, where $\alpha_{\max} \geq \alpha_{\text{loss}}$, assuming that the loss cone is devoid of particles. Here waves are in resonance with electrons at the loss cone or outside the loss cone, and provided the interaction length is of sufficient magnitude, strong interaction between waves and particles may take place.

There are several peaks in l_i, which reaches a maximum of 635 km at 1592 km altitude for α_{loss} and 952 km at 1050 km altitude for $\alpha_{\max}$. Note that the peaks in $l_i(\alpha_{\text{loss}})$ correspond to altitudes where $\alpha_{loss} = \alpha_{max}$. However, the profiles of $\alpha_{\max}$ and l_i are very sensitive to the variation of f_p with altitude. Thus the peaks in $l_i(\alpha_{\max})$ are a result of the small bulge in the plasma frequency profile shown in Figure 6.

Figure 8 shows the resonance in terms of l_i at 1592 km altitude, where l_i maximizes for α_{loss} and $\alpha_{\text{loss}} = \alpha_{\max}$. It is shown as a function of wave frequency (Figure 8*a*), for electrons at the loss cone pitch angle (in second-order resonance at this altitude, pitch angle (Figure 8*b*), for a 10-kHz wave and a constant electron energy, which equals the energy of loss cone electrons in second order resonance with the wave; and energy (Figure 8*c*), for electrons at the loss cone angle and a 10-kHz wave. The peaks in l_i correspond to values of α and E for which waves and particles are in second-order resonance.

The interaction length increases with decreasing wave frequency reaching 1224 km at 1 kHz. The shoulder at ~18 kHz is a consequence of the specific model parameters. The interaction length, as a function of pitch angle, peaks as expected at the loss cone pitch angle. The width in pitch angle at half maximum is $\simeq 1.2^\circ$ while the width in energy is $\simeq$ 12 keV.

The model used for the cyclotron resonance is shown in Figure 9. It is a typical daytime model for magnetically quiet periods [*Rawer et al.*, 1978], with a peak plasma frequency ~5 MHz and the ion transition height at 1500 km.

Plots similar to the ones shown for the Landau resonance are presented in Figures 10 and 11 for a 100 kHz wave for the case of cyclotron resonance. Since the scale height in the plasma frequency varies more rapidly at the ion transition height than in the previous model, and the cyclotron resonance, in general, requires smaller scale heights; the maximum altitude of second-order resonance, shown in Figure 10, is now lower and at 1270 km; and the maximum altitude for which $\alpha_{\max} \geq \alpha_{\text{loss}}$ is 1030 km.

The major differences from the case of Landau resonance are the smaller interaction lengths and higher electron energies. Furthermore, the peaks in l_i as function of pitch angle and energy shown in Figure 11 are more narrow and around 0.06° and 1.5 keV, respectively. Thus we expect the cyclotron resonance interaction to be even more sensitive

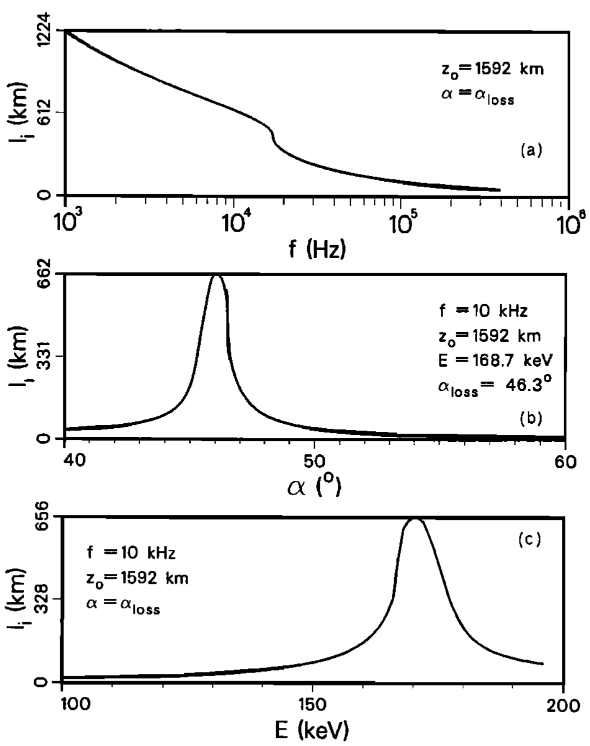

Fig. 8. The interaction length for loss cone electrons in Landau resonance at 1592-km altitude as (*a*) function of wave frequency, (*b*) function of pitch angle, and (*c*) function of electron energy.

