JOURNAL OF GEOPHYSICAL RESEARCH, SPACE PHYSICS VOL. 73, NO. 13, JULY 1, 1968

Artificial Production of VLF Hiss

T. F. BELL

Radioscience Laboratory, Stanford University
Stanford, California 94305

INTRODUCTION

In the summer of this year an experiment is planned by the Goddard Space Flight Center in which an electron accelerator will be flown aboard an Aerobee 350 rocket from Wallops Island. The details of this experiment have been discussed in a recent report by *Hess* [1967]. The general purpose of the experiment is to generate an aurora by injecting an intense stream of electrons into the geomagnetic field parallel to a field line in such a way that the electrons will enter the atmosphere, deposit their energy, and produce an auroral spot.

It can be envisioned that an experiment of this type, if successful, would prove of importance to geophysicists for a number of reasons. Not only would it be of great interest to study auroral spot production, per se, but it would be also of great interest to consider extensions of the original experiment to other purposes, such as field line mapping, large-scale electric field detection, etc. A number of these important ramifications have been discussed by *Hess* [1967].

In the present Letter, we limit ourselves to the discussion of what may be an observable side effect of the Wallops Island experiment, namely the production of broadband VLF hiss by means of the Cerenkov radiation process.

CERENKOV RADIATION

In a broad sense, Cerenkov radiation can be defined as the electromagnetic radiation emitted by charged particles in rectilinear and uniform motion through, or adjacent to, a refracting medium. The general Cerenkov effect has been discussed in a comprehensive monograph by *Jelly* [1958]. For our own purposes we will be interested in Cerenkov radiation produced by charged particles moving within a quasi-homogeneous refracting medium, i.e., the upper ionosphere. For this case it can be shown that coherent Cerenkov radiation will be emitted in the medium whenever a direction can be found such that the components of the particle and wave phase velocity in that direction are equal. If it is assumed that the particle moves rectilinearly in a given direction, for instance parallel to a magnetic field line, the coherence condition can be written

$$v_{\parallel} \cos \theta = V_{ph} \qquad (1)$$

where $v_{\parallel}$ is the particle velocity parallel to the field line and θ is the angle between the field line and the direction of wave propagation. Since the refractive index for propagating VLF waves in the upper ionosphere (and magnetosphere) can vary over wide ranges as a function of wave frequency and direction, it has been suggested by various authors that condition (1) could be fulfilled for low-energy particles precipitating into the ionosphere. In this case the Cerenkov radiation emitted by these particles might appear as broadband hiss at VLF frequencies.

Apparently the first author to suggest that VLF hiss might result from Cerenkov radiation generated in the upper ionosphere or magnetosphere was Ellis in 1957. This work was soon followed by a number of other papers (see references) in which the Cerenkov effect in a magnetoplasma was considered in more detail.

Despite much study, a link between Cerenkov radiation and VLF hiss has so far been neither proved nor disproved, and the connection between hiss and Cerenkov radiation remains unresolved.

Consequently, considering the fact that VLF hiss is an important but unexplained VLF phenomenon, and considering the fact that the Cerenkov process may be an important loss mechanism for low-energy particles in a magnetoplasma, it is of interest to look upon the Wallops Island electron accelerator rocket experiment from the point of view of it being a Cerenkov radiation generator rather than an auroral spot producer. This will be the point of view taken in the remainder of this paper, wherein we will attempt to estimate the steady-state spatial distribution of the Cerenkov radiation within a few hundred kilometers of the electron beam.

In the next section we discuss the experimental parameters necessary for this calculation.

EXPERIMENTAL PARAMETERS

(See *Hess* [1967] for additional information. Only facts necessary to the present discussion are given here.)

The backbone of the Wallops Island experiment is the electron accelerator. This device will consist of a set of nine electron guns each with a single accelerating grid. The guns will be pulsed with voltages of four values 1.25, 2.5, 5, and 10 kev, and the controlled currents will vary from 1.5 to 500 milliamperes. The majority of the pulses will be 0.1 second in duration, but approximately every 10 pulses a long pulse of 1 second at full power will be generated to give the observers on the ground a better chance to see the auroral spot. Pulses will be put out once every 3.5 seconds.

