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Unducted Whistler Evidence for a Secondary Peak in the Electron Energy Spectrum near 10 kev

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Resonant particle interactions between unducted, magnetospherically reflected (MR) whistler-mode radiation and the ambient plasma have been investigated. Because the frequencies of such waves are small compared to the local electron gyrofrequency and the wave normals practically perpendicular to the magnetic field, Landau electron interactions dominate. Resonant ion energies are well above an Mev and can consequently be ignored. The upper and lower frequency cutoffs, as well as the high-frequency region of growth of the multiple bounce MR traces observed near $L = 2.5$, can be accounted for by an ambient electron distribution that has a secondary peak in the vicinity of 10 kev.

1. INTRODUCTION

Magnetospherically reflected (MR) whistlers, first predicted by the ray trace computations of *Kimura* [1966] and later substantiated by the analytic propagation study of *Thorne and Kennel* [1967], have now been observed on OGO 1 and OGO 3 [*Smith and Angerami*, 1968]. There is a major distinction between the MR whistler waves and those which for many years have been detected at the earth's surface (see *Helliwell* [1965] and references therein). The latter are assumed to propagate within ducts of ionization that constrain their propagation vector **k** to remain aligned along the ambient magnetic field $\mathbf{B}_0$. The MR's are however *unducted* and in general propagate with **k** almost perpendicular to $\mathbf{B}_0$. Ion concentrations can then drastically affect their propagation. Waves moving in toward the earth are expected to turn around soon after their frequency falls below the local lower hybrid frequency. This reflection process has been given further support by the high-latitude ($|\lambda| > 30°$) detection of ν whistlers [*Smith and Angerami*, 1968]. An example of these is shown in Figure 1*a*. The sharp cutoff at the ν is attributed to the low-frequency components reflecting at a lower latitude than the satellite location.

The multiple bounce MR's, whose intensity variations we wish to consider, have several characteristic features. They are seen only at lower latitudes $|\lambda| < 30°$ and on L shells between 1.5 and 3.0, the maximum number of observations being around $L = 2.0$ and 2.5. Typical examples of the detected MR signals are shown in Figures 1*b–d*. Each successive continuous trace corresponds to an additional wave bounce as depicted by the typical wave trajectory sketched in Figure 2. The multiple hop traces show a sharp cutoff at high frequencies and a somewhat more diffuse attenuation at the low-frequency end. Also, there is often an intense region of growth near the high-frequency cutoff. As the number of bounces increases, both the upper cutoff and the region of growth move toward lower frequency, whereas the more diffuse lower cutoff rises. This tends to produce a very narrow, but often intense, passband in the region of a few kc as shown in Figure 1*d*.

Using the Stanford ray tracing program, the dispersion characteristics of a chain of MR traces have been fitted by *Edgar and Smith* [1967]. The time delays have furthermore been found consistent with a generating atmospheric disturbance as seen by ground detectors. The noticeable flattening of the multiple hop waves above their nose frequency (which incidentally occurs at a far lower frequency than that of a corresponding ducted wave) is attributed primarily to the longer path lengths of the high-frequency components, although the group velocity at these higher frequencies may also be reduced considerably when the wave is close to its resonance cone.

