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Measurements of Siple Transmitter Signals on the DE 1 Satellite: Wave Normal Direction and Antenna Effective Length

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A new experimental technique is developed to simultaneously measure the wave propagation direction and the effective length of a small ($L << \lambda$) electric dipole antenna on a spin stabilized satellite in the magnetosphere. The technique relies on the near simultaneous measurement of single components of the electric and magnetic fields of a coherent VLF signal injected into the medium from a ground-based source. The spin fading characteristics of the signal received by the electric dipole and the magnetic loop antenna permit the measurement of the wave normal direction assuming whistler-mode propagation. In situ and remote measurements of the local cold plasma density are used to determine the refractive index. The wave electric field is then inferred from the wave magnetic field as measured on the loop antenna, the refractive index and the direction of propagation. Comparing this electric field with the measured voltage across the dipole antenna leads to the determination of the effective length of the receiving electric dipole. The technique is applied to data from the Dynamics Explorer 1 satellite observations of whistler mode signals injected into the magnetosphere from the Siple, Antarctica, VLF transmitter. In one case, with the measured background cold plasma density being 15 el cm^{-3}, the effective length of the 200 m long electric dipole antenna is found to be 222±56 m, i.e., about twice the conventional value.

1. INTRODUCTION

We report the measurements in the magnetosphere of the wave normal direction of coherent VLF whistler mode signals injected from Siple Station, Antarctica (76° S, 84° W, $L \simeq 4.2$) and received on the Dynamic Explorer 1 (DE 1) satellite. The technique developed for the wave normal measurement also leads to a direct estimation of the effective length of an electric dipole antenna on the satellite. A 200-m-long electric dipole antenna and a $\sim$1 m^2 single-turn loop are used in conjunction with the Stanford University linear wave receiver (LWR) to measure, respectively, the wave electric and magnetic fields. While the main purpose of this experiment is to study controlled and naturally occurring wave-particle interactions in the earth's magnetosphere, in this paper we report on the propagation characteristics of fixed-frequency Siple transmitter signals received at high altitudes on the DE 1 satellite. We introduce a new method to measure the wave propagation direction and the antenna effective length in the magnetosphere. This technique relies on a newly formulated technique for analysis of satellite data acquired on a spinning satellite [*Sonwalkar et al.*, 1984; *Sonwalkar*, 1985] and is based on inferring the wave normal directions from the observed fading characteristics of single measured components of the electric and the magnetic signals. The application of the method to DE 1 satellite data provides the first in situ measurement of the effective length of the 200-m-long electric dipole antenna aboard the satellite.

Small ($L << \lambda$) eletric dipole antennae are frequently used on satellite borne experiments to measure electric fields. The wave electric field is related to the voltage at the antenna terminals as

$$V_e = L_{eff}\mathbf{E} \cdot \mathbf{l} \tag{1}$$

where V_e is the open circuit voltage induced across the terminals of a small electric field dipole in the presence of an electric field $\mathbf{E}$, L_{eff} is the effective length of the antenna, and $\mathbf{l}$ is the unit vector parallel to the antenna length. In the past, experimenters have used $L_{eff} = L$ [*Neubert et al.*, 1983] or $L_{eff} = L/2$ [*Gurnett and Frank*, 1978; *Bell et al.*, 1981], where L is the physical length of the antenna. In free space, $L_{eff} = L$ if the incident field is assumed to induce a uniform current distribution along the antenna length, and $L_{eff} = L/2$ for a triangular current distribution [*Jordan and Balmain*, 1971].

A dipole antenna placed in a magnetoplasma interacts with the incident electromagnetic field in a complicated manner. The electrons and ions form a plasma sheath [*Balmain*, 1983] around the conductor so that the voltage induced across the antenna terminals is not entirely due to the incident electric field. Thus, on physical grounds, it is plausible that the relation between the voltage measured and the incident electric field may differ from that for free space. Neubert and coworkers [*Neubert et al.*, 1983] reported that the electric field calculated with $L_{eff} = L$ leads to abnormally small values of refractive index $\mu = c|\mathbf{B}|/|\mathbf{E}_\perp|$. They suggested that this discrepancy could be due to an overestimation of the electric field by a factor of 2 to 6. Accurate absolute value of electric field is important to direction finding experiments [*Lefeuvre et al.*, 1982] as well as to wave-particle interaction studies. For the latter, the magnitude of electric field is needed to determine thresholds for various instabilities, or for transition from linear to non-linear regimes. To deduce the absolute value from the measurement of only one electric field component requires the knowledge of the wave normal direction as well as L_{eff}.

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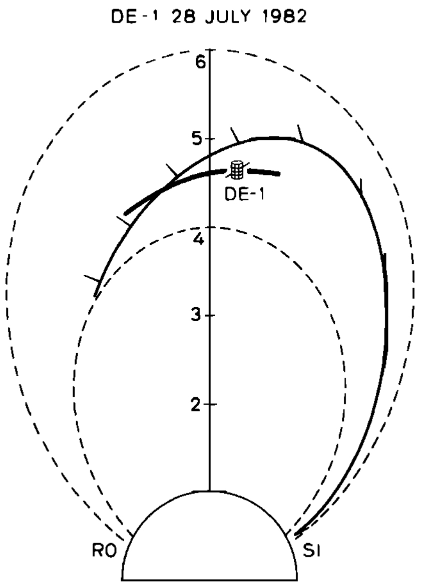

Fig. 1. Projection of the satellite trajectory onto the magnetic meriodinal plane on July 28, 1982. A sample propagation path from the Siple transmitter (SI) to the satellite is illustrated. The point magnetically conjugate to Siple is at Roberval, Canada (RO).

The need for a better understanding of the behavior of electric dipole antennae in a magnetic plasma is underscored by the widespread use of such sensors to study plasma waves in both the terrestrial and planetary magnetospheres and ionospheres [*Scarf et al.*, 1981]. They range in size from a few meters long with spherical elements at the ends [*Gurnett et al.*, 1969] to thin wire antennae of a few hundred meters in length [*Shawhan et al.*, 1981]. Both satellite and rocket borne experiments make use of such elements, and due to the relative ease of deployment and low cost they are usually preferred over magnetic loop antennae. The behavior of electric antennae in plasmas must be understood on a quantitative basis in order to realize the full benefit of these measurements.

