Derivation of Magnetospheric Electric Fields From Whistler Data in a Dynamic Geomagnetic Field

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The whistler method of determining magnetospheric electric fields has until recently been applied on the assumption of a static dipole geomagnetic field. We consider the effect on the whistler analysis of including both departures of the field from a dipole and temporal variations in the field. Departures from a dipole field appear to have relatively small effects on the analysis or can be relatively easily taken into account when it is necessary. Temporal changes present a more serious problem. In the presence of a changing geomagnetic field the temporal change in equatorial gyrofrequency of a drifting whistler path will consist of two parts: the variation due to radial drift of the path in the presence of the inhomogeneous B field and the variation due to the changes in B with time. Given knowledge of the temporal variation of B, it is possible to infer the total (induced plus potential) electric field associated with the radial drift. A substorm event is analyzed in which cross-L inward drifts near $L = 4$ occurred as the night side B field (as reflected in the low-latitude H component) exhibited a rapid increase. It was found that on a time scale of 15 min or greater the inferred total east-west electric field differed by ~20% from the field estimated on the assumption of a static magnetic field. It is possible that effects of $\dot{B}$ may be relatively more important on a shorter time scale or during disturbances much larger than those thus far investigated. Also, in one quiet period preceding a substorm it appeared that all or a major part of the inferred cross-L motions of the plasma could have been due to fluctuations in the magnetic field. It is stressed that both equatorial and ionospheric measurements of electric fields are needed, since certain types of distortions of the high-altitude magnetosphere are not readily observed at ionospheric heights.

For several years VLF whistler data have been used to determine the westward component of the equatorial electric field in the magnetosphere between $L \sim 2.5$ and $L \sim 5$ [*Carpenter and Stone*, 1967, 1968; *Carpenter*, 1970; *Carpenter et al.*, 1972]. This determination is possible because the whistler frequency of minimum travel time, or nose frequency, is nearly proportional to the minimum magnetic field in the duct guiding the whistler [*Smith*, 1960, 1961]. Hence a variation in nose frequency during a period of repeated whistler propagation along a duct reflects a corresponding variation in the minimum magnetic field in the duct. The westward electric field component can then be determined if the duct magnetic field variation is due to radial $\mathbf{E} \times \mathbf{B}$ drift and if the magnetic field is known with reasonable accuracy.

The analysis of whistler data has for the most part been based on the assumption that the geomagnetic field is a static dipole field. This approximation should certainly be good as long as the ring current is weak and the magnetic field is quiet. On such occasions the electric field is also expected to be weak. However, stronger electric fields occur mainly during disturbed times, when the ring current is strong or strongly variable. Thus the magnetic field fluctuations and the deviations from dipole geometry introduce certain systematic errors in a calculation of the electric field if the magnetic field is assumed to be dipolar.

The purpose of the present paper is to estimate the magnitude of these errors and find ways to make the appropriate corrections. We point out some particular advantages of the whistler method for studying the total magnetospheric electric field, including the effects of $\partial \mathbf{B}/\partial t$, and present some preliminary results on the relative importance of $\partial \mathbf{B}/\partial t$ effects in substorms.

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THEORY

Smith [1960, 1961] and *Angerami* [1966] have shown that the nose frequency f_n and the minimum electron gyrofrequency f_{ge} in a whistler duct are nearly proportional; that is,

$$f_n \cong k f_{ge} \tag{1}$$

where k is a constant. (In the $f_n - f_{ge}$ relation there is actually a slight dependence on the distribution of plasma along the field lines and on the geometry of the field [*Smith*, 1960; *Sagredo and Bullough*, 1972; *Likhter and Molchanov*, 1968; *Park*, 1972].) For a diffusive equilibrium model of the field line distribution of ionization in a dipole field, $k \approx 0.38$. For a dipole field, f_{ge} is the equatorial gyrofrequency. When the expression for the gyrofrequency is used, (1) can be written

$$f_n \cong k(e/2\pi m_e)B_m = B_m/k_1 \tag{2}$$

where B_m is the minimum magnetic field along the duct and $k_1 = 0.94 \times 10^{-10}$ T Hz^{-1} (SI units).

