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# An Interpretation of Transverse Whistlers

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## INTRODUCTION

Recently, *Carpenter and Dunckel* [1965] reported a new type of whistler observed in the Alouette 1 satellite VLF recordings. It appeared in the frequency range of 1–8 kc/s and was observed late at night (2100 ~ 0100 local time) from moderate to low latitudes (44° ~ 30° dipole latitudes). One peculiar characteristic of this whistler is that the variation of time delay with frequency, observed at 1000-km altitude, is given by the sum of the Eckersley law time delay, characteristic of normal whistlers, and an additive correction term, as illustrated in Figure 1. The solid line in Figure 1*a* shows an example of the measured frequency versus time curve of one such whistler which was observed at 38°N geomagnetic latitude during the Alouette run of March 18, 1963. The dashed line in this figure is the theoretical curve for a constant dispersion with $D_0 = 17$. Figure 1*b* shows that the difference in travel time ($\Delta t$) between the two curves is almost constant (independent of frequency).

Thus the time delay for the whistler may be written as

$$t = (D_0/f^{1/2}) + \Delta t \qquad (1)$$

where $D_0$ is constant (about 17 sec$^{1/2}$) independent of frequency $f$. As a result of this added time delay, the dispersion ($D = t/f^{1/2}$) of these anomalous whistlers increases markedly with increasing frequency.

*Carpenter and Dunckel* [1965] also noted that, when a series of these whistlers was observed over a range of latitudes, the constant dispersion $D_0$ did not change, but the additive time delay increased almost linearly with latitude from 0 at 30° latitude to 0.22 second at 44°.

In view of the explanation given below, we suggest that this new type of whistler be called a 'transverse whistler.'

## POSSIBLE EXPLANATIONS

As a first thought, it might be suggested that the increase in time delay at the higher frequencies is due to the nose whistler effect. This explanation, however, would require that the upper frequencies be of the order of half the minimum gyrofrequency along the propagation path. A nose frequency of 10 kc/s would indicate a path latitude of roughly 60°, for which a low-frequency dispersion of about 70 would be expected. As these anomalous whistlers are observed only at low latitudes and have much smaller low-frequency dispersions, it seems highly unlikely that the anomalous dispersion results from nose whistler effects.

An alternative explanation is that the additive time delay $\Delta t$ results from propagation transverse to the earth's magnetic field over part of the path. The constant dispersion $D_0$ is, of course, readily explained as normal whistler mode propagation [*Storey*, 1953]. It was pointed out by *Hines* [1957] that, when ion effects are included, propagation transverse to the earth's magnetic field in the ionosphere is possible at whistler mode frequencies. The upper frequency limit for this transverse propagation is the lower hybrid resonance frequency [*Smith and Brice*, 1964].

For a plasma containing electrons and a single ionic species, the refractive index for quasi-transverse (QT) propagation is given by [*Smith*, 1964]

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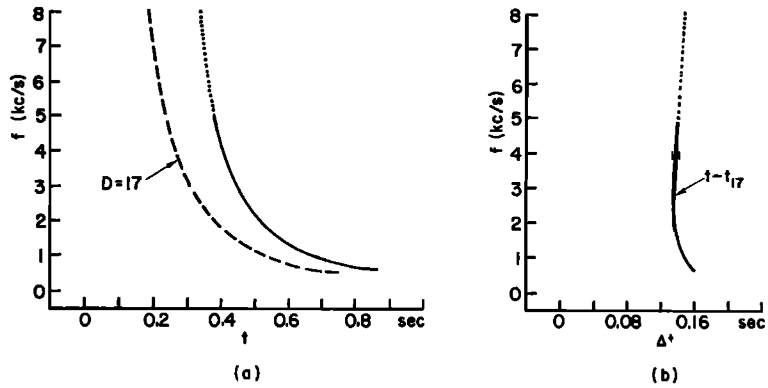


Fig. 1. (*a*) Frequency versus time curve of an observed transverse whistler (solid line) and the Eckersley law curve (dashed line). (*b*) Difference in time delay $\Delta t$ between the observed and the theoretical (Eckersley law) curves [*Carpenter and Dunckel,* 1965.]