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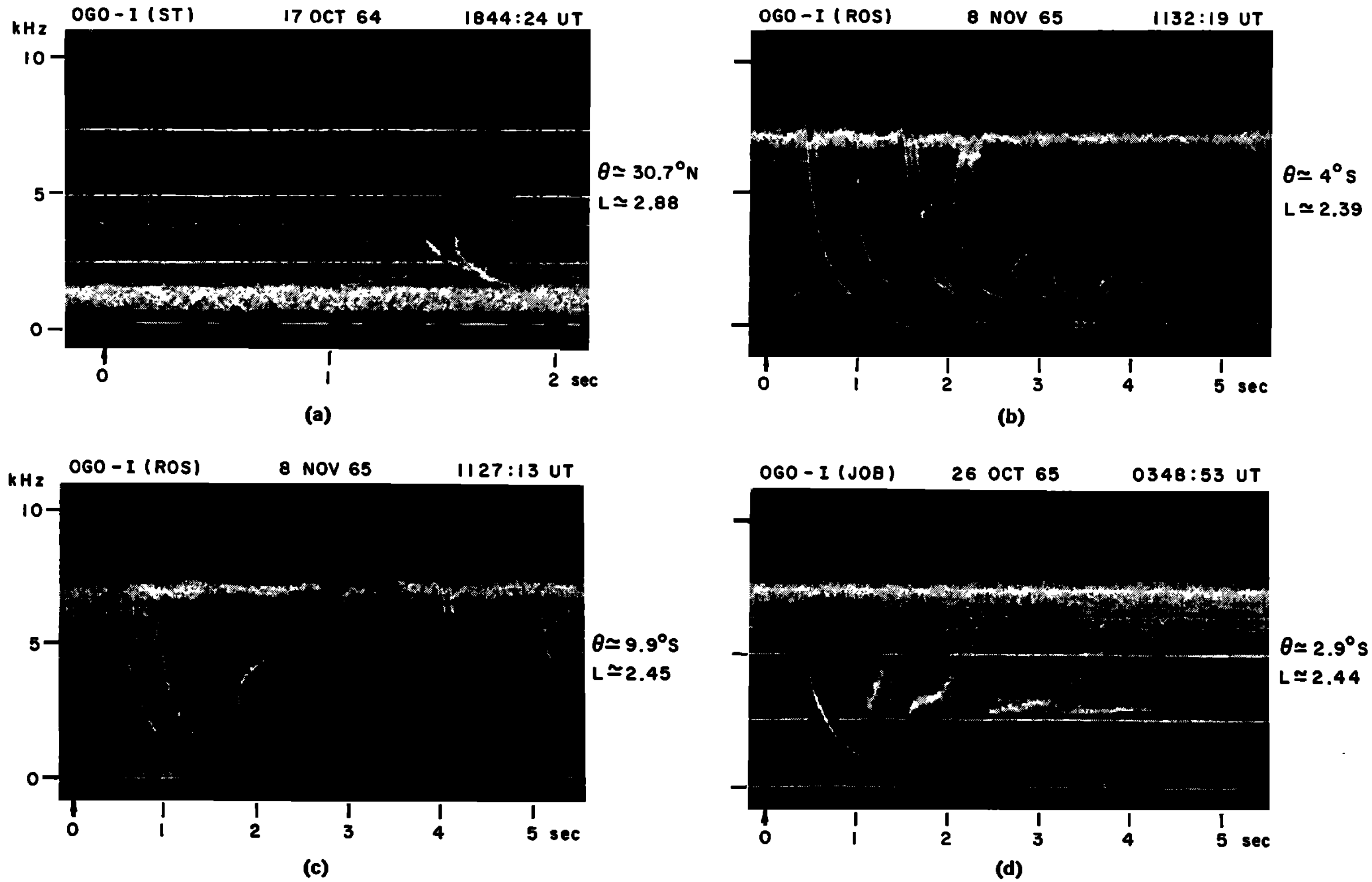

Fig. 1. Examples of magnetospherically reflected MR whistlers observed on OGO 1. (Courtesy of R. L. Smith and J. J. Angerami, *J. Geophys. Res.*, *73*, pp. 1–20, 1968.) (*a*) ν whistler observed at high latitude. Lower frequencies reflect before the satellite location. (*b*) An MR train exhibiting eleven bounces. (*c*) The third and fourth members of this trace exhibit a strong growth just before the sharp upper frequency cutoff. The lower frequency cutoff is more diffuse. (*d*) Example of strong growth. Notice the passband effect at a few kc.

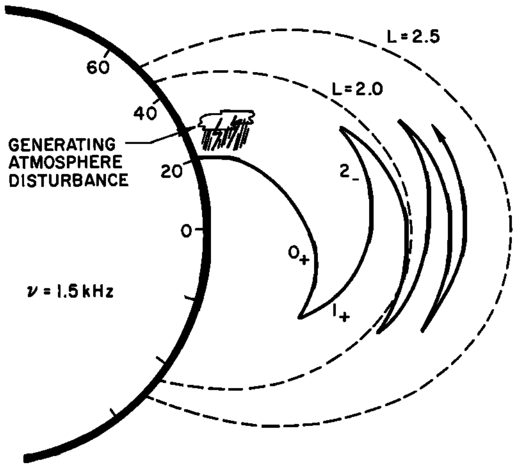

Fig. 2. A typical MR ray path starting with an atmospheric disturbance.

It would perhaps be tempting to associate the upper cutoff with a propagation effect, were the higher frequency turn-around point to move below the ionosphere, allowing the wave energy to be scattered there. The observed cutoffs are however at too low a frequency to be compatible with this idea. Furthermore, while propagation can readily account for the lower cutoff of ν whistlers (Figure 1*a*), no such explanation can be given for the low-frequency MR cutoffs seen near the equatorial plane. Finally, propagation could certainly not account for the high-frequency growth region. We therefore propose that wave-particle interactions are responsible for the observed intensity features. The investigation then leads us to expect an electron distribution having a secondary peak near 10 kev or just above a typical resonant Landau energy.