In this paper, we assume that the relation (1) holds and develop a method to measure the wave normal direction and the effective length of a dipole antenna in a magnetoplasma in a self-consistent manner. The method is applied to DE 1 electric and magnetic field measurement of Siple transmitter signals in order to calculate the effective length of a 200 m long dipole antenna aboard the satellite and the wave normal direction of the Siple signals received on the high altitude DE 1 satellite. The latter constitutes the first direct wave normal measurement of a coherent whistler mode signal using only single electric and magnetic sensors and is in itself important to our understanding of the propagation characteristics and interactions with energetic particles of nonducted whistler-mode waves in the magnetosphere [*Bell*, 1984]. A previously reported wave normal measurement on the ISEE-1 satellite with only one electric field antenna required the additional assumption that the waves were confined to the magnetic meridional plane [*Sonwalkar et al.*, 1982, 1984]. This assumption is not required in the case of DE 1 satellite since the fading characteristics of the magnetic field measurement provides the additional information needed to find the wave normal in three dimensions.

Before we introduce the method, a few general remarks about relationship (1) are in order. First, the relation implies that the voltage induced is related only to the component of the electric field parallel to the antenna orientation (or length) vector. This assumption is justified since the antenna wire thickness is much smaller than its length. Second, the relationship (1) is linear, an assumption well satisfied in practice. For example, constant frequency signals transmitted from ground based VLF transmitters are received at the same frequency (except for a small doppler shift due to the motion of the spacecraft) on ISEE-1 and DE 1 satellites [*Bell et al.*, 1981, *Inan and Helliwell*, 1982] with no higher harmonics being observed. Third, the relationship (1) is isotropic. Since the magnetoplasma is anisotropic with an axis of symmetry (the static magnetic field), it is possible that L_{eff} is a function of the angle θ_a between the static magnetic field and the antenna orientation vector **l**. This aspect is treated in detail elsewhere [*Sonwalkar*, 1985], where it is shown that any anisotropy in L_{eff} would necessarily produce harmonics of the spin frequency in the measured voltage envelope. ISEE-1 and DE 1 observations analyzed so far generally indicate that such harmonics, if present, are at least 20 dB below the fundamental component. Therefore our assumption that the dipole receiving properties are isotropic as implied by (1) is justified.

The calibration of a 200-m-long electric wire antenna in a magnetoplasma at VLF frequencies is itself a difficult problem, primarily because the calibration requires an independent estimate of the wave electric field for comparison with the response of the antenna. Suppose for example that a fixed-frequency pulse of known power is transmitted from the ground up to a high-altitude satellite. To estimate the wave electric field in the magnetosphere one needs to take into account the losses within the earth-ionosphere waveguide, the absorption during the propagation through the lower ionosphere and the spreading loss during propagation through the magnetosphere, a medium which is both anisotropic and inhomogeneous [*Helliwell*, 1965]. This, in general, requires a ray tracing analysis which in turn depends critically on the background plasma densities throughout the region of propagation.

Another method for estimating the electric field is to infer it from a measurement of the wave magnetic field. The magnetic field measurement is considered more reliable be-

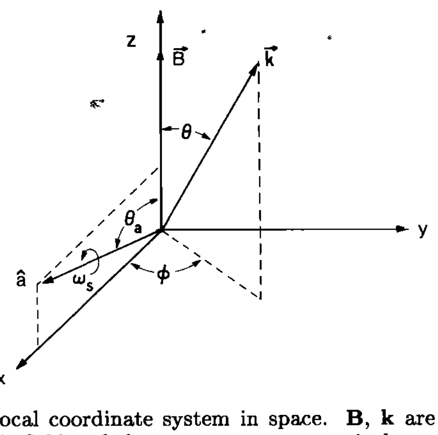

Fig. 2. Local coordinate system in space. **B**, **k** are the local geomagnetic field and the wave vector, respectively, and **a** is the spin axis of the spacecraft.

cause (1) small magnetic loops ($\sim$1 m^2) can be calibrated on ground, (2) magnetic field measurement is less likely to be affected by plasma sheath problems. However, the estimation of electric field from the magnetic field requires knowledge of the wave normal direction and the refractive index, as well as the mode of propagation. In this paper, we apply this method to a 4.025 kHz signal observed at high altitudes near the magnetic equatorial plane. Since the frequency is lower than the electron gyrofrequency and higher than the proton gyrofrequency, the propagating mode can be assumed to be whistler mode [*Stix*, 1963]. We show later that this assumption is strongly supported by the consistency of the whistler mode relationships as well as the comparison of the measured group time delay with that expected for whistler-mode propagation. The refractive index for a plane whistler mode wave propagating through a homogeneous and uniform magnetoplasma is a function of the propagation direction of the wave with respect to the geomagnetic field. To apply the method in such a case requires that (1) a single plane wave be present at the time of measurement, (2) the wave normal direction be determined by means which do not involve absolute values of the electric field, and (3) the local values of plasma frequency and gyrofrequency be measured. The electric field calculated using this method can be compared with the measured voltage across the dipole antenna terminals in order to estimate L_{eff}. Note that this method is based on local measurements, and therefore detailed data on the global ionospheric and the magnetospheric parameters are not required.

The characteristics of antennae in plasmas have been studied extensively since the 1950's (for reviews see *Wait* [1968], *Ohnuma* [1978], and *Balmain* [1979, 1983]). However, as Balmain [1983] concludes in his review paper, there currently exist no generally applicable treatment of dipole antennae in a magnetoplasma for determining the current distribution, input impedance and radiation fields. While a detailed discussion of past research is beyond the scope of this paper, two facts are relevant to our study. First, most of the studies on antennae in plasmas have been directed towards the radiation properties of a dipole when placed in a magnetoplasma. Second, while the reciprocity theorem has been formulated by *Ishizone* et al. [1976] for a multifluid isotropic plasma represented using scalar-pressure theory, no corresponding treatment is available for an anisotropic medium. Our study is specifically directed toward the receiving electric field dipole antenna in an anisotropic medium. The results we describe in this paper were first reported at the 1985 summer conference of American Geophysical Union in Baltimore [*Sonwalkar and Inan*, 1985].

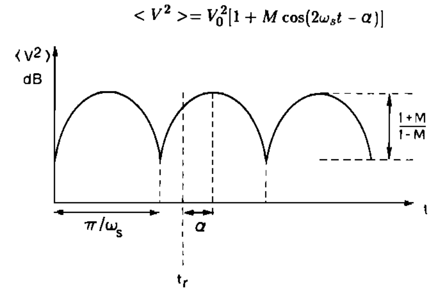

Fig. 4. Schematic of the voltage induced at the antenna terminals on a spinning satellite for an incident plane wave. The measured fading parameters are M and α.