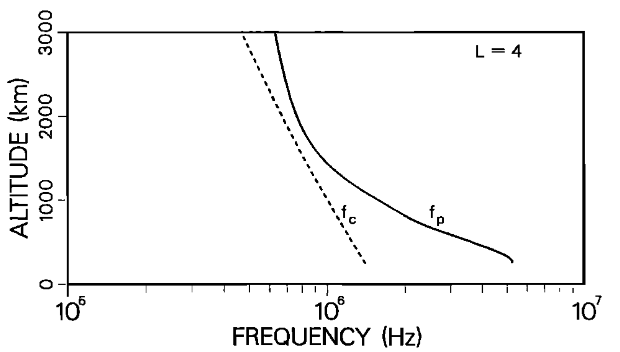

Fig. 9. Model ionosphere used for calculating the cyclotron resonance parameters shown in Figures 10 and 11.

than the Landau resonance interaction to variations in f_p. Finally, since the cyclotron interaction for parallel propagating waves is a more complex function of the wave frequency, the interaction length has a maximum, corresponding to second-order resonance (10), at a particular value of the wave frequency, which in this case is 100 kHz.

We end this section by presenting three-dimensional plots of $l_i(\alpha_{\text{loss}})$ and $t_i(\alpha_{\text{loss}})$ as a function of wave frequency and altitude as calculated using the model in Figure 6 for Landau resonance and Figure 9 for cyclotron resonance. The results for Landau resonance are shown in Figure 12, and for cyclotron resonance are shown in Figure 13.

4. DISCUSSION

It is of interest to apply the foregoing results to determine the pitch angle scattering that might be produced by presently existing VLF, LF, and MF transmitters. In the VLF range the Omega navigational transmitters operate in the 10.2 to 13.6-kHz range with a 10-kW power output. At slightly higher frequencies (17.8–24 kHz), VLF communications transmitters such as NAA and NLK radiate power in the 10^2 to 10^3-kW range. Wave magnetic field amplitudes directly overhead the transmitters at night at altitudes between 500 and 3000 km range from 3 to 30 mγ [*Inan et al.*, 1984]. Nighttime amplitudes in the conjugate regions have a similar range, although higher amplitudes here can be expected when the waves are amplified through the whistler mode instability [*Helliwell and Katsufrakis*, 1974, *Bell et al.*, 1981; *Bell*, 1985].

Because of the higher refractive index of the ionosphere at VLF frequencies the wave normals of the input waves over the transmitter will be roughly vertical. Thus the wave normal angle θ, with respect to the earth's magnetic field lines, will be roughly equal to the complement of the local magnetic field dip angle. In the conjugate region the wave normals of nonducted waves from the transmitter will lie between the Gendrin angle and the resonance cone angle, $\theta_r \approx \cos^{-1}(f/f_c)$ [*Edgar*, 1976; *Bell et al.*, 1981]. On the other hand for ducted waves, $\theta \sim 0$ in the conjugate region.

There are numerous transmitters operating in the LF and MF range, many of them at MW output power levels [*Frost*, 1984]. However, whistler mode waves can be produced near the ion transition height (~1000 km) only if locally, $f < f_c$, or roughly, $f < 800$ kHz.

Typical nighttime wave amplitudes at 1000 km over the LF and MF transmitters are presently unknown, but calculations [*Helliwell*, 1965] indicate that the nighttime ionospheric absorption loss is roughly 10 dB higher at 100 kHz than at 10 kHz. This suggests that MW transmitters at 100 kHz could produce roughly 10 mγ wave amplitudes at 1000-km altitude over the transmitter.