$$\mu_\perp = \frac{f_0}{f_H(1 - f^2/f_r^2)^{1/2}} M^{1/2} \qquad (2)$$

where $f_0$ and $f_H$ are the plasma and gyrofrequency of the electron and $M$ is the mass ratio of the ion to the electron; $f_r$ is the lower hybrid resonance frequency, given by

$$\frac{1}{f_r^2 M} = \frac{1}{f_0^2} + \frac{1}{f_H^2} \qquad (3)$$

If the frequency $f$ is much less than $f_r$, $\mu_\perp$ becomes almost independent of $f$. Then the group refractive index for QT propagation becomes identical with the refractive index $\mu_\perp$; that is,

$$\mu_{g\perp} = \mu_\perp = (f_0/f_H)M^{1/2} \qquad (4)$$

It is worth noting that the above expression is also valid for a multi-ion case, in a limited frequency range, where $M$ is taken to be the effective mass $M_{eff}$, given by

$$1/M_{eff} = \sum \alpha_i/M_i$$

where $M_i$ is the mass ratio of each ion to electron and $\alpha_i$ is the relative concentration of each ion [*Smith and Brice,* 1964].

On the other hand the group refractive index for quasi-longitudinal (QL) propagation is approximately given by (Eckersley's law)

$$\mu_{g\parallel} \cong \tfrac{1}{2}f_0/\sqrt{ff_H} \qquad (5)$$

Substitution from equation 4 yields

$$\mu_{g\parallel} \cong \tfrac{1}{2}\mu_{g\perp}(f_H/Mf)^{1/2}$$

It is apparent that for frequencies well above the ion gyrofrequency ($f > f_H/M$), $\mu_{g\perp}$ is greater than $\mu_{g\parallel}$.

In Figure 1, the group refractive indexes (which are proportional to the propagation delays) are illustrated for the QL and QT propagation.

The QL group refractive index plotted in this figure is obtained from the Eckersley dispersion law and the QT from equation 2 above. It is seen that the QT propagation delay (proportional to $\mu_{g\perp}$) is almost constant but increases slightly with increasing frequency, as does the $\Delta t$ in Figure 1*b* at frequencies above about 2 kc/s. The curves in Figure 2 are not plotted for frequencies below 1 kc/s because, at sufficiently low frequencies, the group refractive index for QL propagation departs from that given by the Eckersley law because of ion effects. Also, for a plasma containing more than one ion species, the QT group refractive index is changed at the lower frequencies from that given by equation 2. The presence of a multiple ion cutoff for trans-

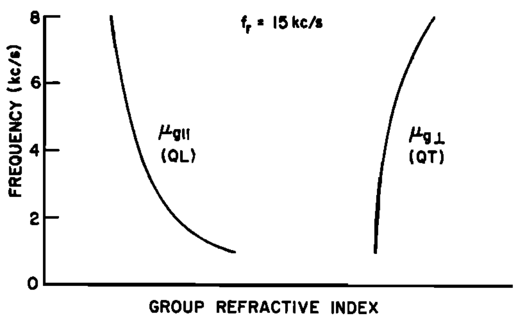


Fig. 2. Group refractive indices for the QL and QT propagation.

verse propagation causes the QT group refractive index to increase at the low frequencies and can thus explain the slight increase in $\Delta t$ at the lower frequencies seen in Figure 1*b*.

From Figure 2 it is apparent that a whistler resulting from QL propagation over part of the path and QT over the remainder of the path would have the dispersion characteristics of the anomalous whistlers. The constant dispersion would result from the QL propagation, and the additive time delay from the QT propagation. In this case the observation that the additive time delay is nearly independent of frequency implies that the highest frequencies of the whistler are less than the lower hybrid resonance frequency. If the proposed model is correct, the highest frequency of the transverse whistler should always be below the lower hybrid resonance frequency. There is some suggestion in the data that this is, in fact, the case [*Carpenter and Dunckel,* 1965]. It remains to be shown that a propagation path such as that described above may exist.

POSSIBILITY OF PURELY TRANSVERSE PROPAGATION

We have seen that transverse propagation may be invoked to explain the characteristics of the 'transverse' whistler. However, the conditions for transverse propagation must persist over a path of at least several hundred kilometers in length in order to explain the observed amount of $\Delta t$ in equation 1.

