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Electron density and path latitude determination from v.l.f. emissions

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Abstract—A new technique has been developed for determining nose frequencies and time delays at the nose frequency for propagation paths in the outer magnetosphere. The computed normalized whistler shape is plotted on a log-log scale, allowing the nose frequency and nose time delay to be found very simply by an overlay method, from measurements of time delays at any two other frequencies. The versatility and simplicity of this method make it particularly suited for use with periodic v.l.f. emissions.

1. Introduction

Whistlers have been extensively used to determine electron densities in the outer magnetosphere by Smith (1960) and Carpenter (1963). The usefulness of whistlers arises from the fact that the frequency of minimum time delay or "nose" frequency, (f_n) indicates the latitude of the path of propagation and the time delay at the nose frequency or "nose time delay" (t_n) is a weighted measure of the electron density distribution along this path.

Smith (1960) showed that the normalized shape of a whistler (i.e. the ratio of frequency to nose frequency plotted against the ratio of time delay to nose time delay) was almost independent of the distribution of electron density along the propagation path. Smith and Carpenter (1961) used this fact in deriving a method of obtaining the nose frequency and nose time delay from whistlers for which the highest frequency observed was less than the nose frequency. This greatly increased the number of whistlers from which path latitude and electron density information could be obtained.

The Smith–Carpenter method may be summarized as follows: For two frequencies, f_u (the upper frequency) and f_l (the lower), the ratio of the whistler-mode group delays t_u (measured at f_u) to t_l (measured at f_l) may be computed as a function of f_u/f_n, for a given ratio f_u/f_l. The ratio f_u/f_l typically used for whistlers is two. For a whistler, then, the time delay is measured at two frequencies, the upper frequency being twice the lower. From the ratio of the time delays, the ratio of the nose frequency to the upper frequency is obtained from a computed curve. Knowing the nose frequency and the time delay at another (known) frequency, the nose time delay can be found.

2. Consideration of Individual Emissions

Let us now consider the applicability of this technique to finding the magnetic field line paths on which v.l.f. emissions are generated. Brice (1962) showed that

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for one particular type of emission called a "hook" (GALLET, 1959), the whistler-mode group delay for the path could be obtained as a function of frequency if it could be assumed that the hook was generated symmetrically about the top of the path, as had been suggested by DOWDEN (1962). However, measurements made on hooks which echoed in the whistler mode showed no agreement between the computed group delays (using the assumption of symmetrical generation) and measured delays between the hooks and their whistler-mode echoes (BRICE, 1962). It appears then that the assumption of symmetrical generation is not valid so that it is unlikely that information regarding the propagation path can be obtained in this manner from a single v.l.f. emission.

3. PERIODIC EMISSIONS

From examination of two recordings containing periodic emissions and whistlers, HELLIWELL (1963) found excellent agreement between the emission period and the two-hop whistler-mode group delay. This observation led HELLIWELL (1963) to suggest that, as a whistler may stimulate or "trigger" the generation of emissions, so the emission, echoing in the whistler mode, may "trigger" new emissions, giving rise to a series of periodic emissions in which the emission period is the same as the whistler-mode group delay for the frequency at which triggering occurs. Much additional support for this hypothesis has subsequently been found by HELLIWELL and BRICE (1964). It appears then that whistler-mode group delays may be obtained for the path of propagation of periodic emissions. For a long enduring set of periodic emissions, by measuring the total duration of the set, counting the number of emissions and then dividing, the emission period may be accurately determined as was illustrated by HELLIWELL (1963).

Alternately, if each set contains only a few emissions, but there are many sets, the period may be accurately determined by an auto-correlation technique developed by BRICE (1965). HELLIWELL (1963) described two types of periodic emissions, dispersive and non-dispersive. For the former, the emissions are predominantly whistler-mode echoes of previous emissions, with relatively little newly-generated emission observable. For the latter, the period for emissions of a given set is the same at all frequencies. As was noted above, this period is the two-hop whistler-mode group delay for the frequency at which triggering occurs. Since we need to know the whistler mode group delay at two frequencies, we will be concerned only with dispersive periodic emissions or non-dispersive emissions in which there are two or more sets of emissions with different triggering frequencies. Both types of emissions may be seen in Fig. 2.