If B_m changes with time and if the whistler duct is simultaneously convected perpendicular to the magnetic field,

$$k_1\, df_n/dt = dB_m/dt = \partial B_m/\partial t + \mathbf{v} \cdot \nabla B_m \tag{3}$$

where $\mathbf{v}$ is the convection velocity

$$\mathbf{v} = B_m^{-2}(\mathbf{E} \times \mathbf{B}_m) \tag{4}$$

Physically, (3) shows that f_n is changed by two effects: local changes in B_m (first term) and convection of the duct through a region where B_m is inhomogeneous. The second term involves the electric field $\mathbf{E}$, which we desire to determine. Note that $\mathbf{E}$ is the total electric field, i.e., the sum of the potential and the induced fields and not the potential field only. Equations (3) and (4) can be combined to give

$$E \cdot (\mathbf{B}_m \times \nabla B_m) = B_m^2\left(k_1 \frac{df_n}{dt} - \frac{\partial B_m}{\partial t}\right) \tag{5}$$

Obviously, only the component E_a of $\mathbf{E}$ directed along $\mathbf{B}_m \times \nabla B_m$ can be obtained. In a cylindrically symmetric field this is the azimuthal (westward) component. Furthermore, since B_m is the minimum field strength, $\nabla B_m \perp \mathbf{B}_m$. Hence (5) reduces to

$$E_a = \frac{B_m}{|\nabla B_m|}\left(k_1 \frac{df_n}{dt} - \frac{\partial B_m}{\partial t}\right) \tag{6a}$$

or

$$E_a = E_{f_n} - E_B \tag{6b}$$

where E_{f_n} is the 'apparent' westward field determined from the total variation in f_n and E_B is a 'fictitious' field needed as a correction to E_{f_n} for the reason mentioned above. In a dipole field

$$E_a = E_w \tag{7}$$

where E_w is the westward field. Equation (6) is then reduced to

$$E_w = E_{f_n} = \frac{\zeta R_E k_1}{3}\frac{df_n}{dt} = 2.00 \times 10^{-4}\, \zeta \frac{df_n}{dt} \tag{8}$$

where R_E is the earth's radius and ζ is the radial distance in earth radii (in a dipole field $\zeta = L$, of course). Since $f_n \mathrel{\tilde{\propto}} \zeta^{-3}$, (8) can be written

$$E_w = \frac{R_E k_1}{2}(kf_{go})^{1/3}\frac{d(f_n^{2/3})}{dt} = 2.07 \times 10^{-2}\frac{d(f_n^{2/3})}{dt} \tag{9}$$

where f_{go} is the electron gyrofrequency for the surface equatorial geomagnetic field. The units in (8) and (9) are hertz for f_n and volts per meter for E_w.

From (6) and (8) it follows that in a dipole field $E_w = 1$ mV/m corresponds to approximately $df_n/dt \approx 1$ Hz/s at $\zeta = 5$. In a nearly dipole field, when effects of $\partial B_m/\partial t$ are important, a value of $E_B = 1$ mV/m corresponds to $\partial B_m/\partial t \approx 0.1$ γ/s at $\zeta = 5$.

It is clear from (6) that deviations from a dipole field affect determinations of the electric field in three ways: through time variations in B_m, through deviations from dipole values of B_m, and through deviations from dipole values of ∇B_m.

As was noted above, the time variation in B_m can be accounted for by adding a correction term to the time variation in nose frequency.

$$\Delta\left(\frac{df_n}{dt}\right) = -\frac{1}{k_1}\frac{\partial \mathbf{B}_m}{\partial t} \tag{10}$$

The deviations in magnitude and gradient of B_m can be taken care of by a correction factor

$$G = 3B_m/\zeta R_E|\nabla B_m| \tag{11}$$

which in the case $\partial B_m/\partial t = 0$ gives the ratio of the real electric field to that based on the dipole approximation.

According to (5), only the component E_a of $\mathbf{E}$ directed along an equi B contour can be obtained. In the dipole approximation this means the longitudinal component. However, any asymmetry (e.g., an asymmetric ring current) changes the direction of E_a and makes it partly radial. The angular deviation of E_a from the purely longitudinal direction is

$$\varphi_a = \arctan\left(\frac{\partial \mathbf{B}_m}{R\partial\lambda}\bigg/\frac{\partial \mathbf{B}_m}{\partial R}\right) \tag{12}$$

where λ is the longitude and R is the geocentric distance.