The electron guns are to be placed in the nose of the rocket, and an attitude control system on board is designed to align the rocket (and guns) parallel to the magnetic field during flight. An 80-foot diameter aluminized mylar foil will be deployed about the axis of the rocket to act as an electron collector to balance the beam current. The electron guns will be in operation from an altitude of 200 km up to apogee (~350 km) and back down to 200 km.

It is estimated that the electrons will be injected into the magnetic field with initial pitch angles between ±7° and will penetrate to an altitude between 100 km (10 kev) and 150 km (1.25 kev).

In Table 1 some of the important electron beam parameters are listed as functions of the particle energy. The *maximum gyroradius* is calculated by assuming that the particle velocity is purely transverse to the dipole magnetic field at an altitude of about 200 km. In practice, the actual gyroradius will be less than the quantity shown, since the particles are to be injected approximately parallel to the magnetic field; thus the electrons will be tightly bound to a given magnetic field line as long as space charge effects are negligible.

The *transit time* is calculated for a 200-km path and assumes that the particle is injected parallel to a field line. Since the average rocket velocity is about 1 km/sec, the electron guns will change their position by less than 10 meters during the time for a single particle to travel from the gun to a height 200 km lower in altitude. Thus the beam can be considered to a good approximation to always move parallel to the field lines, which over these altitudes can be considered to be straight lines.

The *kinetic power* from the gun and the *beam density* entries are calculated on the basis of a ½-amp current output from the accelerator.

Additional data needed for the Cerenkov calculation concerns the ionospheric parameters over Wallops Island during the rocket shot. These cannot be predicted with accuracy, and the best that can be done is to attempt to devise a model that (it is hoped) will represent the ionosphere over Wallops in some average sense.

The model we will use for calculations is shown in Figure 1. This model was constructed, using CRPL ionosonde charts for Washington, D. C., in August 1967, and represents a parabolic distribution with height according to the relation

TABLE 1. Electron Beam Parameters

	1.25 kev	2.5 kev	5 kev	10 kev
Maximum gyro-radius	1.2 meters	1.7 meters	2.4 meters	3.4 meters
Transit time, 200 km	9.4 msec	6.7 msec	4.7 msec	3.3 msec
Kinetic power from gun, 1/2 amp	625 watts	1250 watts	2500 watts	5000 watts
Beam density, electrons/meter	1.4×10^{11} m^{-1}	10^{11} m^{-1}	7×10^{10} m^{-1}	5×10^{10} m^{-1}

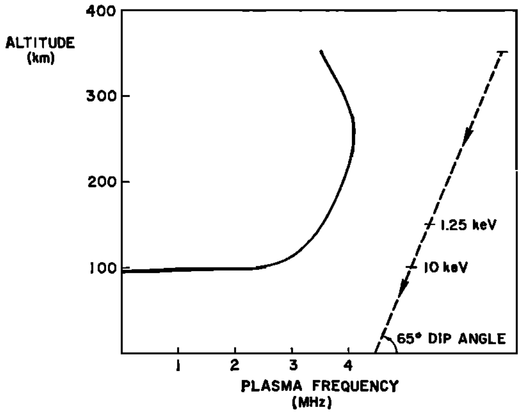

Fig. 1. Model ionosphere over Wallops Island. Also depicted are estimated auroral spot altitudes for both 1.25- and 10-kev electrons.

$$f_0 = 4 \text{ MHz} \left[1 - \left(\frac{h - 250}{230}\right)^2\right]^{1/2}$$

The above distribution is one appropriate to about 0400 LMT. Since the range in f_oF_2 between upper and low quartiles is 4.9–3.6 MHz for the CRPL data, it may be expected that deviations from this model could be large.

Regarding the ambient magnetic field, it is assumed that a dipole model is appropriate, and the magnetic latitude of Wallops Island is taken as 50°N, with a resultant dip angle at Wallops of 65°.