2. Wave-Particle Interactions

We will adopt the linear wave-particle interaction theory developed by *Kennel* [1966] for a temperate, low $\beta = p/(B^2/8\pi)$ plasma. This can readily be applied to the inner magnetosphere where β is typically $0(10^{-4})$. Physically this restricts all resonant interactions to particles in the high-energy tail of the distribution and allows us to treat temperature effects as a small perturbation. Assuming a real propagation vector **k**, the imaginary component of the wave frequency can then be expressed as a double sum over all particle species (α) and their Doppler-shifted cyclotron resonances. For each species the resonant energy (parallel to $\mathbf{B}_0$) of a particle with mass $M_{(\alpha)}$ may be expressed as

$$E_{(\alpha)}^{(m)} = \frac{M_{(\alpha)}c^2}{2}\left\{\frac{1 + m[\Omega_{(\alpha)}/\omega]}{n \cos\theta}\right\}^2 \quad (1)$$

Here ω is the wave frequency, $\Omega_{(\alpha)} = [q_{(\alpha)}B_0]/[M_{(\alpha)}c]$ is the particle gyrofrequency, θ is the angle between the propagation vector **k** and the ambient magnetic field $\mathbf{B}_0$, n is the wave refractive index, and m is any integer.

A typical magnetospherically reflected (MR) whistler has θ close to $\pi/2$ and frequency bracketed by the proton and electron gyrofrequencies,

$$\Omega_p \ll \omega \ll \Omega_e \quad (2)$$

Furthermore, ray path calculations [*Edgar and Smith*, 1967] suggest a typical value of $n \cos\theta \sim 0(10)$. These values imply that resonant ion energies are all above an Mev, and we shall subsequently ignore their effect. Also, resonant electrons would be relativistic in all but the $m = 0$ interaction. However, for this Landau resonance we would require only kev electrons. The flux of such particles could be appreciable in the region where MR whistlers have been seen. A further scaling factor that tends to favor the Landau resonance at large wave normal angles reinforces our contention that for unducted whistler mode waves only *Landau electron* interactions are important.

The wave amplitude growth rate γ can thus be simplified from equation 3.9 of *Kennel* [1966] to the form

$$\gamma = 8\pi^2\left[\frac{\omega_{p,e}}{k_\parallel}\right]^2\left[\frac{\partial D^0}{\partial\omega}\bigg|_{\omega^*}\right]^{-1} \cdot \int_0^\infty V_\perp M^{(1)}(V_\perp)\left[\frac{\partial F_e(V)}{\partial V_\parallel}\right]_{V_\parallel=\omega/k_\parallel} dV_\perp \quad (3)$$

Here $\omega_{p,e} = 4\pi N_e e^2/M_e$ is the electron plasma frequency and $F_e(V)$ is the unperturbed electron distribution function. D^0 is the *cold* dispersion relation

$$D^0 = 4\{[P\cos^2\theta + S\sin^2\theta]n^4 - [RL\sin^2\theta + PS(1 + \cos^2\theta)]n^2 + PRL\} \quad (4)$$

where the coefficients P, R, L, and S are defined as in *Stix* [1962] or *Kennel* [1966].

$$P = 1 - \sum_\alpha \left[\frac{\omega_{p,(\alpha)}}{\omega}\right]^2$$

$$R = 1 - \sum_\alpha \frac{\omega^2_{p,(\alpha)}}{\omega[\omega + \Omega_{(\alpha)}]} \quad (5)$$

$$L = 1 - \sum_\alpha \frac{\omega^2_{p,(\alpha)}}{\omega[\omega - \Omega_{(\alpha)}]}$$

and

$$S = (R + L)/2$$

The remaining undefined quantity is finally given by

$$M^{(1)}(V) = -\left[\frac{V_\perp}{\omega/k_\parallel}\right]^2 \cdot J_1^2(\ )[P(n^2 \cos^2\theta - S) + Sn^2 \sin^2\theta] + \left[\frac{V_\perp}{\omega/k_\parallel}\right] J_0(\ )J_1(\ )(R - L)n^2 \cos\theta \sin\theta + J_0^2(\ )[n^4 \cos^2\theta + RL - Sn^2(1 + \cos^2\theta)] \quad (6)$$

where $J_0(\ )$ and $J_1(\ )$ denote zero and first-order Bessel functions of argument $K_\perp V_\perp/\Omega_e$.