In the following we first discuss the analysis method and the theoretical considerations in section 2 and give a brief description of the Stanford experiment on the DE 1 satellite in section 3. The application of the method to the electric and magnetic field data received on the DE 1 satellite is given in section 4, and discussion and summary are presented in sections 5 and 6.

2. DESCRIPTION OF THE METHOD

Consider a constant frequency (ω) signal transmitted from a ground based source (Siple transmitter), that is received by single electric and magnetic dipole antennae on a spinning satellite (DE 1) at high altitudes in the magnetosphere (Figure 1). The coordinate system is described in Figure 2, where the z axis is taken to be along the earth's magnetic field $\mathbf{B}_0$, the spin axis $\mathbf{a}$ is contained in the xy plane, and the y axis is given by the right-hand rule. Figure 3 shows the position of the antennae in the spacecraft coordinate system $(\mathbf{p}, \mathbf{q}, \mathbf{a})$. The unit vectors along three axes are given by

$$\begin{aligned} \mathbf{p} &= \cos\theta_a \mathbf{x} - \sin\theta_a \mathbf{z} \\ \mathbf{q} &= \mathbf{y} \\ \mathbf{a} &= \sin\theta_a \mathbf{x} + \cos\theta_a \mathbf{z} \end{aligned} \tag{2}$$

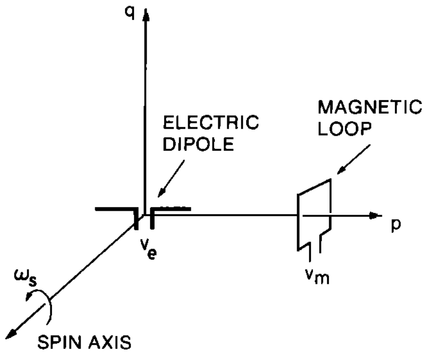

Fig. 3. Schematic of the antenna orientation in the spin plane. The spacecraft coordinate system is defined by equation (2). Electric and magnetic antennae spin at the frequency ω_s rad/s in the spin plane measuring the components of electric and magnetic fields. The length of the electric dipole and the axis of the magnetic loop are pointing in the same directions as they spin.

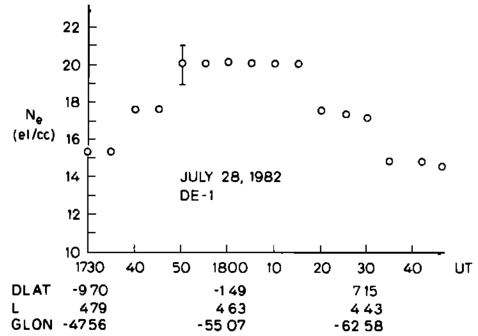

Fig. 5. In situ electron density values deduced from the measured upper hybrid resonance frequency. The geomagnetic latitude (λ_m), the L value and the geographic longitude (ϕ_g) are indicated as a function of time.

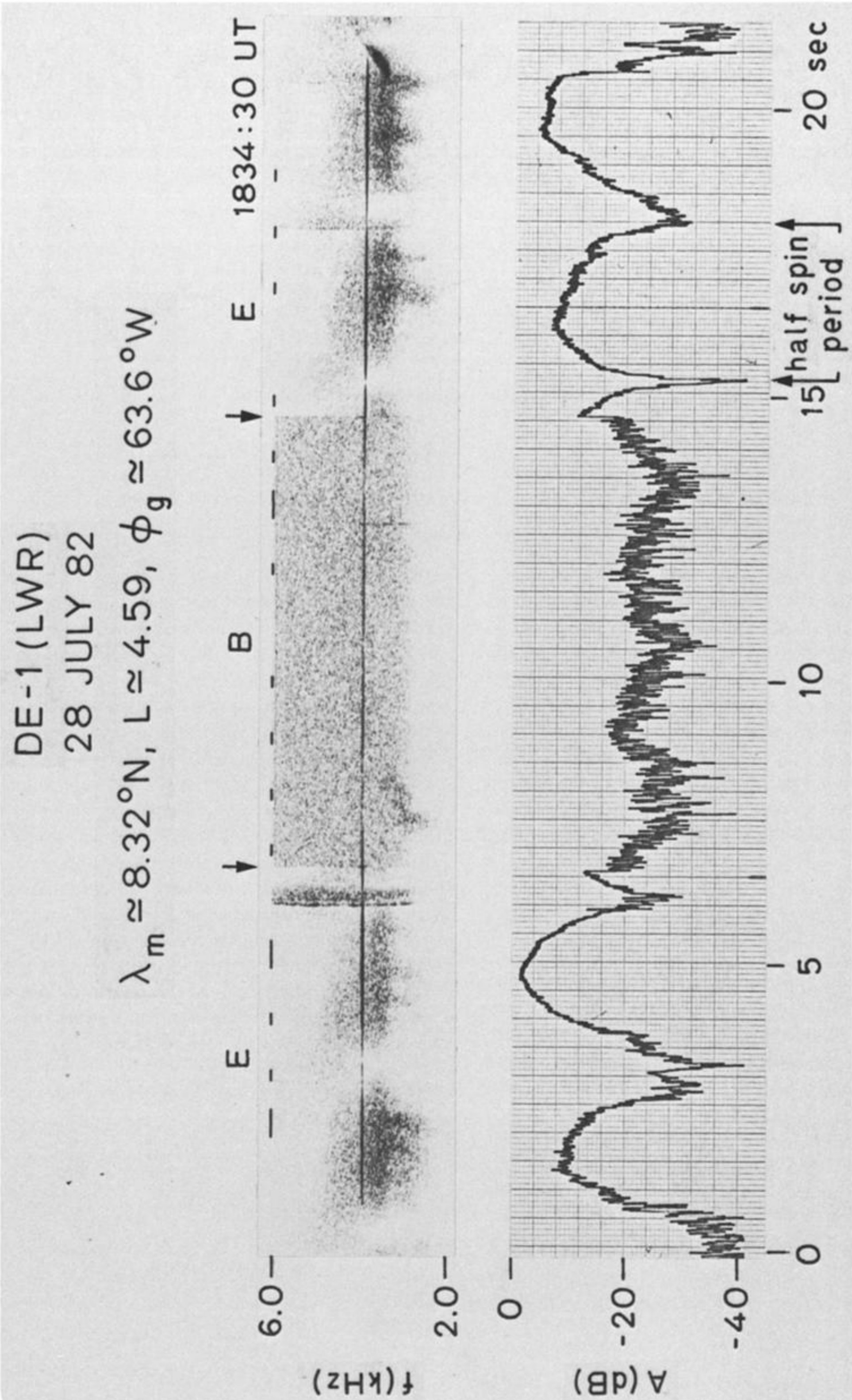

Fig. 6. An example of 20-s-long Siple transmitter pulse received on DE 1 satellite when the receiver was toggled between E and B antennae