Now E_a and φ_a can be calculated from (6) and (12), if appropriate information from satellite magnetometers is available or if a good model of the geomagnetic field is adopted.

EQUATORIAL DISTANCE OF THE WHISTLER PATH

In recent papers *Likhter and Molchanov* [1968] and *Sagredo and Bullough* [1972] have discussed the effect of a symmetric ring current on the equatorial radius of a whistler path. The diamagnetic effect of the ring current decreases the magnetic field and hence also the nose frequency. When the ring current or *Dst* index reaches $-100\ \gamma$, the equatorial distance may be overestimated by about 0.4 R_E near $\zeta = 4\ R_E$, according to Sagredo and Bullough. This overestimation must, of course, be corrected for before the electric field is calculated from (6). The correct equatorial distance corresponding to a certain nose frequency is found from (2) once a good model of B_m as a function of distance ζ or R is available.

THE FACTOR G

This factor, defined by (11), can be written as

$$G = \alpha_{\mathrm{real}}/\alpha_{\mathrm{dipole}} \tag{13}$$

where

$$\alpha = B_m/|\nabla B_m| \tag{14}$$

Thus G is a measure of the combined effects of the instantaneous deviations in both B_m and grad B_m from the dipole field.

It is not necessary here to adopt a particular model field and compute G at several points in the geomagnetic equatorial plane. Let it suffice to say that the ring current makes $G < 1$ out to about 4–5 R_E owing to the decrease in B_m. Beyond that, grad B_m is usually decreased below its dipole value, in particular on the day side, so that G may exceed unity. It is conceivable that in some regions of the distant magnetosphere grad B_m may be very small, and thus G will be quite large. However, a detailed study of many published satellite magnetometer data has convinced us that even during large disturbances, G very rarely falls outside the range $0.7 < G < 1.4$ in the regions where whistlers occur, i.e., at $L < 6$, approximately.

This means that the error due to the factor G is less than about 30% when the dipole model is used. In general, we find that $G \approx 1$ at $L < 2.5$ and $L \approx 4$–5, $G < 1$ at $2.5 < L < 4$–5, and $G > 1$ at $L > 4$–5, the deviations from unity increasing with increasing ΔH at low latitudes.

LONGITUDINAL ASYMMETRIES

Asymmetries in the magnetic field give rise to small deviations from the purely westward or azimuthal direction of E_a. Consider, for example, the noon-midnight asymmetry due to solar wind compression on the day side and the extended tail field on the night side. This does not cause any change in direction of E_a at midnight or noon, but at dawn E_a is partly made up of a small inward radial component if the main direction is westward. At dusk the reverse is true. On the other hand, a partial ring current centered at 1800 hours deflects E_a in the same direction at midnight as the noon-midnight asymmetry at dawn.

Quantitatively, the effect of the noon-midnight asymmetry is negligible. From the data published by *Sugiura* [1972] it can be estimated to be of the order of only 1° in angle at $L = 4$ at dawn and dusk. At $L = 6$ it may perhaps reach about 30° during very disturbed conditions, but whistlers would then be infrequently observed at that distance. Thus we conclude that this asymmetry is unimportant.

The partial ring current may give much larger deviations. According to *Crooker and Siscoe* [1971], the amplitude of the first diurnal harmonic of the geomagnetic disturbance field asymmetry (peaked around 1800 hours) can exceed 100 γ during great storms, as has been observed on the ground. With the same amplitude at $L = 4$ in the equatorial plane, as suggested by satellite observations, E_a would deviate by 5° from the purely azimuthal direction at midnight. Higher harmonics of the asymmetry may possibly add to this effect.

Even 5°–10° is rather insignificant in view of other uncertainties. Thus deviations from the azimuthal direction can normally be neglected.

Satellite observations [*Sugiura et al.*, 1971] are consistent with the above conclusions for $Kp \leq 3$. For higher Kp, good satellite results have not yet been obtained owing to the difficulty in discriminating between spatial and temporal variations.