For the case of Landau resonance we can use equations (42), (48), and (51) of *Bell* [1984] to determine the pitch angle change of resonant electrons at the position of second-order resonance shown in Figures 7 and 8. Using the values $f = 10$ kHz, $z_o = 1592$ km, $E = 170$ keV, $\alpha = \alpha_{\text{loss}} = 46°$, and $l_i = 635$ km, we find that $\Delta\alpha_{rms} \simeq 4° \times 10^{-3}$ for a wave with a wave normal angle at the Gendrin angle and with a wave magnetic field amplitude of 10 mγ. Repeating this calculation for a 100 kHz wave with $l_i \simeq 130$ km (from Figure 8), we find $\Delta\alpha_{rms} \simeq 10^{-2}$ deg.

These same electrons can also experience pitch angle scattering near the magnetic equatorial plane through a cyclotron resonance interaction with ELF plasmaspheric hiss [*Lyons et al.*, 1972; *Thorne et al.*, 1973]. Typical wideband amplitudes of the plasmaspheric hiss lie in the range 5–50 mγ, and the total bandwidth is a few hundred hertz [*Thorne et al.*, 1973]. To estimate the pitch angle scattering due to the hiss, we assume that the hiss extends with uniform spectral power density over the range 100–500 Hz and has a wideband amplitude of 25 mγ. We also assume that the scattering takes place near $L \sim 4$, with $f_p \simeq 200$ kHz and $f_c = 14$ kHz. In this case we can use an expression given by *Roberts* [1968] to calculate the local pitch angle diffusion coefficient; $D_\alpha \sim 1$ deg^2/s. The particle can be in resonance with a component of the hiss band over the latitude range

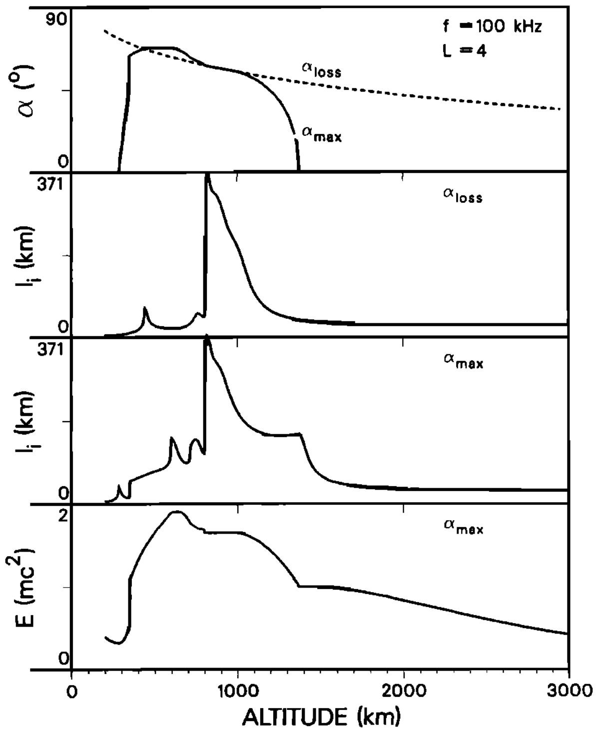

Fig. 10. Cyclotron resonance in the model ionosphere shown in Figure 9. This figure is equivalent to Figure 7 for Landau resonance.