In attempting to use the emission periods and the Smith–Carpenter method to obtain information about the propagation path, some difficulty is encountered. For a large number (over 100) of periodic emissions, measurements were made of the maximum and minimum frequencies at which the emission period could be measured. The average ratio of these frequencies was only 1·24. It is apparent then that to obtain the nose frequency for the propagation path with maximum accuracy, we will want to use the maximum available ratio of frequencies. In addition, instances may be found in which the emission period can be conveniently measured only at two discrete frequencies. For both these reasons, we require a convenient method of obtaining f_n and t_n for the propagation path for any ratio of frequencies, f_u and f_l, at which whistler-mode group delays are measured. A suitable method has been devised and is illustrated in Fig. 1. Measurements of frequencies and periods were made from the emissions shown in Fig. 2 to demonstrate the method. These emissions were exceptional in that periods could be measured for an unusally wide range of

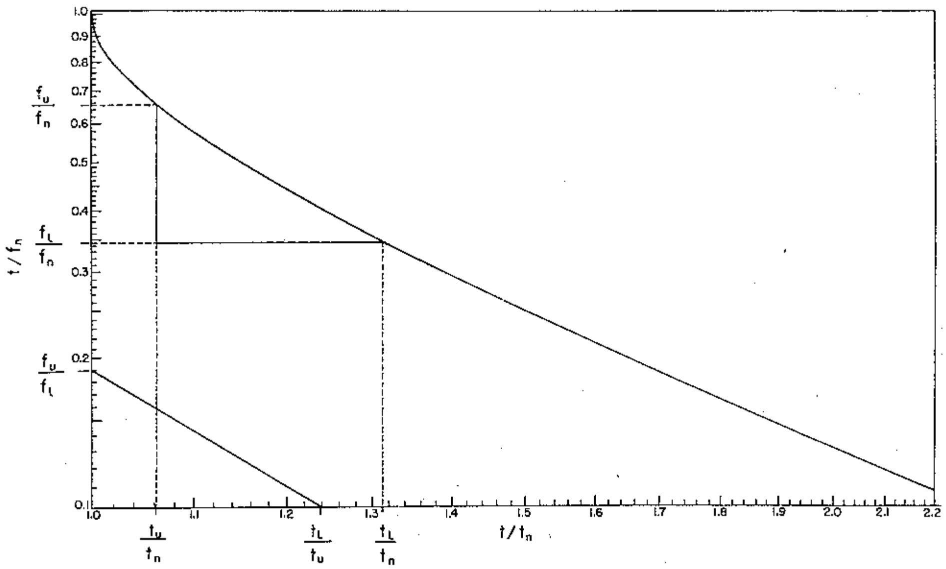

Fig. 1. The computed normalized whistler shape, ratio of frequency to nose frequency plotted against the ratio of time delay to time delay at the nose using a log-log scale.

frequencies. For these emissions, periods of 1·086 and 0·877 sec were measured at 4·08 and 7·75 kc/s respectively, so that

$$\frac{f_u}{f_l} = 1{\cdot}9,$$

$$\frac{t_l}{t_u} = 1{\cdot}238.$$

The method of obtaining f_n and t_n is now described. The whistler shape is computed, normalized, and the ratio f/f_n plotted against t/t_n using log-log graph paper. A piece of transparent paper is then placed on this graph, a vertical line drawn with a length corresponding to f_u/f_l (1·9) and a horizontal line drawn with a length corresponding to t_l/t_u (1·238) as illustrated in the lower left hand corner of Fig. 1. The overlay is then moved, keeping the vertical line vertical and the horizontal line horizontal, until the end points of these lines lie on the plotted curve. The ratios f_u/f_n (or f_l/f_n) and t_u/t_n (or t_l/t_n) are then read from the vertical and horizontal scales respectively, as is also illustrated in Fig. 1.