Physical Model and Formulas

Two facts suggest that the accelerator experiment should be considered as a 'steady-state' experiment insofar as the production of VLF Cerenkov radiation is concerned:

1. Once the rocket is above the 300-km level, the fractional change in the total beam length due to the rocket's motion will be less than 1% during any given pulse period, and it is a reasonable approximation to consider that the beam length is essentially constant during these periods.

2. The theory of whistler mode propagation indicates that the group velocity of VLF waves is approximately equal to their phase velocity [*Helliwell,* 1965]. Thus, according to (1), within a few transit times after the pulse initiation it can be expected that all transient fields will have propagated out of the region within 200 km of the beam, and only steady-state radiation will be observed in this region during the major portion of the duration of any given pulse.

Assuming that the above arguments are correct, the intensity of VLF noise near the beam will depend upon the steady-state output of Cerenkov radiation from the beam.

If the current density along the beam is constant, the steady-state of the system will be one in which there is no net Cerenkov radiation from the beam. This can be seen from the fact that a beam of constant density is equivalent to a uniform current element, which in the steady-state cannot produce a time-dependent magnetic field. Thus, a steady-state radiation output from the beam can arise only from fluctuations in the beam current.

In the present problem, due to the complexity of the beam interaction with the ambient plasma, we cannot hope to predict the statistical behavior of the beam. Consequently, it is somewhat superfluous to attempt to do detailed calculations of the Cerenkov radiation output from the beam using complicated statistical models. Instead, we intend merely to show the range of values for the radiation that could be expected on the basis of two crude but extremely simple statistical models. These two models are described below:

Beam with uncorrelated fluctuations. Here we assume complete lack of correlation between different parts of the electron beam up to the scale of interparticle distances. In this case each electron acts as an independent radiator, and the power density at a given point is just the sum of the power densities of each electron at that point. Thus this is an instance of incoherent radiation from the particles.

Beam with partially correlated fluctuations. Here we assume that at any one time the beam consists of a given number of separate sections of length Δ in each of which the fluctuations are completely correlated. However, it is assumed that there is no correlation between these sections. The distance Δ is taken to be much larger than the interparticle spacing but less than a wavelength at the frequencies considered. The number of separate sections of length Δ is taken to be approximately $2l/\Delta$ (where $2l$ = beam length), and it is assumed that these sections are more or less uniformly distributed along the beam.

It is anticipated that use of the first model will give an estimate of the minimum amount of Cerenkov radiation that could be expected from the beam, while use of the second model will give an estimate of the maximum amount.

In view of the crude nature of our statistical models, we will be satisfied with order-of-magnitude estimates and will neglect complicating features of the radiation problem such as finite plasma temperature, boundary conditions at the lower ionospheric boundary, and the inhomogeneity of the ionospheric plasma.

Introducing the above simplifications, it is possible to show that the expression for the spatial distribution of Cerenkov radiation from the beam can be cast into two forms, corresponding to the two statistical models adopted above:

$$P_A(\bar{r}) = P(\omega)N_b(4\pi r)^{-1} \cdot \left[\tan^{-1}\left(\frac{l - z}{r}\right) + \tan^{-1}\left(\frac{l + z}{r}\right)\right]F(\bar{r}, \omega) \tag{2a}$$

$$P_B(\bar{r}) = P(\omega)N_b^2\,\Delta(4\pi r)^{-1} \cdot \left[\tan^{-1}\left(\frac{l - z}{r}\right) + \tan^{-1}\left(\frac{l + z}{r}\right)\right]F(\bar{r}, \omega) \tag{2b}$$

In the above equations, N_b is the average number of beam particles per unit length (given in Table 1), $P(\omega)$ is the single particle power spectral density function, $F(\bar{r}, \omega)$ is a gain factor resulting from the anisotropy of the medium, and the cylindrical coordinate system is as shown in Figure 1.

To use (2) we must know $P(\omega)$ and $F(\bar{r}, \omega)$. The power spectral density function $P(\omega)$ is a complex function of wave frequency, plasma frequency, and gyrofrequency. Detailed studies of this function have been made in a number of papers in recent years. For the calculations reported here, we have made use mainly of the results of three of these papers [*McKenzie*, 1963; *Mansfield*, 1964; *Liemohn*, 1965].