Fortunately, the conditions for propagation through the ionosphere exactly transverse (and exactly parallel) to the direction of the centered-dipole-magnetic field have been determined by *Hoffman* [1960]. In his study, the general forms of the refractive indices required were obtained by integrating *Haselgrove's* [1955] partial differential equations for ray tracing. The form of the transverse refractive index required to maintain transverse propagation is given by

$$\mu_\perp^2 = A \cos^6 \theta f_H^2 F(\cos \theta / r^{1/2}) \qquad (6)$$

where $\theta$ is the dipole latitude, $r$ is the radial

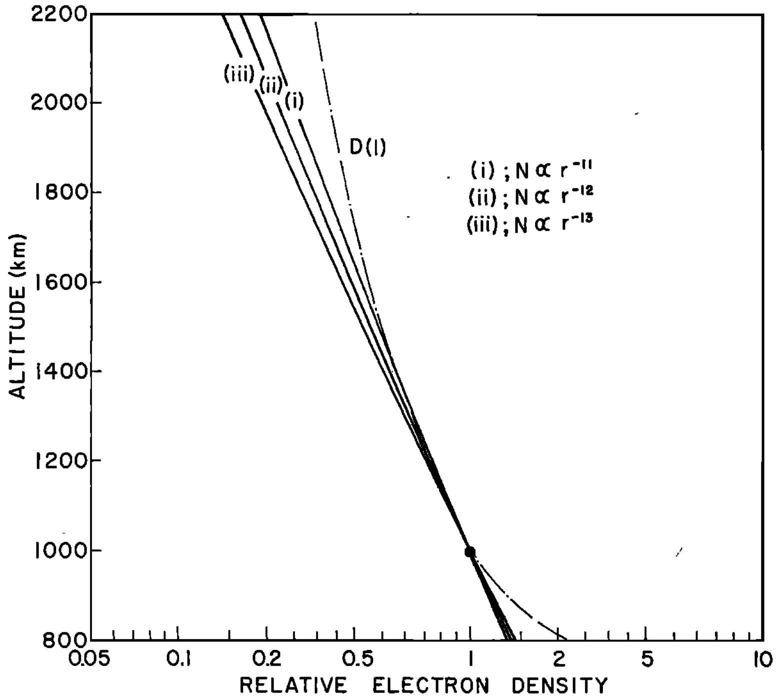


Fig. 3. Relative electron density derived from Hoffman's functions with altitude normalized by the value at 1000 km. Curves (i), (ii), and (iii) correspond to $n = -2, 0, +2$, respectively, for $F = (\cos \theta / r^{1/2})^n$.

distance, $F(\cos\theta/r^{1/2})$ is an arbitrary analytic function of $\cos\theta/r^{1/2}$, and $A$ is a constant. We shall determine whether condition (6) can be attained for an adequate path length by using a realistic model of the ionosphere.

Let us assume that the transverse refractive index is given by equation 4. From Hoffman's condition (6)

$$A \cos^6 \theta f_H^{\,2} F\left(\frac{\cos\theta}{r^{1/2}}\right) = \frac{f_0^{\,2}}{f_H^{\,2}} M_{eff} \qquad (7)$$

or

$$f_0^{\,2} = \frac{A}{M_{eff}} \cos^6 \theta f_H^{\,4} F\left(\frac{\cos\theta}{r^{1/2}}\right) \qquad (8)$$

which determines the required electron density distribution.

Since the function $F$ is arbitrary, let us examine a number of different functions and compare the resulting density distributions with those observed in the ionosphere. For functions of the form $F = (\cos\theta/r^{1/2})^n$, and letting $B = A/80.6$, we obtain the following electron density models for $n = -2$, 0, and $+2$, respectively.

$$\text{(i)}\ N = \frac{B}{M_{eff}} \frac{1}{r^{11}} \cos^4\theta(1 + 3\sin^2\theta)^2 \qquad (9)$$

$$\text{(ii)}\ N = \frac{B}{M_{eff}} \frac{1}{r^{12}} \cos^6\theta(1 + 3\sin^2\theta)^2 \qquad (10)$$

$$\text{(iii)}\ N = \frac{B}{M_{eff}} \frac{1}{r^{13}} \cos^8\theta(1 + 3\sin^2\theta)^2 \qquad (11)$$