EFFECTS OF TIME-VARYING MAGNETIC FIELDS

The $\partial B_m/\partial t$ or E_B term in (6) is found to provide the most important correction to the dipole-calculated electric field during disturbed times. Both ground-based and satellite magnetometer data show fast changes in B, particularly during sudden commencements and substorm expansion and recovery phases. *Wang and Kim* [1972] have discussed the decaying ring current and the electric field that may be associated with that decay.

Several authors have shown that the H component at low latitudes on the ground and in space out to at least synchronous orbit varies essentially coherently and with similar amplitudes (within a factor of 2) if the data are taken at nearly the same magnetic longitude [*Cahill*, 1966; *Cummings and Coleman*, 1968; and several others]. Caution must, of course, be taken to make sure that the ground magnetograms are not influenced by the equatorial electrojet, but they should refer to low latitudes, at least lower than 40°.

This coherent variation in H out to large distances makes it possible to use ground magnetograms to obtain the $\partial B_m/\partial t$ correction to the time variation in nose frequency, within a factor 2.

As an example of a very large variation, we take the magnetic storm of January 7–8, 1967, which has been studied by *Cummings and Coleman* [1968]. During the recovery phase of a substorm at about 1010 UT on January 8 the magnetic H component at Honolulu rose by 28 γ in 7.5 min, or ~0.06 γ s^{-1}. At the same time ATS 1 recorded a rise of about 29 γ in 6 min, or ~0.08 γ s^{-1}. These variations correspond to values of E_B in (6) of the order of 0.6–0.8 mV/m. Such values are as high as the larger substorm fields derived from whistler data by using E_{f_n} without correction. However, this example is extreme and should not be taken as typical. A more typical case is analyzed in the next section.

It should be pointed out that the variation in nose frequency can be seen only as an average over several minutes, and so it is appropriate to take similar averages for $\partial B/\partial t$, as has been done in the example from Honolulu and ATS 1 above.

A SUBSTORM ON JULY 6, 1965

During a substorm near 0500 UT on July 6, 1965, well-defined changes in whistler nose frequency were accompanied by a relatively large increase in the geomagnetic H component at low latitudes. Figures 1*a* and 1*b* show the AE index and the San Juan H component for the interval 0300–0800 on July 6. This interval was early in a period of developing disturbance that followed two days of quiet; the Kp indices on July 5 and 6, 1965, were 11110011 and 25443222, respectively. San Juan is at 18°N, 66°W, $L \sim 1.4$; the Eights, Antarctica, whistler station is at 75°S, 77°W, $L \sim 3.9$.

Figure 1*c* shows E_{f_n}, the westward field calculated from the whistler nose frequency, during an ~1-hour period near 0500 and during a brief interval near 0700. The plotted values are 15-min averages based on either synoptic 1-min recordings at 15-min intervals (near 0500–0600) or continuous recordings (near 0700). The various symbols identify individual whistler paths that were tracked for varying periods of time near $L = 3.1$, $L = 3.7$, and $L = 4.5$. Results from tracking during two or more successive 15-min intervals are connected by a dashed line.

Figure 1*c* also shows an estimate of the E_B correction term of (6). The values are based on $\partial B/\partial t$ at San Juan, which was determined from measurements of H at 15-min intervals and scaled to units of millivolts per meter according to (6), dipole values of B_m and its gradient being assumed. A very small L dependence in the predicted effect is not shown. On the preceding quiet day, July 5, the San Juan H trace remained within ~±2 γ of a constant level in the period 0300–0800.

The solid symbols in Figure 1*c* show the corrected westward electric field E_a formed by subtracting the values of E_B from those of E_{f_n} (no adjustment was made near 0700, when San Juan H was relatively quiet). The main effect is to obtain values of E_a about 20% less than E_{f_n} during the period 0450–0520, when San Juan H was recovering rapidly. An error bar near 0510 gives a rather conservative estimate of the uncertainty in E_a. This uncertainty includes a roughly ±50% uncertainty in estimating the magnetospheric quantity E_B on the basis of San Juan H.

The results on E_a (and E_{f_n} near 0700) are consistent with previous results on substorm-associated westward fields within the plasmasphere [*Carpenter et al.*, 1972; *Carpenter and Akasofu*, 1972]. There is a moderately high level of ~0.4 mV/m near 0440 and then a substantial increase near 0500 as a significant increase in substorm activity occurred. From the AE index (Figure 1*a*) it is evident that significant activity was underway near 0440 but apparently at some distance to the east of the meridian of San Júan. This is suggested by the continuing decrease of San Juan H for some minutes past 0440. At about 0450 a new expansion phase developed, as is evidenced by the rapid recovery of H at San Juan.