The important features of $P(\omega)$, appropriate to the plasma parameters of Figure 1, are listed below; a few of these major features can be deduced from Figure 2, which has been prepared using the model of Figure 1.

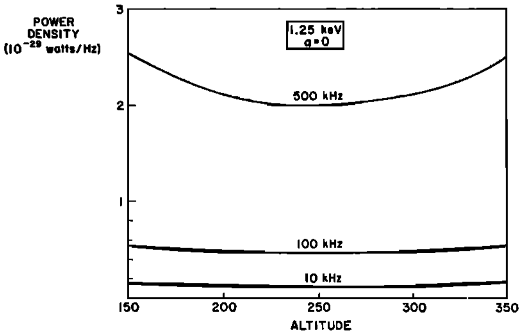

Fig. 2. Single particle Cerenkov power density as a function of altitude at zero pitch angle. Curves are plotted for three center frequencies, 10, 100, and 500 kHz. The particle energy is 1.25 kev, and the pitch angle (α) is zero.

1. $P(\omega)$ is essentially independent of electron pitch angle from 0 to 30° pitch angle.
2. $P(\omega)$ varies little with altitude for frequencies between 10 and 500 kHz.
3. $P(\omega)$ increases with frequency between 10 and 500 kHz. The maximum output (not shown on the graph) occurs at about 800 kHz (giving a few per cent more output than at 500 kHz), and thereafter the power decreases for higher frequency.
4. For wave frequencies below the gyrofrequency, $P(\omega)$ varies approximately inversely as the square root of the particle energy up to 5 kev, while at 10 kev there is no Cerenkov radiation. For wave frequencies between the plasma and upper hybrid frequencies, $P(\omega)$ varies inversely as the square root of the particle energy up to 10 kev. Thus the energy output is always largest for the 1.25-kev electrons.
5. $P(\omega)$ for 1.25-kev electrons is much higher for frequencies above the plasma frequency than for frequencies below the gyrofrequency, being of the order of 10^{-28} watts/Hz over most of the range between the plasma and upper hybrid frequencies.

The final function in (2) that must be discussed is the gain factor $F(\bar{r}, \omega)$. This factor results from the anisotropy of the medium and is normalized in such a way that setting $F(\bar{r}, \omega) = 1$ in (2) yields the radiation that would be observed if all the beam electrons were isotropic radiators. In general, this gain factor is a complicated function of the plasma parameters, the wave frequency, the position vector, and the beam electron velocity distribution. It is possible for $F(\bar{r}, \omega)$ to have a value as high as 10^4, but only for special values of the plasma frequency [*Bell*, 1967]. Under normal circumstances for wave frequencies below 800 kHz, we will have $F(\bar{r}, \omega) \lesssim 10$, while for frequencies between the plasma and upper hybrid frequency $F(\bar{r}, \omega) \sim 1$.

With the above information, we are now in a position to estimate the spatial distribution of Cerenkov radiation within a few hundred kilometers of the beam for the situation in which the rocket is near apogee.

INCOHERENT RADIATION

First consider the beam with uncorrelated fluctuations. From Figure 2, Table 1, and the foregoing discussion of $P(\omega)$, it can be seen that the product of beam density N_b and power density $P(\omega)$ has its maximum value when the particle energy is 1.25 kev and the wave frequency is approximately 500 kHz. This value is approximately

$$N_b(1.25 \text{ kev}) \times P(500 \text{ kHz}) \cong 3 \times 10^{-18} \text{ watts/Hz m} \tag{3}$$

Using (3) in (2a) (along with $F(\bar{r}, \omega) \sim 10$), the Cerenkov power density at a distance of 1 km from the beam can be estimated as

$$P_A(r = 1 \text{ km}) \cong 10^{-20} \text{ watts/Hz m}^2 \tag{4}$$

Considering that the noise level of present-day VLF receivers is about 10^{-18} watts/Hz m^2 [*Helliwell,* 1965], it must be concluded that the power output shown in (4) would not be detectable.