The variations of $N$ with respect to $r$ (or altitude) are shown in Figure 3 for the above functions, where $M_{eff}$ is assumed constant. $D(I)$ in Figure 3 is the diffusive equilibrium model of *Angerami and Thomas* [1964] for a temperature of 1000°K. This model fits the topside sounder data well up to an altitude of 1000 km. We can see that the change of $N$ with altitude is not very great for the three Hoffman functions shown, and not very different from $D(I)$ in the height range of 1000 to 1600 km. Moreover, if we consider the tendency of $M_{eff}$ to decrease with altitude, Hoffman's functions appear to be a reasonable approximation to $D(I)$. Figure 4 shows the dependence of electron density with latitude ($\theta$) for the three models and also one example of the actual variation with latitude at a height of 1000 km. We see that the $\theta$ dependence can change markedly with the choice of $F$. Of the three cases, (ii) fits best to the actual electron density for geomagnetic latitudes higher than 35°.

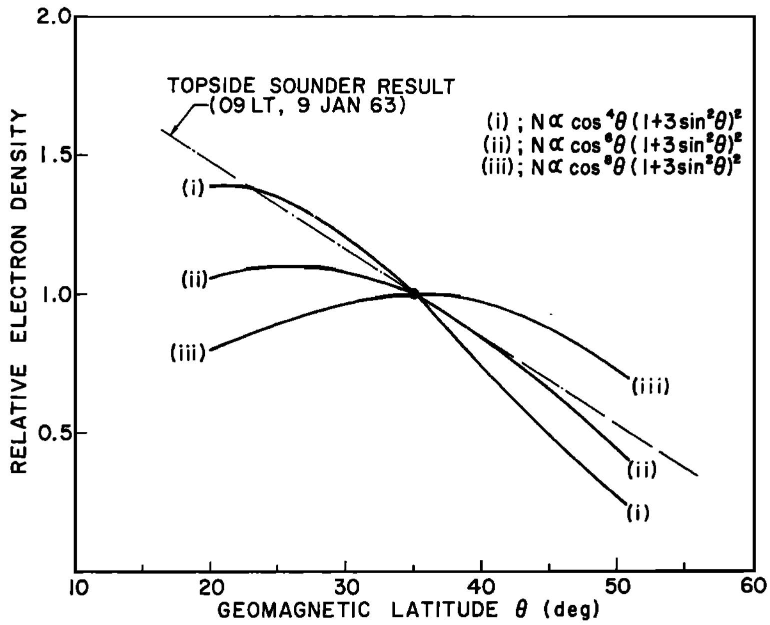


Fig. 4. Relative electron density derived from Hoffman's functions with latitude normalized by the value at latitude 35°. Curves (i), (ii), and (iii) correspond to $n = -2, 0, +2$ as in Figure 3.

From these examples it appears that the Hoffman condition for maintaining transverse propagation may sometimes be realized over a limited range of altitudes and latitudes.

## A Model for the Transverse Whistler

In the previous section we examined some electron-density distributions which would maintain transverse propagation and found that favorable conditions could exist in a limited region at about 1000-km altitude and moderately low latitudes. This observation leads us to the following model for the transverse whistler. It is suggested that this whistler is initially a normal ducted whistler [*Smith,* 1960], propagating quasi-longitudinally. The observation that the constant dispersion part of the time delay does not change with latitude indicates that all the whistlers travel in a single field-aligned duct. After a transition from QL to QT propagation, the whistler propagates quasi-transversely for a significant distance.

Two possible types of transitions labeled (*a*) and (*b*) are illustrated in Figure 5. In (*a*), the wave normal of the downcoming wave is gradually refracted towards 90°. Unfortunately, electron-density distributions favorable both for refracting the wave toward 90° and for maintaining QT propagation are unrealistic.

In (*b*), the downcoming whistler is reflected from the base of the ionosphere or an irregularity, such as a sporadic-*E* layer. For such a reflected ray, the angle between the wave normal and the magnetic line of force would be quite large for latitudes of interest. For this case the electron densities required are not unreasonable. It has been verified by ray-path calculations for realistic electron- and ion-density profiles that, after a reflection in the *E* layer, the ray can be launched into the transverse direction [*Kimura,* 1966], as could be expected from the explanation of the subprotonospheric whistler [*Smith,* 1964].