3. An Approximate Analytic Solution

We will first estimate the magnitude of the damping produced by a purely power law electron spectrum. It is then convenient to express the distribution function $F(E) \sim E^{-P}$ in terms of the electron velocity:

$$F_e(V) \simeq 3.78 \times 10^{-3}[0.0039]^P \cdot \left[\frac{c^{2p-3}}{V^{2p}}\right] \frac{J_e(1\text{ kev})}{N_e} \quad (7)$$

where the normalizing flux J_e (1 kev) is measured in electrons cm^{-2} sec^{-1} ster^{-1} ev^{-1} and the number density N_e in electrons cm^{-3}.

With the restriction that the proton gyrofrequency

$$\Omega_p \ll \omega \ll \Omega_e \ll \omega_{p,e} \quad (8)$$

the Stix coefficients approach the values

$$P \Rightarrow -\left(\frac{\omega_{p,e}}{\omega}\right)^2 \quad \text{and} \quad \frac{R}{L} \Rightarrow \pm\frac{|P|\,\omega_{GF}}{1 \mp \Lambda} \quad (9)$$

where

$$\omega_{GF} = \frac{\omega}{|\Omega_e|} \quad (10)$$

and

$$\Lambda \simeq \omega_{GF}\left[1 - \frac{|\Omega_e|\,\Omega_p}{\omega^2} + \left(\frac{\Omega_e}{\omega_{p,e}}\right)^2\right]$$

Inside the plasmapause, the equatorial value of $\omega_{p,e}/\Omega_e$ varies from about 10 at $L = 4$ to only 3 at $L = 2$, both numbers being reduced at higher latitudes. Thus the final inequality in (8) is a little questionable in the region where most MR whistlers have been observed. This objection is, however, removed by our exact computations in section 5. Also, for quasi-longitudinal propagation [*Booker*, 1935], the derivative of the cold dispersion relation is essentially given by

$$\left.\frac{\omega\partial D^0}{\partial\omega}\right|_{\omega^*} = 8\,|P|^3\,\omega_{GF}^2 \quad (11)$$

while the refractive index can be expressed [*Thorne and Kennel*, 1967] in the form

$$n^2 \Rightarrow \frac{|P|\,\omega_{GF}}{\cos\theta - \Lambda} \quad (12)$$

These approximate values, together with the distribution function (7), lead to a *damping* rate

$$\frac{\gamma}{\omega} \Rightarrow K_p\left[\frac{\omega_{p,e}}{|\Omega_e|}\right]^{2p-3}\left[\frac{|\Omega_e|}{\omega}\right]^{p-5/2} \cdot \frac{J_e(1\text{ kev})}{N_e}\,\frac{\cos^{2p-4}\theta\,\sin^2\theta}{(\cos\theta - \Lambda)^{p-3/2}}\,I_p \quad (13)$$

Here

$$K_p \simeq 0.086[0.0039]^p \quad (14)$$

$$I_p = p\int_0^\infty \frac{xM^{(2)}(x)\,dx}{(1 + x^2)^{p+1}} \quad (15)$$

$$M^{(2)}(x) \Rightarrow \left(\frac{x}{\epsilon}\right)^2 J_1^2(\epsilon x) + 2\left(\frac{x}{\epsilon}\right) J_0(\epsilon x)J_1(\epsilon_1 x)\left[\frac{\cos\theta}{\cos\theta - \Lambda}\right] + J_0^2(\epsilon x)\,\frac{\Lambda}{\omega_{GF}}\cdot\left[\frac{\cos\theta}{\cos\theta - \Lambda}\right]^2 \quad (16)$$

and

$$\epsilon = \omega_{GF}\tan\theta \quad (17)$$

The above expression for $M^{(2)}(x)$ is valid for almost all wave normal angles consistent with the quasi-longitudinal approximation and has been simplified only by dropping terms of order $[\omega_{GP}/\cos\theta]^2$. Noting first that the integrand of (15) peaks strongly close to $x \sim 0(1)$, or physically when the electron's perpendicular velocity $V_\perp \sim 0(V_{\parallel,\,res} = (\omega/k_\parallel)$, and also that ϵ is always small under our assumptions, allows us to further approximate the Bessel functions by the first term in their Taylor expansion. The integral (15) obviously converges for $p > \frac{1}{2}$ and then takes the simpler form

$$I_p = p \int_0^\infty \left[\frac{x^4}{4} + \left(\frac{\cos\theta}{\cos\theta - \Lambda}\right)x^2 + \frac{\Lambda}{\omega_{GP}}\left(\frac{\cos\theta}{\cos\theta - \Lambda}\right)^2\right]\frac{x\,dx}{(1+x^2)^{p+1}} \quad (18)$$

which can readily be seen to have a value close to unity.