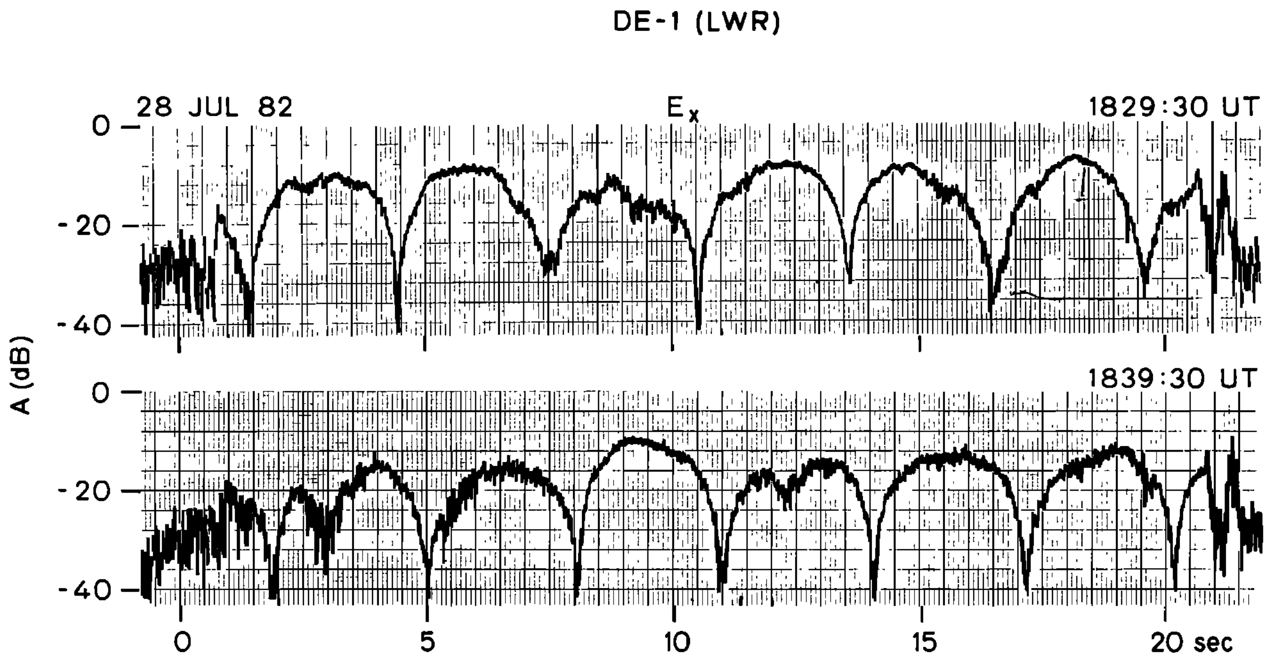

Fig. 7. Two examples of near plane waves received on the DE 1 satellite. The receiver was connected to the E antenna for these cases.

Both antennae spin about the spin axis at an angular velocity ω_s. Also note that at any given time both antennae measure the field components along the same direction. The reference time t_r is taken to be the time when the electric dipole (and the axis of the magnetic loop) is aligned along the p axis. The phase α_{sp} of the antenna in the spin plane is the angle between $\mathbf{l}$ and $\mathbf{p}$. It can be shown [*Sonwalkar*, 1985] that if the signal arrived at the satellite via a single ray path, with the wave normal direction given by (θ, ϕ), then the envelope of the received voltage across the electric field antenna is

$$
\begin{aligned}
< V_e^2(t) > &= V_{0e}^2(1 + M_e \cos(2\omega_s t - \alpha_e)) \\
V_{0e}^2 &= \frac{1}{4} L_{eff}^2 |E_0|^2 A_e
\end{aligned} \tag{3}
$$

where E_0 is a complex constant proportional to the electric field strength and ω_s is the spin freqency. The parameters A_e, M_e, and α_e are known functions of the wave normal direction (θ, ϕ), the spin axis orientation vector $\mathbf{a}$, the phase α_{sp} of the antenna length vector in the spin plane at a specified time, and the medium parameters (i.e., plasma frequency ω_p and the electron gyrofrequency ω_H) when the wave is assumed to be propagating in the whistler mode.

Similarly, the envelope of the measured voltage across the magnetic field antenna is given by

$$
\begin{aligned}
< V_m^2(t) > &= V_{0m}^2(1 + M_m \cos(2\omega_s t - \alpha_m)) \\
V_{0m}^2 &= \frac{1}{4} S_{eff}^2 \frac{k^2}{\omega^2} |E_0|^2 A_m
\end{aligned} \tag{4}
$$

where S_{eff} is defined by the following relationship.

$$V_m = S_{eff} \mathbf{B} \cdot \mathbf{n} \tag{5}$$

where V_m is the voltage measured across the magnetic loop antenna in the presence of the wave magnetic field $\mathbf{B}$, $\mathbf{n}$ is unit vector normal to plane of the loop, and S_{eff} is the proportionality constant. Here S_{eff} is the magnetic counterpart of L_{eff}, and is given as $S_{eff} = \omega A$, where ω is the signal frequency and A is the area of the loop.

In equation (4) A_m, M_m, α_m are known functions of (θ, ϕ), $\mathbf{a}$, α_{sp}, and medium parameters, i.e., ω_p and ω_H, whereas k, the wave vector magnitude is a known function of (θ, ϕ) and ω_p and ω_H. Figure 4 shows an ideal plane wave voltage envelope received by a spinning satellite. The parameters M_e and M_m are called the "depth of fading" and the parameters α_e and α_m are termed the "phase of fading".