In the above model the additional time delay due to QT propagation increases with increasing latitude of the satellite location, as is observed. Although the transverse trajectory does not coincide with the trajectory of the satellite, the energy of the wave can easily leak away from the QT trajectory, because this propagation path is not stable. The situation may be explained as follows. The ray direction may be changed from nearly transverse to nearly longitudinal by a relatively small change of the wave normal angle. Consequently small irregularities may easily scatter the wave energy away from the QT-ray path. The scattered energy will be propagated in the QL mode to the satellite. This propagation path does not add much time delay, since the path is short and $\mu_{g\parallel}$ is much less than $\mu_{g\perp}$. Since the transverse path is somewhat unstable, this places a limit (unknown at present) on the distance over which the transverse path can reasonably be expected to persist.

As an example, assume that at 1500-km altitude

$$f_0 = 1000 \text{ kc/s} \qquad f_H = 700 \text{ kc/s}$$
$$M_{eff} = 2000$$

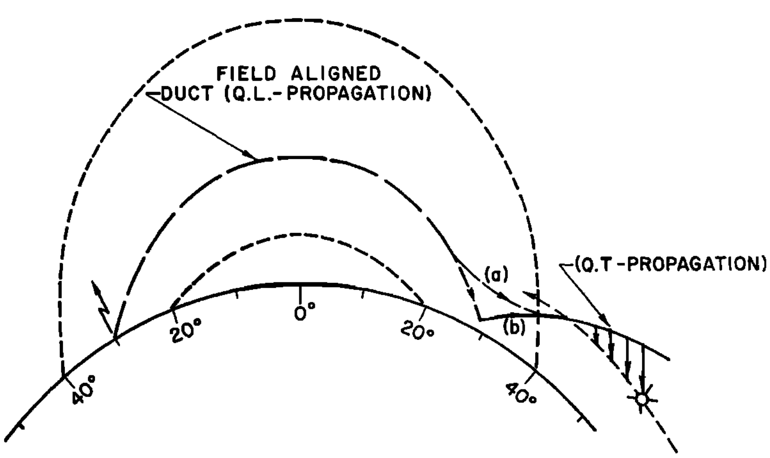


Fig. 5. A model of the transverse whistlers.

then $\mu_{g\perp}$ is given by (4), as

$$\mu_{g\perp} = \frac{f_0}{f_H}\sqrt{M_{eff}} = \frac{1000}{700}\sqrt{2000} = 63.9$$

independent of frequency. For the same conditions and a frequency of 4 kc/s, $\mu_{g\parallel}$ is given by

$$\mu_{g\parallel} \cong \frac{1}{2}\frac{f_0}{\sqrt{ff_M}} = \frac{1000}{2\sqrt{4 \times 700}} = 9.66$$

For QT propagation, the time delay for a 1000-km propagation path length (corresponding to about a 10° change in latitude) is 0.213 second, an amount comparable to the largest observed $\Delta t$.

## Conclusion

The whistlers with anomalous dispersion reported by *Carpenter and Dunckel* [1965] are interpreted as follows: The constant dispersion (independent of latitude) arises from QL propagation in a field-aligned duct. The time delay, which is approximately independent of frequency, arises from subsequent QT propagation. It varies linearly with latitude because the length of the QT-ray path varies linearly with latitude.

Previously reported phenomena such as the subprotonospheric whistler [*Smith,* 1964] and lower-hybrid-resonance emissions [*Brice and Smith,* 1965] have drawn attention to transverse propagation of VLF in the ionosphere. However, the name transverse whistler seems particularly appropriate for the whistlers discussed above since their unusual characteristic arises from persistent QT propagation. Since, as was pointed out by *Hines* [1957], transverse propagation is possible only when ion effects are included, these whistlers, by providing evidence of transverse propagation, implicitly provide evidence of the importance of ion effects on propagation.

*Note added in proof.* Additional ray tracings have been made which indicate that the attenuation loss in the transverse mode for the paths and frequencies illustrated are very low—of the order of 0.1 db/1000 km.

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