Before attributing a numerical value to (13) it is instructive to notice that the rate of damping varies as $N^{p-5/2}/|\Omega_e|^{p-1/2}$ and thus, for $p > \frac{1}{2}$, attains a peak value as the wave crosses the equatorial plane. For the region of the magnetosphere inside the plasmapause, both the electron concentration and the magnetic field strength decrease roughly as L^{-3}, so that the damping rate should increase as L^6 for *any* value of the spectral index. The noticeable decrease in the number of observed multiple bounce MR whistlers exterior to $L \sim 3$, even at quiet times when the knee should be well outside this value [*Carpenter*, 1966], could be evidence of just such a severe damping effect taking place. To make sure that the damping rate is sufficiently rapid, we shall evaluate (13) in the vicinity of $L \sim 4$, taking a typical spectral index $p = 3$. (This would imply that the particle flux fall off as E^{-3}, consistent with the observations of *Frank* [1967].) At this L shell the gyrofrequency is $\simeq 15$ kHz, and from the ducted whistler observations of *Angerami and Carpenter* [1966] we take $N_e \simeq 300$, suggesting a plasma frequency $\simeq 150$ kHz. Neglecting Λ compared to $\cos\theta$ and observing that the angular dependence $\cos^{1/2}\theta \sin^2\theta$ changes by only a factor of 2 within the limits $45° < |\theta| < 80°$ gives an equatorial wave *intensity* loss rate

$$2\gamma(L = 4) \simeq 4.10^{-4}\nu^{1/2}(kc)J_e(1\text{ kev})\cdot\text{sec}^{-1} \quad (19)$$

Thus, even a relatively weak flux, J_e (1 kev) $= 10^4$ electrons cm^{-2} sec^{-1} ster^{-1} ev^{-1}, should produce severe attenuation within a few tenths of a second, which itself is comparable to the time that a whistler takes to cross the equatorial plane, $|\lambda| < 10°$. We could then hardly expect the waves at $L = 4$ to exhibit the low L, multiple bounce traces shown in Figure 1. In passing, we note that the $L = 3.9$ observations of Frank [1967] give a flux at 1 kev of almost 10^5 electrons cm^2 sec^{-1} ster^{-1} ev^{-1}. Although these particles may be outside or very close to the plasmapause, their damping effect would be catastrophic, forbidding the detection of even the fraction hop whistlers.

The majority of the observed MR whistlers have been seen at $L \simeq 2.5$. Here we can estimate that the damping is about a factor of 20 weaker than at $L = 4$. Nevertheless, a flux J_e (1 kev) of only 10^5 electrons cm^{-2} sec^{-1} ster^{-1} ev^{-1} would again indicate appreciable damping. It is difficult to imagine how a wave could then survive as many as ten equatorial crossings (Figure 1*b*). There is however an obvious way out of this dilemma. A secondary peak in the electron spectrum, or even merely a region of flattening, would act as a pass filter, allowing only a certain band of frequencies to propagate without severe attenuation. This idea will be developed in section 4.

4. Consequences of a Humpy Spectrum

In the previous section we estimated that the Landau damping expected from a purely power law electron spectrum will be so severe as to exclude the presence of multiple-bounce MR whistlers at $L \geq 2.5$. With such a simple electron distribution, we would also be unable to explain the rather general intensity features that have been observed on OGO 1.

Let us therefore, without specifying its origin, hypothesize the presence of a *secondary peak* in the electron distribution at an energy comparable to that of a typical resonant Landau electron. This resonant energy can be written

$$E_e^{\,0} = \frac{M_e c^2}{2}\left[\frac{1}{n\cos\theta}\right]^2 \equiv \frac{B_0^{\,2}}{8\pi N_e}\cdot\frac{\omega}{|\Omega_e|}\cdot\frac{\cos\theta - \Lambda}{\cos^2\theta} \quad (20)$$