Equations (3) and (4) have a simple physical interpretation. The electric (or the magnetic) field of a whistler mode signal is in general elliptically polarized and the properties of the polarization ellipse are functions of the wave normal direction for known medium parameters [*Stix*, 1963]. In the spin plane the fields map onto another ellipse which now is a function of both the wave normal direction as well as the spin axis. A slowly spinning antenna ($\omega_s << \omega$) samples the properties of this ellipse.

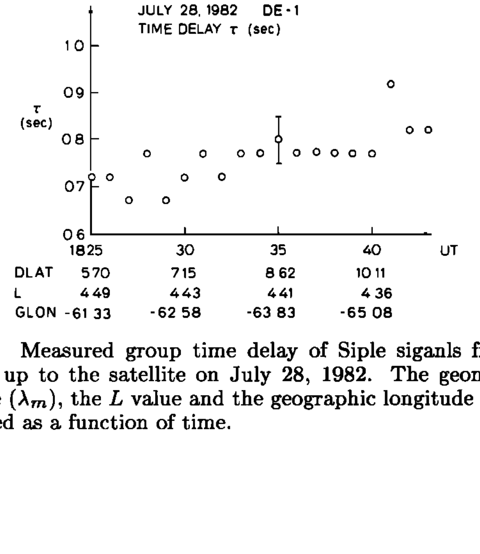

Fig. 8. Measured group time delay of Siple siganls from the ground up to the satellite on July 28, 1982. The geomagnetic latitude (λ_m), the L value and the geographic longitude (ϕ_g) are indicated as a function of time.

TABLE 1. Satellite Attitude and Medium Parameters

θ_a	t_r, s	w_{sp}, rad/s	n_e, el/cm^3	B_0, G
$92.61° \pm .01°$	$44.45 \pm .05$	$1.031 \pm .001$	15 ± 1	$0.0034 \pm .00001$

On this basis wave normal direction (θ, ϕ) can be determined from the measured fading parameters $\overline{M}_e$, $\overline{M}_m$, $\overline{\alpha}_e$, $\overline{\alpha}_m$. We note here that each of these parameters is independent of the absolute magnitudes of the electric or magnetic field. The wave normal angle is obtained by simultaneous solution of the following equations.

$$\begin{aligned} M_e(\theta,\phi) &= \overline{M}_e \\ \alpha_e(\theta,\phi) &= \overline{\alpha}_e \\ M_m(\theta,\phi) &= \overline{M}_m \\ \alpha_m(\theta,\phi) &= \overline{\alpha}_m \end{aligned} \tag{6}$$

Given the satellite attitude information ($\mathbf{a}$, ω_s, α_{sp}), medium parameters ($\mathbf{B}_0$, ω_p), and the wave frequency (ω), the functions on the left hand side of equation (6) are known functions of the wave normal angle (θ, ϕ) based on the whistler mode relationships. Each of the equations in (6) represents a contour in the (θ, ϕ) plane and the intersection point(s) of all the contours represent the graphical solution(s) of the equation (6). Note that existence of at least one common point in the (θ, ϕ) plane for all four contours is a necessary requirement for the wave to be a plane wave propagating in the whistler mode.

Using equations (1), (3), (4) and (5), we obtain

$$E_{max} = \left(\frac{cB_{max}}{\mu}\right)\left(\frac{A_e}{A_m}\right)^{1/2}\left(\frac{1+M_e}{1+M_m}\right)^{1/2} \tag{7}$$

where E_{max} and B_{max} are the peak values of the electric and magnetic field in spin plane and μ is the whistler mode refractive index.

The procedure to estimate L_{eff} is now straightforward. Using the estimated wave normal direction (θ,ϕ), A_e, A_m and μ are calculated, B_{max} is obtained from the measured V_{0m} and equation (5) and E_{max} is calculated from equation (7). Finally, L_{eff} is found from equation (1).

3. EXPERIMENT BACKGROUND

The experimental data used in this paper were obtained from observations of Siple transmitter signals on the DE 1 satellite. The experiment has two main components: (1) A broadband (1–20 kHz) VLF transmitter located at Siple Station, Antarctica [*Helliwell and Katsufrakis*, 1974], and (2) a linear wave receiver (LWR) which is integrated into the plasma wave instrument on the Dynamic Explorer 1 (DE 1) satellite [*Shawhan et al.*, 1981]. The receiver measures wave amplitude in the frequency range 1.5–16 kHz. The gain of the amplifier can be set at 10 dB steps over a 70 dB range and can be varied automatically or can be commanded to remain fixed at any level. In the automatic mode the gain is updated every 8 s. The response is linear over a 30 dB range in any gain position, thus facilitating accurate measurement of signal intensity and temporal growth rate. The LWR can be commanded to cycle between a 200 m long eletric dipole or a 0.8 m by 1.25 m single loop magnetic antenna (threshold sensitivity 6 × 10^{-10} $\gamma/(Hz)^{1/2}$ at 6 kHz). The input impedance of the LWR preamplifier is $\geq 10^9$ Ω. During the DE 1/Siple wave injection experiments several transmission formats are utilized including fixed frequency pulses of variable duration and frequency ramps of both positive and negative slopes. Each format is designed to investigate one or more specific questions concerning the physics of wave propagation and wave particle interactions in the magnetosphere. In this paper we concentrate on the analysis of fixed frequency pulses (4.025 kHz) of 20–s duration.

4. EXPERIMENTAL RESULTS

On July 28, 1982, Siple transmitter signals were continuously received on the DE 1 satellite during the period 1800–1900 UT. Figure 1 shows the projection of the orbital segment of the satellite pass in the magnetic meridional plane. In the following we separately discuss the measurements of the medium and wave parameters and the wave normal analysis.