For frequencies between the plasma and upper hybrid frequency, $F(\bar{r}, \omega) \sim 1$ and $P(\omega) \sim 10^{-26}$ watts/Hz, and from (2a) the power distribution close to the beam is given by

$$P_A(|z| < l, r) \sim 10^{-15}\, r^{-1} \text{ watts/Hz m} \tag{5}$$

At a distance of 1 km from the beam, (5) indicates that the power density has dropped to the receiver threshold level of 10^{-18} watts/Hz m^2.

From the above calculations our general conclusion is that a beam consisting of incoherent radiators will produce detectable Cerenkov radiation only within 1 km of the beam.

PARTIALLY COHERENT RADIATION

The radiation output increases significantly when the beam fluctuations are correlated over a length that is much greater than the interparticle distance, but less than a wavelength at the frequencies of interest. There are a number of physical processes whereby such a correlation distance might be established. We consider one of these processes—the two-stream instability.

General features of the two-stream instability have been discussed by *Stix* [1962]. We are concerned with a special case in which one 'stream' consists of a finite radius beam of the monoenergetic electrons, and the second stream consists of the ambient ionospheric electrons, which are of low kinetic temperature and approximately at rest [*Kislov and Bogdanov,* 1961].

Using the ionospheric model of Figure 1, and assuming that the beam radius is of the order of 1 meter (the maximum gyroradius for 1.25 kev electrons), it can be shown that the e-folding time τ of the fastest growing unstable mode is approximately equal to the period of the ambient electron plasma oscillations, i.e., $\tau \sim 10^{-7}$ sec, and the wavelength of the most unstable wave is approximately $\lambda_u \sim v_\parallel/f_o$, where $v_\parallel$ is the beam velocity parallel to the field lines. Since the transit time of the electrons is about 10 msec, beam fluctuations originating near the accelerator will have of the order of 10^5 e-folding times available in which to grow. In this case it could be expected that nonlinear effects would set in, and the beam would be broken up into small segments of the order of the wavelength of the fastest growing mode. Thus the value for the coherence length becomes

$$\Delta \sim v_\parallel/f_o \sim 10 \text{ meters} \tag{6}$$

For wave frequencies below the electron gyrofrequency $\lambda > 10$ m and (2b) will apply. If we now evaluate (2b), using (6) and assuming $F(\bar{r}, \omega) \sim 1$, $|\bar{r}| = 1$ km, $f = 500$ kHz, and 1.25-kev particle energy we obtain

$$P_B(500 \text{ kHz}) \sim 10^{-10} \text{ watts/Hz m}^2 \tag{7}$$

This is quite a large value for the power spectral density and would be easily measurable. In fact the power spectral density at a distance from the beam of 100 km would be approximately 10^{-12} watts/Hz m^2, and this would be the power illuminating the lower edge of the ionosphere at about 100-km altitude. From this point, even assuming a 20-db loss due to absorption and reflection, the power density on the ground would be of the order of 10^{-14} watts/Hz m^2, a value which would also be easily measurable.

On the basis of the above calculation, the general conclusion is that a beam consisting of partially correlated fluctuations could produce detectable Cerenkov radiation in the ionosphere within 100 km of the beam and also on the ground directly below the beam.

On the basis of the above results, it is possible to suggest at least one way in which the original experiment could be modified, so that the production of hiss would be enhanced. For instance, if the electron guns were programmed to fire microsecond bursts of 1.25-kev electrons rather than long pulses, these small groups of electrons would radiate coherently over a frequency range of 20 to 500 kHz. If the pulse repetition rate were of the order of 10^5/sec, the power levels at 500 kHz would be comparable to those shown in (7), while the power density at 20 kHz would be about two orders of magnitude lower. On the other hand, the Cerenkov wavelength at 20 kHz is about 10 times the Cerenkov wavelength at 500 kHz, and the pulse length could be increased by this factor, resulting in a power density at 20 kHz, which was also about 10^{-10} watts/Hz m^2 at a distance of 1 km from the beam.