where Λ is essentially given by (10). Such a

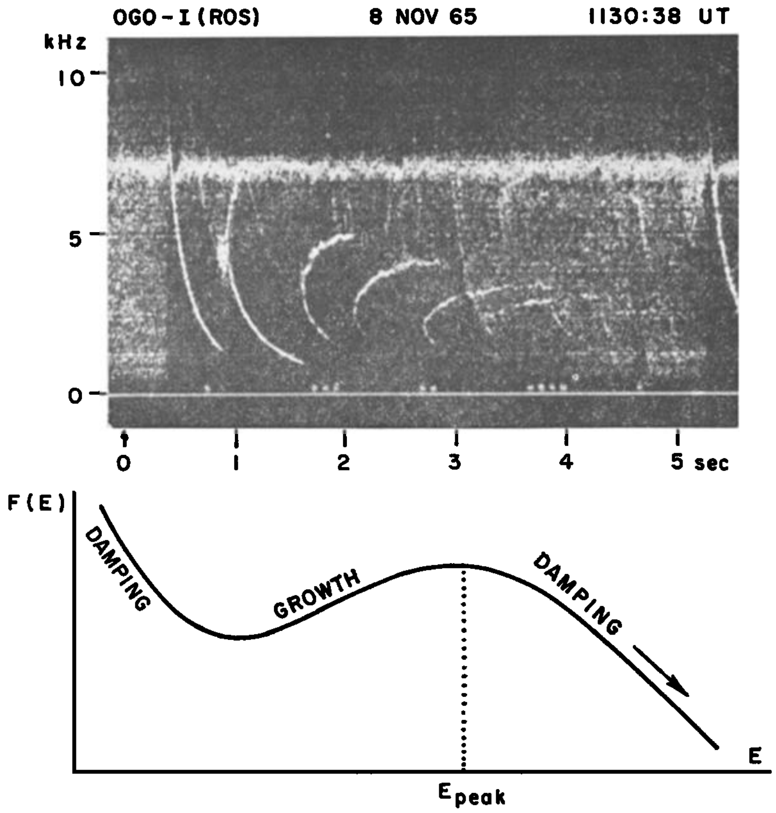

Fig. 3. An example of the electron spectrum required to explain the depicted MR intensity features.

spectrum is sketched in Figure 3. We can now make rather general remarks concerning the Landau interaction of such electrons with the unducted whistler mode waves. Landau *growth* is possible only when the electron distribution function increases with energy. We therefore contend that the region of growth observed near the high-frequency cutoff of the MR whistlers (see Figure 3) is due to resonance with electrons on the rising portion of distribution. To substantiate this claim we observe first that the resonant parallel energy depends only on the value of the wave refractive index in the direction of the ambient magnetic field. We have already mentioned that the computer ray trace calculations of *Edgar and Smith* [1967] give a typical value of $n \cos \theta \sim 0(10)$, suggesting a few-kev resonant energy. For a given MR trace, however, this resonant energy tends to increase monotonically with frequency (see equation 20). Provided the Landau interaction is sufficiently strong (see section 3), we can now qualitatively account for the low-frequency cutoff as Landau damping by the low-energy electrons. As frequency increases, the wave moves first into resonance with the stable electrons in the trough of the distribution (see Figure 3), then into a region of growth, and is finally cut off at high frequencies when the resonant energy approaches E_{peak}.

One further feature of the MR traces can be described by this simple resonant energy picture. There is a definite tendency for the region of growth to move toward lower frequencies as the number of 'bounces' increases (see Figure 3). However, at the same time, we find from ray-tracing calculations that the value of θ also increases with the hop number. The resonant energy (20) would then also be expected to increase with the number of bounces provided the wave is not too close to the resonance condition, $\cos \theta \Rightarrow \Lambda$. This tendency is shown later in Figures 4*a*–*c*. Thus, at a given frequency, a wave that showed strong growth on any given bounce would move into the region of damping on successive bounces. Both the upper cutoff and the region of growth would therefore be expected to move toward lower frequencies at higher bounce numbers and possibly eventually produce a passband as shown in Figure 1*d*.

The origin of this hypothesized hump and its chances of survival against various wave instabilities are strictly a matter for separate study. In the present investigation we merely point to evidence of this feature existing during some intervals of time. To produce a secondary peak in a supposedly smooth background spectrum, it would be necessary to introduce some energy-discriminating acceleration mechanism. Our proposed Landau damping mechanism is obviously one such example, but it is hard to imagine how this relatively weak process could do more than just flatten out the energy distribution in some region. A much more vigorous wave damping (and consequently particle accelerating) process is afforded by cyclotron damping. This has been investigated by *Tidman and Jaggi* [1962] and has been suggested [*Liemohn and Scarf*, 1964] as a mechanism to explain the sharp *ducted* whistler cutoffs observed at close to one-half the equatorial electron gyrofrequency. Although the cutoff itself now seems to be more easily explained [*Carpenter*, 1968] by an unducting process [*Smith*, 1961], severe cyclotron damping could still occur. Near $L = 2$ the cyclotron resonant energy with ducted waves of frequency $\Omega_e/2$ turns out to be approximately 10 kev. It is just these particles that would receive the preferential acceleration and possibly cause a hump in the electron distribution.