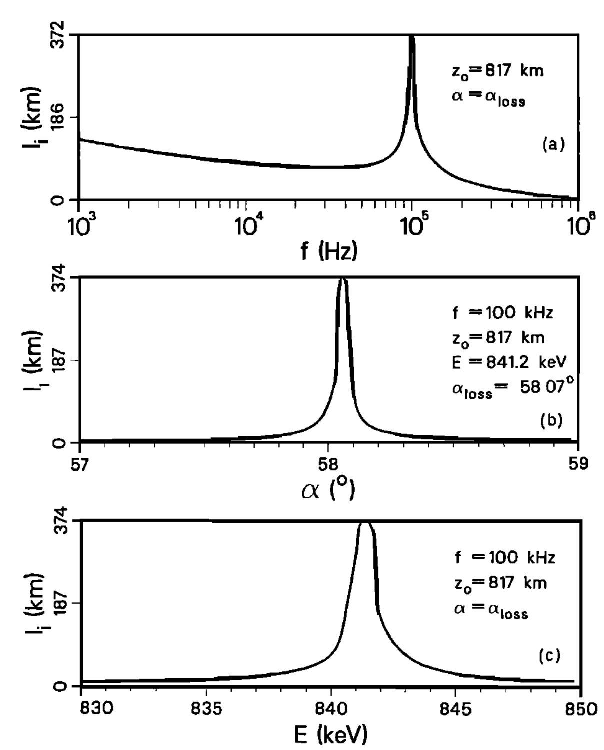

Fig. 11. The interaction length for loss cone electrons in cyclotron resonance at 817-km altitude. This figure is equivalent to Figure 8 for Landau resonance.

between ±20°. Near L ~ 4 this latitude range represents an interaction time of roughly 50 ms. Thus the rms pitch angle change over the entire resonance region has the value $\Delta\alpha_{\rm rms} \simeq 0.2°$.

Since the expected scattering due to plasmaspheric hiss is an order of magnitude larger than that due to Landau resonance at low altitude, we conclude that low-altitude pitch angle scattering due to Landau resonance interactions does not appear to be an important effect for input waves from mid-latitude VLF, LF, and MF transmitters.

For the case of cyclotron resonance interaction we can use equations (47) and (51) of *Bell* [1984] to determine the pitch angle change of resonant electrons at the position of second order resonance shown in Figures 10 and 11. Using the values f = 100 kHz, z_o = 817 km, E = 841.3 keV, $\alpha = \alpha_{\rm loss}$, and l_i = 372 km, we find $\Delta\alpha_{rms} \simeq 0.2°$ for a wave of 10-mγ amplitude. Given the characteristics of the plasmaspheric hiss as described above, it is found that the 841-keV electrons can experience a cyclotron resonance with the hiss over a 5° latitude range centered at roughly 23° latitude near $L \sim 4$. Repeating the calculation described above, we find $\Delta\alpha_{\rm rms} \simeq 0.1°$ for the total rms pitch angle scattering due to the plasmaspheric hiss band. Since this value is less than that due to the low-altitude scattering, we conclude that low-altitude scattering effects could be important for energetic electrons in the 800-keV energy range.

Electrons in second-order cyclotron resonance with a monochromatic wave emitted from a ground-based transmitter are expected to experience scattering in several consecutive bounces. The width in pitch angle of the cyclotron resonance (Figure 11a) is ≃ 0.06° for the model ionosphere chosen and imposes an apparent upper limit on the total pitch angle scattering of the electrons. However, the influence of the wave field on the particle orbits is a second-order effect as compared to the influence of the earth's magnetic field. Thus with a slightly different model ionosphere, similar curves to the ones shown in Figures 10 and 11 would result from a calculation including the $\Delta\alpha_{\rm rms}$ pitch angle scattering in the particle motion. The peaks in energy and pitch angle are then indicative of the part of the electron distribution interacting with the wave, rather than representing the bandwidth of the scattering in pitch angle and energy. When interaction in consecutive bounces is considered, an indication of the upper limit of the total pitch angle scattering possible is the maximum value along the field line of $\alpha_{\rm max} - \alpha_{\rm loss}$. For the cyclotron resonance interaction this value is ≃ 6°, which is reached at ≃ 600-km altitude.