The value of E_{f_n} near 0700 is shown so as to contrast the relatively large westward fields near 0500 with eastward fields that appeared following the substorm. The postsubstorm data are fragmentary but suggest a generally eastward trend in the 0600–0800 period. *Carpenter et al.* [1972] found that following a number of substorms ~1–2 hours in duration the azimuthal electric field in the plasmasphere reversed to become eastward.

The July 6 event appears to be a relatively typical case. Others that we have briefly examined show a similar relation between the corrected and uncorrected values of westward field. When allowance is made for a factor of 2 increase in the E_B correction term, it seems clear that fluctuating magnetic fields represent an appreciable but not a predominant contribution to E_a during substorms, at least for periods of the order of 15 min and greater. Magnetic field variations may be

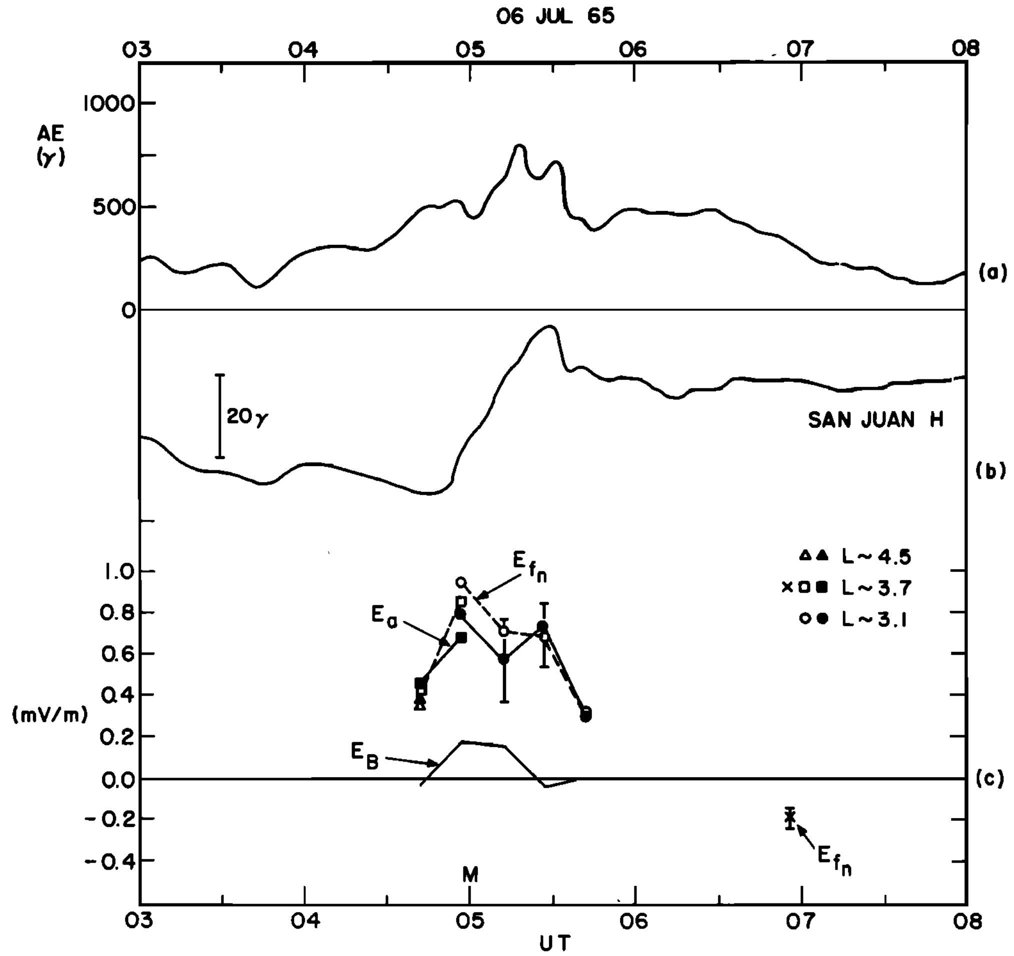

Fig. 1. Analysis based on whistlers of the westward electric field in the magnetosphere during a substorm, including effects due to the fluctuating geomagnetic field. The whistlers were recorded at Eights, Antarctica, on July 6, 1965. In (*c*) E_{f_n} is the field inferred on the assumption of a static dipole field, E_B is the correction term inferred from variations in San Juan *H*, and E_a is the inferred total (induced plus potential) westward electric field. See text for discussion of symbols.