The above ideas are meant only to convey a general physical picture of the rather complicated interactions that can occur. With just one assumption, we do, however, have a means of explaining the gross intensity feature of the observed MR whistlers. Because the relative damping rates at different frequencies are rather sensitive to the position and size of the secondary peak, we feel that this could provide a convenient tool for probing the electron spectra at these low energies.

5. Numerical Ray Path Calculation

The numerical values presented in section 3 were only meant to be indicative and should not be taken too seriously. Our approximations

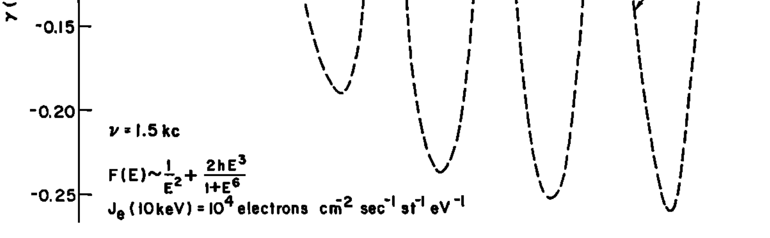

Fig. 4*a*.

Figures 4*a*–*c* show numerically computed ray path growth rates for the case of a purely power law spectrum (dashed lines) and a spectrum exhibiting a secondary peak at 10 kev (solid lines). The arrowheads indicate equatorial crossings where we also give values of L and the resonant Landau energy E_{res} in kev.

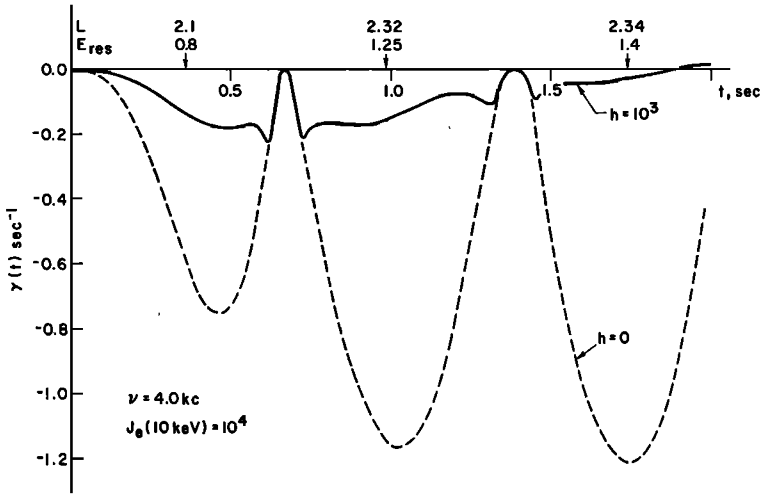

Fig. 4b.

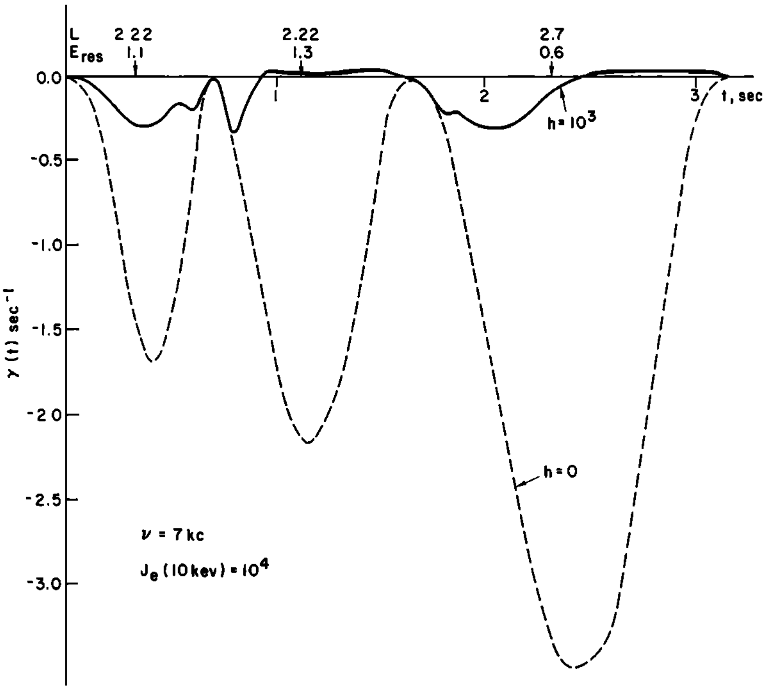

Fig. 4c.