Nonlinear effects can be expected if the interaction endures for a time roughly equal to the "trapping" time, that is, the time required for the resonant particle to complete one oscillation in the potential well of the wave. Using the expressions given by *Bell* [1984], we find that for the parameters given above, the trapping time at low altitude has the value of roughly 0.2 s for the case of Landau resonance and 2×10^{-3} s for the case of cyclotron resonance. Since

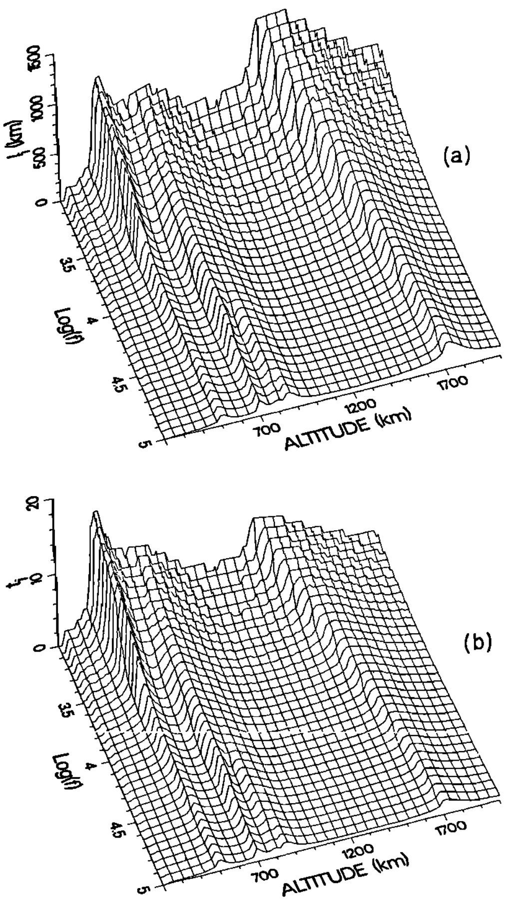

Fig. 12. Three dimensional plots of Landau resonance: (*a*) l_i measured in kilometers and (*b*) t_i measured in milliseconds. The frequency scale is logarithmic from 1 to 100 kHz.

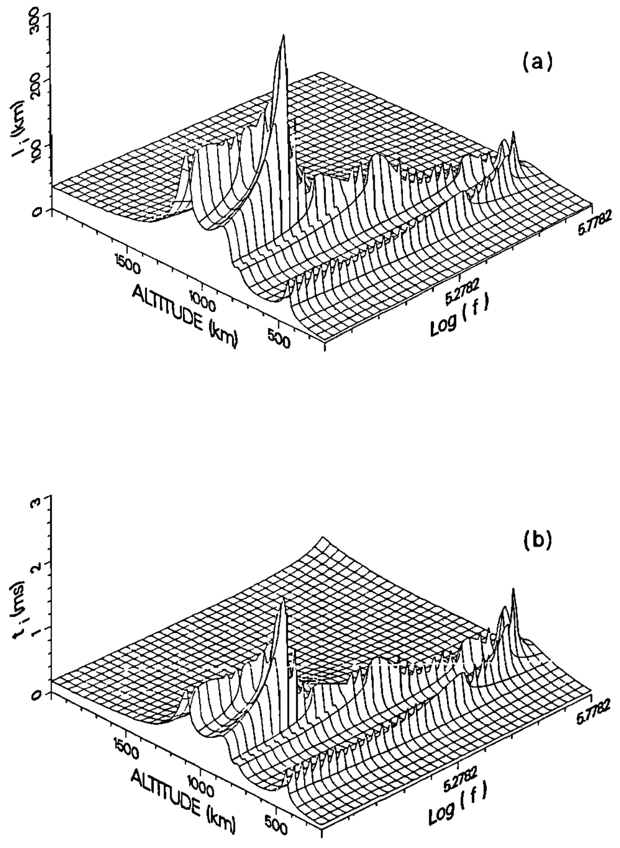

Fig. 13. Three dimensional plots of cyclotron resonance: (*a*) l_i measured in kilometers and, b) t_i measured in ms. The frequency scale is logarithmic from 60 to 600 kHz.

the trapping time for cyclotron resonance is roughly equal to the interaction time defined by (19) of $t_i = 2.4 \times 10^{-3}$ s, it appears possible that a nonlinear interaction could take place at low altitude which might be analogous to the nonlinear whistler instability. In this case it might be possible that wave amplification, enhanced resonant particle precipitation, and VLF emission generation could take place at altitudes near the ion transition height.