Medium Parameters

The relevant medium parameters for the present study are the local geomagnetic field and the local electron density. The geomagnetic field is obtained from the triaxial fluxgate magnetometers [*Farthing et al.*, 1981]. The electron density measurement is performed by two independent methods. First, the electron plasma frequency is obtained from the measured upper hybrid frequency and the computed electron gyrofrequency. The in situ electron density measured with this technique was provided to us by the University of Iowa (courtesy of D. A. Gurnett) and is shown in Figure 5. Second, the equatorial electron density is estimated from ground whistler measurements [*Carpenter*, 1970]. This gives an electron density of 10-15 el/cm^3 at L=4.5, consistent with the in situ measurement. Such low values suggest that the signal was received outside the plasmapause, consistent with the measured high values (~ 4) of K_p index in the preceding 12 hour period, from which an approximate

TABLE 2. Measured Wave Parameters

f, KHz	M_e	α_e	M_m	α_b	$[V_e]_{max}$, mV	$[V_m]_{max}$, mV
4.025 ± 0.001	0.993 ± 0.003	$-70° \pm 10°$	0.82 ± 0.05	$+80° \pm 10°$	0.71 ± 0.08	0.118 ± 0.006

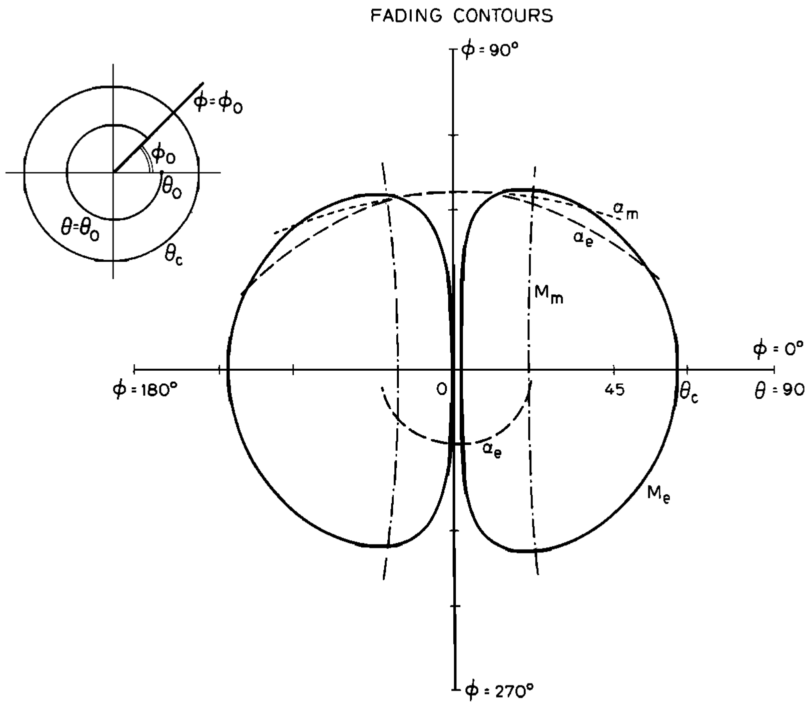

Fig. 9. Fading contours representing a graphical solution to equations (6). The two points where all four contours intersect are the two possible wave normal angles.

estimate of the plasmapause location can be deduced to be $L \simeq 3.5$ [*Carpenter and Park*, 1973].

Wave Measurements

Figure 6 shows a sample of the data received on July 28, 1982. The top panel shows the spectrogram showing the reception of a 20-s-long, 4.025-kHz continuous wave Siple transmitter signal received on the DE 1 satellite. During the reception of this pulse the LWR was toggled between E and B antennae. The second panel shows the amplitude of the electric and the magnetic field within a 100-Hz band centered at 4.025 kHz. Figure 7 shows two other examples of 20-s-long pulses received on DE 1, at times during which the LWR was connected to the electric field antenna. The wave data were analyzed for group time delays and the wave normal directions; the latter analysis is described in the following subsection.

Group travel times from the ground up to the satellite were measured with ±25 ms accuracy both from the spectrogram and the amplitude records. The time delay was found to vary from ~0.72 s to 0.82 s over the period of 1825–1843 UT, as shown in Figure 8. The low values of time delay (along with the ray tracing simulations of wave propagation) suggest that the waves have arrived at the satellite over direct ray paths without involving reflections. This observation allows us to assume that $\mathbf{k} \cdot \mathbf{B} > 0$, a fact that is used in the wave normal analysis.

Wave Normal Analysis and the Measurement of L_{eff}

Three 20-s-long pulses shown in Figures 6 and 7 were analyzed for the wave normal direction. All of these represent a single plane wave propagating in the upward direction ($\mathbf{k} \cdot \mathbf{B} > 0$). That the wave packet consists predominantly of a single plane wave is deduced from the fact that fading is seen dominantly at twice the spin frequency. Superposition of wave components arriving from multiple directions would result in additional frequencies in the amplitude envelope, due to the different doppler shifts [*Sonwalkar*, 1985]. The long time scale (~ 10 s) modulation seen in the amplitude envelopes is attributed to closely spaced direct multiple paths [*Sonwalkar et al.*, 1982, 1984]. It can be shown [*Sonwalkar*, 1985] that for such closely spaced plane waves the fading parameters give the average wave normal angle. For the purpose of this paper the pulses shown in Figures 6 and 7 can be practically treated as plane waves.

We first discuss in detail the 20-s-long pulse received at 1834:30 UT (Figure 6), since both the electric and magnetic field measurements are available for this case.

TABLE 3. Results

θ	ϕ	μ	$\left(\frac{E}{cB}\right)_{max}$	B_{max}, pT	E_{max}, μV/m	L_{eff}, m
$53° \pm 2°$	$65° \pm 2°$	12.39 ± 1.24	0.35 ± 0.03	0.0317 ± 0.0016	3.34 ± 0.45	219.8 ± 53.5
$51° \pm 3°$	$110° \pm 3°$	13.23 ± 1.05	$0.34 \pm .04$	0.0317 ± 0.0016	3.29 ± 0.50	224.6 ± 58.4

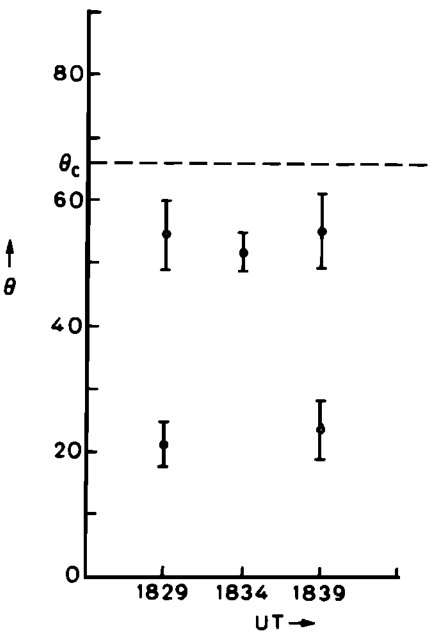

Fig. 10. Wave normal distributions in terms of the elevation angle θ.