begin to break down just in the region where application to MR whistlers becomes possible. Nevertheless we have demonstrated a tendency for the Landau interaction to produce significant damping, especially at large wave normal angles. Detailed investigation of the interaction when the wave is close to its resonant cone is possible only by the use of a computer.

The results of evaluating the damping rate given by the full form of (3) at successive points along a ray path are shown in Figures 4*a*, *b*, and *c*. For the case of a purely power law spectrum (dashed lines), the damping rates peak strongly close to the equator, indicated here by the small arrow heads. The equatorial values, furthermore, increase roughly as L^6 as the wave 'walks out' (see Figure 2) in radius. For a flux of 10^4 electrons cm^{-2} sec^{-1} $ster^{-1}$ ev^{-1} at 10 kev we find the following net intensity changes:

At $\nu = 1.5$ kc, wave intensity falls by approximately $\exp(-0.7)$ after 6 bounces.

At $\nu = 4.0$ kc, wave intensity falls by approximately $\exp(-2.8)$ after 3 bounces.

At $\nu = 7.0$ kc, wave intensity falls by approximately $\exp(-9.1)$ after 3 bounces.

For a purely power law spectrum we would thus expect the attenuation to increase monotonically with frequency. The observation, on the other hand, shows a minimum damping rate at a *finite* frequency (see Figures 1*b–d*). To account for this we shall consider an electron distribution having a secondary peak close to the resonant Landau energy.

The solid curves in Figures 4*a–c* are intended to give an indication of the effects of such a spectrum. We have used an electron distribution function of the form

$$F_e(E) \sim \frac{1}{E^p} + \frac{2hE^n}{1 + E^{2n}} \qquad (21)$$

for the case $p = 2$, $n = 3$, and an effective height of the hump $h = 10^3$. The energy E is here scaled to the value of the secondary peak E_{peak} (see Figure 3), which we have taken as 10 kev. Various other combinations of the four parameters p, n, h, and E_{peak} have been investigated. Provided E_{peak} is larger than the resonant Landau energy (values given in Figures 4*a–c*) there is always a tendency for the over-all damping rate to be reduced by the presence of the peak. As the wave moves out from the earth, its propagation vector becomes more nearly perpendicular to the ambient magnetic field, and the likelihood of wave growth increases. After a few bounces, both the 4-kc and 7-kc curves show regions of positive growth rate, albeit small, while the low-frequency behavior at 1.5 kc is only slightly modified. However, when E_{peak} is made much larger than 10 kev, the possibility of growth in the middle frequencies diminishes.

While this is scarcely a convincing demonstration that the Landau damping mechanism is capable of predicting the intensity features of MR whistlers in detail, we do feel there is sufficient indication to suggest the presence of a secondary peak *near* 10 kev. A more realistic electron spectrum, possibly taking spatial variations into account, could undoubtedly produce a much better fit to the observed data. However, at present time, with little observation information on the electron distribution at these low L values, we feel that such an investigation would be unrewarding.

6. Discussion

We have demonstrated that the unducted MR whistlers can experience significant Landau damping by the ambient electron component of the inner magnetosphere. The damping rates peak strongly at the equatorial plane and increase as L^6 as the wave walks out. The observed flux of 10^5 electrons cm^{-2} sec^{-1} $ster^{-1}$ ev^{-1} at 1 kev on $L \simeq 3.9$ [*Frank*, 1967] would be expected to produce at least 10 inverse e foldings on each pass across the equator and thus forbid the detection of even fractional hop whistlers.

It has also been shown that the upper and lower frequency cutoffs of the multiple bounce MR traces, as well as the high-frequency region of growth, can qualitatively be explained by an ambient electron distribution that has a secondary peak in the vicinity of 10 kev. Because the wave intensity variations with frequency depend rather critically on the size and position of the secondary hump, we suggest that MR whistlers could provide a convenient tool for probing the electron spectra at low energies.

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