This method is quick, simple and accurate, may be used for any frequency ratio, and only one curve need be computed. The advantages of this method arise from the use of logarithmic scales. A vertical distance represents a fixed frequency *ratio*, regardless of its location on the graph and a horizontal distance likewise represents a fixed time delay *ratio*. This allows us to use the overlay method and hence find f_n and t_n simultaneously. It should be remembered that the emission period measured is the two-hop whistler mode group delay, so the t_n found is the nose frequency time delay for two traverses of the path of propagation.

4. SOURCES OF ERROR

There are three possible sources of error in this method (two of them applying also to the Smith–Carpenter method). These arise from errors in measurements, uncertainties in the computed curve and imprecision in correctly locating the overlay. If the measured frequency ratio is $\alpha \pm \varepsilon$ and the time delay ratio $\beta \pm \delta$, the error in nose frequency may be estimated by using first a frequency ratio $\alpha + \varepsilon$ and a time delay ratio $\beta - \delta$, and then a frequency ratio $\alpha - \varepsilon$ and time delay ratio $\beta + \delta$. These will give minimum and maximum values of f_n and maximum and minimum values of t_n respectively.

While the computed whistler shape is almost constant, differences in the assumed path latitude and the assumed electron density distribution along the path do make small changes in this shape. These changes represent differences in the weighting of the time-delay integral near the top of the path (where the "nose" effects are large) relative to the weighting near the bottom of the path (where these effects are very small). For the purposes of determining an upper limit for the variation in the whistler shape, then, we require models of electron density variation with height for which the electron density falls off much more rapidly and much more slowly than expected. For the purposes of computation, the simplest model of electron density distribution along the path is given by

$$f_0 = \cos^{-n} \alpha$$

where f_0 is the plasma frequency,

α the geomagnetic latitude, and n is a variable. For $n = 0$, the electron density is constant, for $n = 4$, the distribution is very similar to the gyrofrequency model

$$f_0 \propto f_H^{1/2}$$

suggested by SMITH (1960) and for $n = 6$, we obtain a model in which the density falls off very rapidly with distance.

For these three values of "n" and for magnetic field line paths which arrive at the earth's surface at geomagnetic latitudes (α_0) of 51°, 56°, and 61°, whistler shapes were computed by the author, and plotted by J. J. Angerami. For each shape, the nose frequency and nose time delay were obtained by the overlay method, using the emission frequency and time delay ratios given above, the results being tabulated in Table 1. It is seen that the variation in f_n is $\pm 10\%$ and in t_n, $\pm 2\%$, and these should be considered as upper limits. The consistency in the variation of f_n and t_n for the different assumed electron density models and path latitudes indicates that errors arising from imprecise location of the overlay are negligibly small.

Table 1 shows that larger values of f_n accompany smaller values of t_n, the same result being found for uncertainties arising from errors in measurement.