Table 1 summarizes the measured satellite attitude information, medium parameters, and fading characteristics for this pulse. The antenna is almost perpendicular to the local $\mathbf{B}_0$ ($\theta_a = 92.61°$). The reference time t_r is the time when the antenna is parallel to p axis. The spin period is 6.08 s resulting in a fading period of 3.04 s. Table 2 summarizes the measured wave parameters.

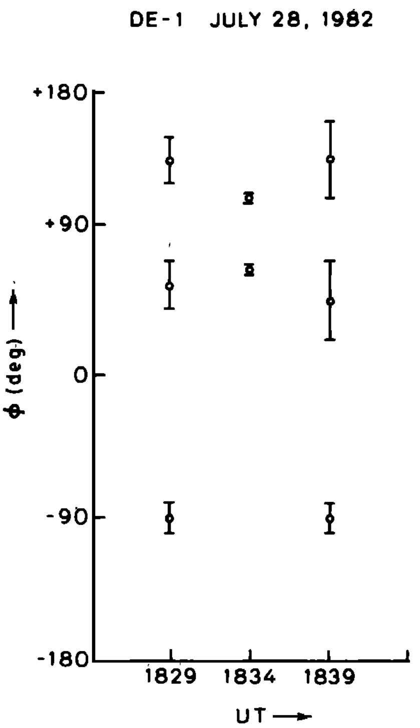

Fig. 11. Wave normal distributions in terms of the azimuthal angle ϕ, showing the orientation in the plane perpendicular to $\mathbf{B}$ (see Figure 2).

Figure 9 shows the results of the application of a computer code that is used to compute and plot the four contours given by equation (6). In this figure, θ is plotted along the radial direction, whereas ϕ is plotted in the polar direction. Note that all four contours intersect at two locations in the (θ, ϕ) plane. The two values result from the fact that an additional symmetry is introduced by virtue of spin axis being almost perpendicular to the local geomagnetic field. The possible directions of propagation have similar θ values, but different ϕ values. The fact that four contours based on four different measurements intersect at one (or more) points confirms that the signal is indeed a single plane wave that propagates in the whistler mode.

Table 3 summarizes the results, including the magnetic field amplitudes determined from the ground calibration and the refractive index estimated using the whistler–mode relationship. Note that either of the two possible wave normal directions lead to similar values of L_{eff}. Considering the extreme values, we estimate that $L_{eff} = 222 \pm 56$ m, or $L_{eff} = (1.11 \pm 0.28)L$. Figures 10 and 11 show the estimated wave normal angles θ and ϕ for all the three pulses shown in Figures 6 and 7. The two pulses shown in Figure 7 lead to three different possible wave normal angles. This is due to the fact that for these pulses we have only the electric field data available and the symmetry introduced by $\theta_a \simeq 90°$. Taking into consideration that the pulse at 1834:30 has yielded high wave normal angles unambiguously, and also based on the ray tracing simulations described below, it is more likely that these two pulses also have large wave normal angles.

Ray Tracing Analysis

The ray tracing calculations were carried out using the Stanford VLF ray tracing program [*Inan and Bell*, 1977]. The plasmapause location was estimated at L=3.5 using an empirical formula given by [*Carpenter and Park* [1973]. Outside the plasmapause an r^{-1} dependence was assumed, with the density at $L = 4.5$ being ~ 10 el/cm^3, consistent with the observed electron densities shown in Figure 5. Figures 12 shows the equatorial electron density profile used in the ray tracing simulations, and Figure 13 shows typical ray paths and the large wave normal angles near the satellite trajectory at the times Siple signal was observed. The ray tracing

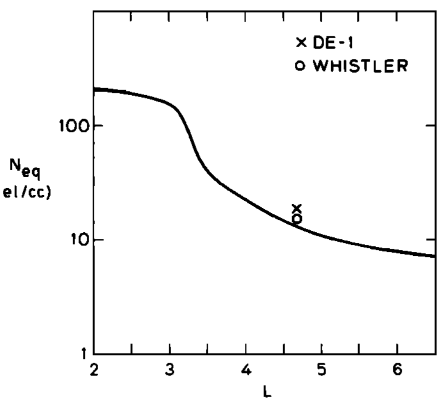

Fig. 12. Equatorial electron density used in the ray tracing calculations. The distribution of density along the field lines in the L=4–5 range was assumed to be governed by a collisionless model [*Inan and Bell*, 1977].

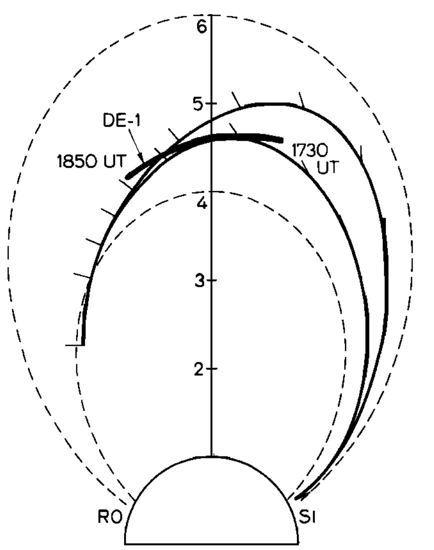

Fig. 13. Example of raytracing calculations. The calculated time delay is ~ 0.75 s and the wave normal is ~ 60°.

analysis results in group time delays of ~ 0.75/ s, a wave normal angle of ~ 60° and wave magnetic field intensity of order 0.1 pT, consistent with our measurements. The wave field intensity quoted here is based on an approximate estimate determined by accounting for the losses during the earth–ionosphere waveguide propagation [*Helliwell*, 1965], the absorbtion during crossing of the lower ionosphere, the spreading of the ray paths and the whistler mode relationship between the power density at the satellite location and the wave magnetic field intensity for a ~ 60° wave normal angle.

5. CONCLUSIONS AND DISCUSSION

We have presented a new method to estimate wave propagation direction for a whistler mode signal and the effective length of a small electric dipole antenna in a magnetoplasma. The method has been applied to data from the DE 1 satellite to measure a wave normal angle of ~ 52° for Siple transmitter signals observed on the DE 1 satellite and to obtain the effective length of 222 ± 56 m for a 200 m long electric dipole on the satellite.