very large during shorter periods, as was suggested in the previous section. In order to evaluate such effects, *E* field measurements with improved time resolution are needed.

Presubstorm Activity on July 15, 1965

As was pointed out in the section on theory,

$$E_a = E_{f_n} - E_B$$

is a component of the total (potential plus induced) electric field. Since E_B is proportional to $\partial \mathbf{B}/\partial t$, induced fields will in general be present whenever $E_B \neq 0$. However, we cannot straightforwardly determine the induced field at points on a whistler duct from information of E_B. It is, in fact, easy to construct a model in which the induced field is zero at the duct even when $\partial \mathbf{B}/\partial t \neq 0$ there. The relation between E_B and the induced field depends on the detailed geometry of $\partial \mathbf{B}/\partial t$ in the magnetosphere.

We have, however, found observations indicating that induced *E* fields may play an important role in some relatively quiet intervals. Such a possibility is suggested in Figure 2, which represents a period of presubstorm activity on July 15, 1965. The top is a plot of $\partial B/\partial t$ scaled from the *H* component of San Juan magnetograms, and the bottom shows low-amplitude fluctuating values of E_{f_n} scaled from the nose frequency of Eights, Antarctica, whistlers (many features of the westward electric field during this event were reported elsewhere by *Carpenter et al.* [1972]). In this case, continuous VLF recordings were available, and fluctuations in both curves with period of ~10 min or greater are resolved. At the time illustrated there was low but increasing magnetic agitation following several relatively quiet days. The *Kp* indices on July 14 and 15 were 21100112 and 22233113, respectively.

Vertical lines connect a number of peaks in $\partial B/\partial t$ to corresponding peaks in E_{f_n}, and it is therefore suggested that each extremum of the uncorrected E_{f_n} may be partly or entirely explained by $\partial B/\partial t$. Further study and improved knowledge of the relation between low-amplitude surface magnetic oscillations and magnetospheric oscillations are needed before a corrected E_a can be obtained in this case of low-amplitude variations. It is unlikely, however, that E_a would differ substantially from the uncorrected E_{f_n}. There was no substantial ring current in this period, and the necessary corrections for changes in the magnetospheric field B_m and its gradient are probably small. If the E_{f_n} fluctuations are not due to the effect of $\partial B/\partial t$, they give a good estimate of a total field as they stand. If $\partial B/\partial t$ effects are important, as is suggested in Figure 2, the corrected field may simply involve a reduction in scale of the order of 40% from the uncorrected values. This reduction is estimated as follows. When a nearly dipole field, no potential field, and uniform $\partial B/\partial t$