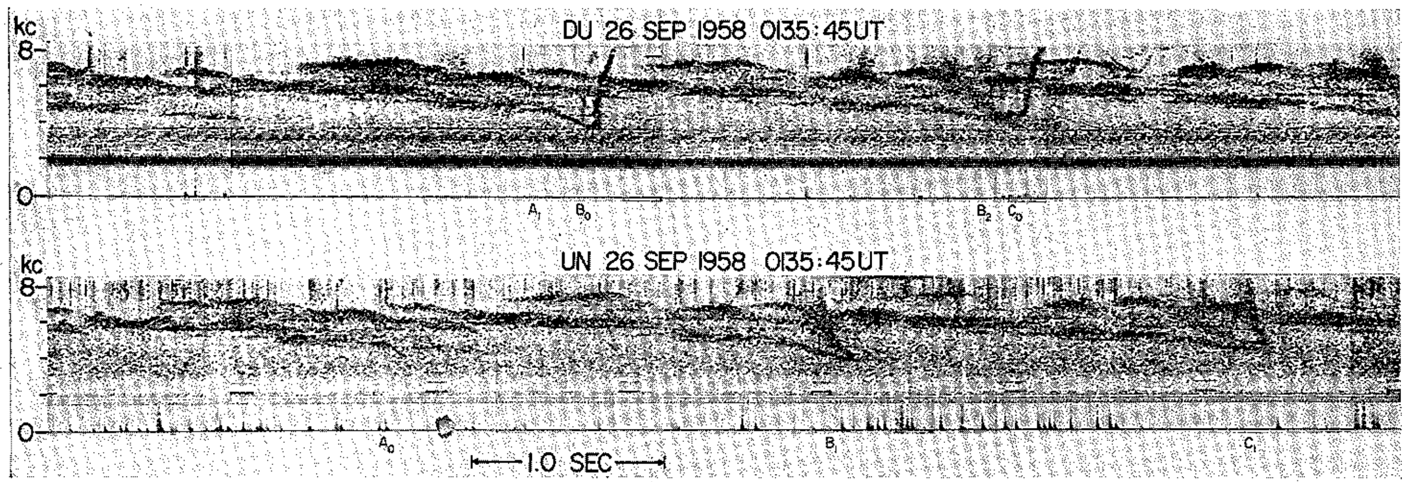

Fig. 2. Conjugate periodic emissions showing hooks echoing in the whistler mode (B_0, B_1, B_2). The first hook is triggered by the echo (A_1) of a previous emission (A_0) and the two hop echo of the hook (B_2) triggers a new emission (C_0) which also echoes (C_1).

Table 1. Nose frequencies and time delays for a given periodic emission as a function of assumed electron density distribution (n) and assumed path latitude (α_0)

Assumed α_0		n = 0	4	6
51°	f_n	13·25	12·36	11·66
	t_n	0·807	0·816	0·825
56°	f_n	13·15	11·82	11·00
	t_n	0·809	0·823	0·834
61°	f_n	12·95	11·56	10·50
	t_n	0·811	0·826	0·838

Carpenter and Smith (1964) showed that, because of this association, the error in the ratio of nose time delay to that expected, on the average, for whistlers with the same nose frequency could be quite small, even though the error in nose frequency was relatively large. Thus the extension method is very useful for determining whether the electron density along the path of propagation for the emission (or whistler) is normal or depressed i.e. for determining whether the path is inside or outside the whistler "knee" found by Carpenter (1963). The method is less accurate, but still useful, for determining the nose frequency for this path of propagation. The discussion of errors given here is intended to be illustrative rather than definitive. If the frequency ratio is small, then small errors in measurement of frequencies or time delays will have a relatively large effect on the estimated f_n and t_n. Relatively large errors may also result if the frequencies are small compared with the nose frequency, since the curve of Fig. 1 approaches a straight line for $f \ll f_n$. These remarks are equally applicable to any extrapolation method for determining f_n and t_n.

The method presented here, while it is particularly suited for use with v.l.f. emissions, may, of course, also be used for whistlers. It provides a simple, quick and convenient method of finding nose frequencies and nose time delays. An additional advantage is its versatility in that only one curve need be computed, instead of one curve for each ratio of frequencies. The uncertainties in the results for this method are essentially the same as for the Smith–Carpenter method, since the errors arising from location of the overlay are negligibly small.

In summary, an improved method of finding nose frequencies and nose time delays has been found. It has been shown that, although path latitudes cannot be determined from individual emissions, they can be determined for some periodic emissions if the emissions show echoing for an unusually large range of frequencies. An examination of possible errors shows that they may be quite small, and that the added convenience of the new method has not been bought at the expense of accuracy.

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