The possibility of second-order resonance effects in the topside ionosphere suggests an explanation for the "fast Trimpi" effect [*Armstrong*, 1983]. In a normal "Trimpi" effect it is found that the amplitude and/or phase of subionospherically propagating MF/LF/VLF signals are perturbed at about the time of reception of a magnetospheric whistler. It has been proposed that the signal perturbations are due to energetic electron precipitation induced in the equatorial region by the correlated whistlers [*Carpenter et al.*, 1984; *Inan and Carpenter*, 1986]. In the "fast Trimpi" effect, VLF wave perturbations are produced close to the time of a lightning stroke rather than at the time of reception of the whistler which generally follows the lightning stroke by about 2 S.

It has been suggested (W. C. Armstrong, private communication, 1985) that the "fast Trimpi" effect might be caused by energetic particle precipitation produced as the wave energy from the lightning stroke propagates upward through the topside ionosphere. The results of the present study appear to lend further plausibility to this hypothesis.

In the work reported above we have concentrated on the electron precipitation caused as a direct result of resonant interactions with signals from transmitters and whistlers within the topside ionosphere. However, it appears possible that scattering can alo be induced in the topside ionosphere by a multistep process such as a parametric interaction. In these interactions the input wave excites two or more additional waves, and in principle, these new waves may produce enhanced pitch angle scattering. For instance, *Neubert* [1982] has considered a three-wave parametric interaction in an inhomogenous plasma and found that the gradients in the plasma parameters may favor interaction in the topside ionosphere. Other works include *Riggin and Kelley* [1982] and *Lee and Kuo* [1984], which discuss the excitation of lower hybrid waves in the topside ionosphere through a parametric interaction with a coherent whistler wave as the pump wave. Under night time conditions, *Lee and Kuo* [1984] estimate that these lower hybrid waves can be generated within a few tenths of a second or less. The enhanced pitch angle scattering due to the lower hybrid waves could possibly produce an effect similar to the "fast Trimpi" events. Numerical calculations of pitch angle scattering in the case of parametric interactions need to be carried out in order to assess the importance of this effect in precipitating energetic particles in the topside ionosphere.

5. SUMMARY

The strength of the Landau and cyclotron interactions of coherent whistler mode waves and electrons has been characterized by the interaction length l_i and the interaction time t_i (equations (19) and (20), strong interactions having large values of l_i and t_i. In an inhomogenous plasma, l_i and t_i are dependent on the electron pitch angle, thus only electrons at certain pitch angles will interact significantly with the wave. When applied to the earth's magnetosphere, the theory predicts enhanced interactions in the topside ionosphere. For certain ionospheric plasma density profiles, electrons with pitch angles at the loss cone angle interact most strongly with the waves and may thus be scattered into the loss cone and precipitate into the atmosphere. We find that scattering of electrons in the topside ionosphere through a Landau resonance interaction with waves from ground based transmitters is insignificant as compared to the scattering the same electrons experience by plasmaspheric hiss. Cyclotron resonance scattering in the topside ionosphere, however, may surpass the scattering induced by plasmaspheric hiss.

The electron energy required for resonance in the topside ionosphere is generally larger than that required for resonance at the magnetic equator. Furthermore, because of the difference in the local f_c the resonant wave frequencies can be higher in the topside ionosphere compared to the equatorial region. To establish a more quantative assessment of the interactions and their significance relative to interactions in the equatorial region, more experimental data on electron distribution functions for electrons in the MeV range and wave field amplitudes above VLF, LF, and MF transmitters are required. In addition, existing computer simulations should be extended to incorporate the case of interactions in the topside ionosphere.

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T. F. Bell, T. Neubert, and L. R. O. Storey, STAR Laboratory, Stanford University, STanford, CA 94305.

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