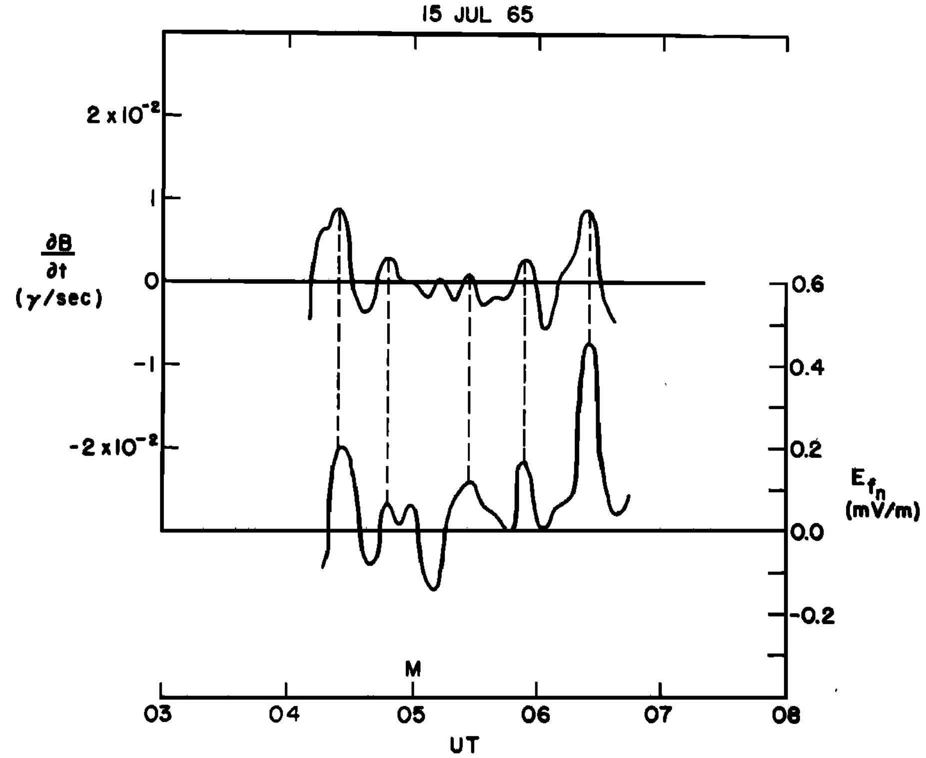

Fig. 2. Comparison of fluctuations of $\partial B/\partial t$, scaled from the H component of San Juan magnetograms with fluctuations of the westward electric field determined from whistlers recorded on July 15, 1965, at Eights, Antarctica. The electric field was calculated on the assumption of a static dipole geomagnetic field. The near coincidences of the extremums of the two curves suggest that under the relatively quiet presubstorm conditions that prevailed the fluctuations in the inferred E field were largely due to compressional oscillations of the magnetosphere.

at the equator are assumed, the induced westward field from the relation

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{A}$$

is just

$$E_a = E_w = (LR_E/2)\ \partial B/\partial t$$

The correction term in (6) is

$$E_B = (LR_E/3)\ \partial B/\partial t$$

so that, using (6), we have

$$E_a = E_w = {}^3/_5 E_{f_n}$$

In this hypothetical case of an induced field only, E_{f_n} is entirely due to the changing magnetic field.

This model, of course, implies strict proportionality between E_{f_n}, E_B, and the practically uniform $\partial B/\partial t$. However, the San Juan $\partial B/\partial t$ and E_{f_n} displayed in Figure 2 do not obey such a proportionality. If we nevertheless wish to apply this model, we must assume that the San Juan field is not representative of the magnetospheric $\partial B/\partial t$ except for the positions of the extremums. Potential fields and more complicated inhomogeneities in $\partial B/\partial t$ may, of course, also be important. However, the remarkable coincidences of the extremums in Figure 2 do suggest to us that E_{f_n} at that time was largely due to compressional oscillations of the magnetosphere qualitatively similar to those described in the above model.

According to *Wang and Kim* [1972], model calculations have revealed that the electric field induced by a rapidly decaying storm time ring current is in good agreement with that deduced from whistler duct studies. With our notation this would imply that $E_a \approx E_{f_n}$ in a case like that shown in Figure 1 since Wang and Kim did not account for E_B and hence did not include the factor ${}^3/_5$. It remains possible that the decaying ring current mechanism can explain the observed drifts of whistler ducts during substorms. As is noted above, however, our present knowledge suggests that other explanations are needed.

RELATION BETWEEN EQUATORIAL AND IONOSPHERIC $E_\perp$

The foregoing sections suggest that induced electric fields may contribute in specific and occasionally important ways to the total westward electric field in the plasmasphere. This contribution complicates the problem of comparing E field measurements at ionospheric heights with similar measurements near the magnetospheric equator. In a static geomagnetic field the equatorial and ionospheric perpendicular electric fields are simply related by a geometric factor given by the magnetic field geometry, provided that no parallel electric field exists. However, if the magnetic field varies with time, the simple relation may be violated.

Consider, for example, the simple case illustrated in Figure 3. The upper part of the figure is a meridional plane, and the lower part is a view from above the north pole. Points A, B, F, and G are in the ionosphere at constant altitude, but C, D, E, H, I, and J are in the equatorial plane.