The in situ calibration of an electric dipole requires that (1) the wave be plane, (2) the wave normal direction be determined idependent of the absolute electric field magnitude, (3) the waves be travelling in the whistler mode, (4) the magnetic antenna calibration on the ground be valid in the magnetosphere. We discuss below each of these points with respect to our data. The theoretical necessity for these requirements are discussed earlier in sections 1 and 2.

The plane wave condition is required so that a single refractive index can be assigned to the incoming wave. A plane wave leads to fading at twice the spin frequency, $2\omega_S$. Presence of multiple paths would lead to addition of signals coming from different directions with different doppler shifts These doppler shifted frequencies beat with each other to produce addional nulls (or periodicities) in the observed amplitude envelope. For our example this differential doppler shift is of the order of 1 Hz. With a 20-s-long pulse, the frequency resolution is 0.05 Hz, and a multipath would be detected in terms of fading at additional frequencies. In fact, it is not possible to decide whether the wave is plane unless the pulse is long enough. This basic problem for short pulses can possibly be tackled by posing the direction finding as a statistical estimation problem [*Storey and Lefeuvre*, 1974; *Shawhan*, 1983; *Sonwalkar*, 1985].

The plane wave condition can be relaxed in cases when more antennae are available on the satellite and an accurate determination of signal frequency (or phase) is possible. The multipath wave structure can then be estimated and a proper connection between **E** and **B** can be made [*Sonwalkar*, 1985].

The analysis above has assumed that the wave mode is known. Numerous ground–based studies have verified that the whistler mode dispersion relation is satisfied for electromagnetic waves at frequencies of a few kilohertz propagating in the inner magnetosphere where such frequencies lie between the electron and proton gyrofrequencies. The relations for the fading parameters take into account the whistler mode polarization and refractive index. The intersection of the four contours in Figure 9 is an alternate check to verify that these relations are satisfied. The consistency of the measured group time delay with that estimated from the ray tracing analysis is another independent verification of the whistler mode propagation.

The method assumes that the magnetic field measurement is correct. As mentioned before this assumption is based on the fact that an accurate ground calibration for a small loop is possible, and that the magnetic field measurement is not likely to be affected by the plasma sheath problems.

Our result indicate that 166 m $< L_{eff} <$ 283 m. This is a very conservative estimate that takes into account propagation of all the measurement errors. The physical length of the antenna is 200 m. Within experimental errors we have shown that the effective length, L_{eff}, of a small electric dipole is equal to its physical length, L. Neubert and coworkers have used $L_{eff} = L$. They found a descrepancy of a factor of 2 to 6 in the measurement of the refractive index using the relation $\mu = c|\mathbf{B}|/|\mathbf{E}_\perp|$. The cause of the discrepancy was attributed to overestimation of the electric field by the same factor. Our result with its upper extreme value accomodates a factor of 1.4. The factor of 6 is not consistent with our result as it would imply an effective length of ~ 1200 m. The relation $\mu = c|\mathbf{B}|/|\mathbf{E}_\perp|$ holds only for a plane wave case. Neubert and coworkers noticed that the amplitude envelope of the Omega transmitter pulses received on Geos 1 satellite showed time variation representative of multiple paths. Thus it is possible that the discrepency found by Neubert and coworkers is a result of incomplete account of the wave normal angles (multiple paths) and plasma density. We draw the reader's attention to equation (7) which relates the peak values of the electric and magnetic field in the spin plane. Note that this relationship involves both a dependence on the wave normal direction (θ, ϕ) as well as the geometrical factors arising out of the measurement process. Therefore incorrect evaluation of either of these factors may lead to incorrect estimation of the ratio $cB/E_\perp$. Morover as mentioned before, the simple relationship as in equation (7) holds only for a single wave normal case. When the waves arrive from more than one direction, the relationship is far more complex and determination of all the wave normals will be required before any connection can be made between the electric and the magnetic field [*Sonwalkar*, 1985].

As described by [*Shawhan et al.*, 1981], the electric antenna on the DE 1 satellite has in insulated inner section and a conducting tip. Because of the insulation over the wire, the effective length at lower frequencies where the antenna is resistively coupled to the plasma, is longer than at higher frequencies where the antenna is capacitively coupled to the plasma. At low frequencies (dc) the effective length is usually assumed to be the distance between the center of the conducting section (given by Shawhan et al. as 173.1 m). At high frequencies (ac) the effective length is taken to be one-half of the tip to tip length, being 101.4 m for the DE 1 electric field antenna [*Shawhan et al.*, 1981]. The measured effective length of 222 ± 56 m is consistent with the dc effective length 173.1 m. Thus it is possible that the antenna is operating in the low frequency resistive-coupled region, even at frequencies of a few kilohertz.

We conclude the discussion by drawing reader's attention to the generality of the method. First, the method can be applied in any region of the magnetosphere or in the ionosphere. In the latter case, the electron densities are high, and so pulses shorter than 20 s will suffice to discriminate between multiple paths. Second, even though in this paper we have assumed whistler mode propagation, the method holds for any other mode of propagation if the appropriate polarization relationship are included in equation (6). The analysis for more than two antenna is also a straightforward extension.

6. SUMMARY

We have presented a detailed analysis of a coherent whistler mode signal received on a spinning satellite and deduced the wave propagation direction from the measurement of only two field components. We have also confirmed that the signals arriving at the satellite satisfy the whistler mode polarization relationship. This has led to the first direct estimation of the effective length of a small electric field dipole antenna in a magnetoplasma. It appears that for a background electron density of $\sim$15 el cm^{-3} the dipole antenna effective length is approximately equal to its physical length. While the dependence of L_{eff} on plasma density as well as wave frequency remains to be determined, this result implies that absolute electric fields inferred using the conventional $L_{eff} = 0.5L$ may be in error by a factor of ~ 2.

The quantitative results can be briefly summarized as follows. Large wave normal angles of $\sim 52°$ were observed on July 28, 1982, when DE 1 was at L=4.6 and at a geomagnetic latitude of $\lambda_m = 8.32°$. The measured wave magnetic and electric fields were, respectively, 0.03 mγ, and 3.3 μV/m. For a propagation path outside the plasmapause ($L_{pp} \sim 3.5$) in a region where the local electron density was $\sim$ 15 el/cm^3, the effective length for the 200–m–long electric dipole antenna on DE 1 satellite is measured to be 222$\pm$56 m. Finally, we note that the ray tracing simulations are in good agreement with the experimental results.

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