Conclusion

In conclusion, we summarize the main findings of this paper:

(1) During the Wallops Island electron accelerator rocket experiment, electrons traversing the field line path between the accelerator ($h > 200$ km) and the auroral spot location ($h \sim 100$ km) will emit Cerenkov radiation that will appear as broadband VLF hiss.

(2) In the event that the fluctuations in the accelerated electron beam are uncorrelated over the field line path, the Cerenkov hiss will be of observable intensity only within about 1 km of the beam.

(3) In the event that the fluctuations in the electron beam are correlated over a distance that is much larger than the interparticle spacing, but less than a wavelength at the frequency of interest, the Cerenkov hiss could approach an intensity of about 10^{-12} watts/Hz m^2 within 100 km of the beam and an intensity of about 10^{-14} watts/Hz m^2 on the ground at Wallops. These values are much above the VLF receiver noise levels of 10^{-16} watts/Hz m^2 and should be easily observable.

(4) A modification of the Wallops Island experiment is proposed that would enhance the hiss intensity. It is suggested that the accelerator be programmed to produce microsecond bursts of particles at pulse repetition rates of about 10^5 sec. In this case hiss power levels could approach the values mentioned above.

Acknowledgments. The author wishes to express his thanks and appreciation to Dr. T. S. Jørgensen for his generous contributions to this paper and to thank his other colleagues at the Radioscience Laboratory for their helpful criticism.

This research was supported by the National Aeronautics and Space Administration under grant NsG 174-SC/05-020-008 and under contract NAS 5-3093.

References

Bell, T. F., The artificial production of VLF hiss, paper presented URSI Meeting, Ottawa, Ontario, Canada, May 23–26, 1967.

Cohen, M. H., Radiation in a plasma, 1, Cerenkov effect, *Phys. Rev., 123,* 711, 1961.

Eidman, V. Ya., The radiation from an electron moving in a magneto active plasma, *Soviet Phys. JETP, 34,* 91, 1958.

Ellis, G. R., Low frequency radio emission from aurorae, *J. Atmospheric Terrest. Phys., 10,* 302, 1957.

Gallet, R. M., and R. A. Helliwell, Origin of very-low-frequency emissions, *J. Res. NBS, 63D,* 21, 1959.

Ginzburg, M. A., Electromagnetic radiation from solar corpuscular streams, *Phys. Rev. Letters, 7,* 399, 1961.

Helliwell, R. A., *Whistlers and Related Ionospheric Phenomena,* Stanford University Press, Stanford, California, 1965.

Hess, W. N., The Wallops Island electron accelerator rocket experiment, *Goddard Space Flight Center Rept.,* Greenbelt, Maryland, April 1967.

Jelly, J. V., *Cerenkov Radiation,* Pergamon Press, New York, 1958.

Kislov, V. J., and E. V. Bogdanov, Interaction between slow plasma waves and an electron stream, in *Proc. Symp. on Electromagnetics and Fluid Dynamics of Gaseous Plasma,* Polytechnic Press, Polytechnic Institute of Brooklyn, April 1961.

Liemohn, H. B., Radiation from electrons in magnetoplasma, *Radio Sci., 69D,* 741, 1965.

Mansfield, V. N., Cerenkov and cyclotron radiation as VLF emission sources, *Sci. Rept., Radio-*

physics Lab., Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire, 1964.

McInnes, B. A., A study of ionospherics at Macquarie Island, *Australian J. Phys., 14,* 218, 1961.

McKenzie, J. F., Cerenkov radiation in a magnetoionic medium, *Phil. Trans. Roy. Soc. London, A255,* 585, 1963.

Ondoh, T., On the origin of VLF noise in the earth's exosphere, *J. Geomagnet. Geoelec., 12,* 77, 1961.

Stix, T. H., *The Theory of Plasma Waves,* McGraw-Hill, New York, 1962.

Tuan, H. S., and S. R. Seshadri, Radiation from a uniformly moving charge in an anisotropic plasma, *IEEE Trans. MIT, 11,* 462, 1963.

(Received March 22, 1968.)