Suppose first longitudinal symmetry, constant B, and a westward potential electric field E_i in the ionosphere and

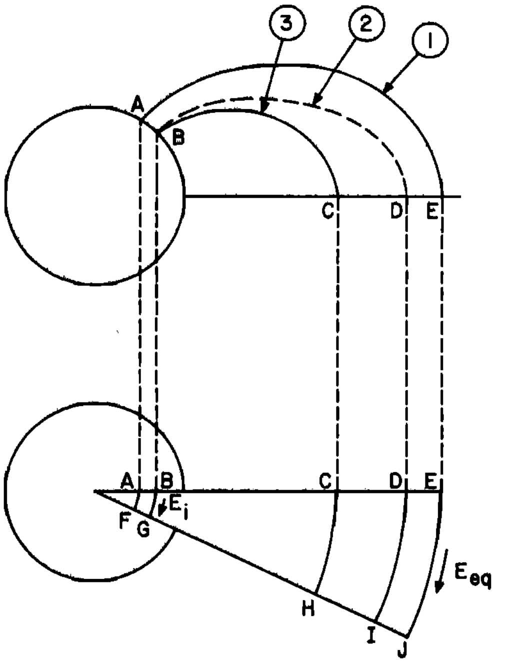

Fig. 3. Model illustrating the problem of comparing electric field measurements at ionospheric heights with similar measurements near the magnetospheric equator.

E_{eq} in the equatorial plane. This implies that the magnetic field line 1 is convected to position 3 in a certain time. We then have

$$\Phi_{ABGF} = \Phi_{CEJH} \qquad (15)$$

$$E_{i1} S_{AF} = E_{eq1} S_{EJ} \qquad (16)$$

$$E_{i3} S_{BG} = E_{eq3} S_{CH} \qquad (17)$$

where Φ_{ABGF} is the magnetic flux through the surface bounded by the closed contour ABGFA, E_{i1} is the ionospheric electric field at field line 1, and S_{AF} is the arc length AF. Equations (16) and (17) imply the validity of the simple relation between E_i and E_{eq} referred to above.

Suppose then that a symmetric ring current is growing while the potential electric field discussed above is present. The plasma drifts toward the earth as before, but the magnetic field is inflated simultaneously. Assume, for example, that the plasma on field line 1 drifts to field line 2, implying that an ionospheric drift from A to B occurs when the equatorial plasma drifts from E to D. Hence during this process E_{eq} is considerably weaker than would be expected from the observed E_i.

For the sake of argument, assume now that when the plasma originally on field line 1 has reached position 2, the potential field disappears, and at the same time the extended magnetic field collapses due to some ring current breakdown. Field line 2 then moves to position 3, perhaps faster than when it moved from 1 to 2. Hence a sudden increase in the westward electric field would be seen at the equator, and the ionospheric field would disappear at the same time.

The hypothetical situation above is only meant to show that during dynamic events, such as the substorm-associated changes in magnetospheric current systems, the equatorial electric field cannot be inferred from the ionospheric field or vice versa, even if the magnetic field lines are equipotentials.

The argument can be given in a more condensed form as follows. If the magnetic flux through any closed fixed contour (such as AEJF in Figure 3) varies with time,

$$\oint \mathbf{E} \cdot d\mathbf{s} \neq 0$$

even if $E_{\parallel}$ vanishes everywhere. Hence

$$\int_A^F \mathbf{E}_i \cdot d\mathbf{s} \neq \int_E^J \mathbf{E}_{eq} \cdot d\mathbf{s}$$

and E_i cannot be inferred from E_{eq} or vice versa.

On the other hand, discrepancies between simultaneous measurements of E_i (by balloons, barium clouds, or rocket-borne double probes) and E_{eq} (by whistlers or satellite-borne double probes) may yield new insight into the interrelations between the magnetic field dynamics and the electric fields, provided, of course, that the whistler ducts remain field aligned and therefore can function as ducts during these dynamic events. This will evidently be the case as long as $E_{\parallel} = 0$, since then the ducts and the field lines will be deformed in the same way. However, if significant parallel electric fields occur, the ducts become skewed relative to the magnetic field, and no whistlers can propagate. The disappearance of whistlers in a certain region of space may therefore indicate the onset of anomalous resistivity or double layers.

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