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An Explanation of Subprotonospheric Whistlers

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Abstract. In recent investigations, Carpenter et al. have found a new type of low dispersion whistler which is multiply reflected between the lower ionosphere and a region around 1000 km. The explanation suggested here requires the presence of hydrogen ions and a substantial tilt of the *F* layer. The hydrogen ions above 1000 km allow the whistler energy to be refracted through the region of transverse propagation; in the absence of the ions, such refraction would be forbidden. The hypothesis gives a lower limit for the plasma frequency near the region of refraction.

Recent work by *Carpenter et al.* [1964] has shown the existence of a new type of whistler. The first example of the new whistler was reported by *Barrington and Belrose* [1963], who made their observations with the Alouette satellite. The whistler, as seen by Alouette, consists first of a very short whistler with dispersion of about 3 sec$^{1/2}$ corresponding to that expected for propagation through the ionosphere to 1000 km, the height of the satellite. However, the whistler was unexpectedly followed by echoes which apparently result from multiple reflections between an altitude close to the satellite and the lower ionosphere. In another experiment with an Aerobee rocket, carried out by Stanford Research Institute, short whistlers and echoes with dispersions of roughly 5 and 10 sec$^{1/2}$ were seen much lower in the ionosphere, between 100 and 200 km. Recently many whistlers have also been seen on the ground with dispersions of roughly 5 sec$^{1/2}$. The observations from the rocket and on the ground offer additional experimental evidence that whistlers may, on occasion, be reflected from altitudes of roughly 1000 km. Since this altitude is near the base of the protonosphere, the region where hydrogen ions begin to predominate, the new whistlers are called 'subprotonospheric' or 'SP' whistlers. As discussed in the companion paper [*Carpenter et al.,* 1964], SP whistlers are observed mostly during the night, the maximum of activity occurring before midnight. In this paper we shall attempt to explain the apparent reflection at 1000 km. The reflection of the SP whistler in the lower ionosphere is presumably the same as the reflection of normal whistlers.

A reflection near 1000 km due to a discontinuity in refractive index seems unlikely on physical grounds. Such a reflection would require a change in electron density of an order of magnitude in a distance much less than a wavelength (approximately 10 km), whereas scale heights are typically greater than 100 km in this region. The only alternative is that the reflection must be considered a refraction phenomenon.

Refraction at first appears difficult, because the wave must pass through a region of transverse propagation. In the usual formulation of whistler propagation in which only electrons are considered, the refractive index becomes infinite in the transverse direction. However, as *Hines* [1957] pointed out, whistler frequencies may propagate at all angles when ions are present. A ray tracing program prepared by *Hines et al.* [1959] has shown the possibility of refraction through the region of transverse propagation in the presence of ions.

Finally, it is necessary to take horizontal gradients of refractive index in the ionosphere into account. If the ionosphere were strictly horizontally stratified, application of Snell's law would show that a wave incident on the ionosphere from below could not be refracted to a direction parallel to the boundary in a region where the refractive index is greater than unity. However, the presence of horizontal gradients of refractive index allows the wave to refract through a direction parallel to the boundary and further, until the energy flows downward.

Let us estimate roughly the gradients required by the above hypothesis. For simplicity assume that the earth's magnetic field is vertical and that near 1000 km the gradient of refractive index is vertical. Assume a wave frequency of 3 kc/s and an electron gyrofrequency of 1 Mc/s. At 1000 km assume a nighttime electron and proton density of 2000/cc [*Thomas and Sader,* 1963]. At the *F* layer assume an electron density of 3 × 10^5/cc, and ignore the effects of ions. Using the technique of refractive index surfaces [*Poeverlein,* 1948; *Brandstatter,* 1963; *Smith et al.,* 1960], we can deduce that a wave normal angle of 15° from the vertical in the *F* region would allow the wave to be reflected in a region where the electron and proton density is 2000/cc.

To obtain a wave normal making an angle of 15° with the vertical at the top of the *F* region, consider a simplified model of the *F* region in which the electron density is constant over an altitude range of say 400 km in the vertical direction but varies with horizontal position. The phase velocity at whistler frequencies varies directly as the plasma frequency. Hence if the plasma frequency changes by 0.05% in 1-km horizontal distance, the wave normal would be 15° from the vertical after traveling for 400 km. The integrated plasma frequency change of 0.05%/km corresponds to an integrated electron density variation of 0.1%/km, which is a typical value [*Lawrence et al.,* 1964] as measured by satellites. Wave normal angles as measured in a rocket *below* the maximum of the *F* layer have shown deviations of ±10° from the vertical [*Storey et al.,* 1964]. Since most of the *F*-layer ionization is above the maximum of ionization, the effect on wave normal deviations from horizontal gradients above the *F*-layer maximum must be considerably greater than that below the maximum. Thus there is adequate support for the rough calculations above to indicate that wave normals may deviate substantially from the vertical in the *F* region.

For reflection to occur, the refractive index must decrease with height in the reflection region. The refractive index is proportional to the plasma frequency and inversely proportional to the square root of the gyrofrequency. If we assume that the ion constituent at the reflection region is hydrogen and that hydrostatic equilibrium applies, the refractive index should increase with height unless the temperature is below 1100°K. However, a small amount of helium, say 20%, would allow higher temperatures without substantially changing the refractive index at a given electron density (Angerami, private communication).

For a given horizontal gradient in the *F* layer the refractive index in the direction perpendicular to the magnetic field in the region where hydrogen ions begin to dominate will determine whether a reflection can occur. From the equations given by *Hines* [1957], and assuming only hydrogen ions and electrons such that $f_0^2 \gg f_H F_H$, the transverse refractive index can be shown to be approximately:

$$\mu_t = \frac{F_0}{F_H(1 - f^2/f_r^2)^{1/2}} \tag{1}$$

where

$$\frac{1}{f_r^2} = \frac{1}{F_0^2} + \frac{1}{f_H F_H} \tag{2}$$

F_0 is the *ion* plasma frequency, f_H is the electron gyrofrequency, F_H is the ion gyrofrequency, and f is the wave frequency. From (1) and (2) we can deduce that for a given gyrofrequency and wave frequency there is a value of plasma frequency which yields a minimum value of transverse refractive index. This minimum transverse refractive index increases with wave frequency. We can conclude that reflection is more easily obtained at the lower frequencies, since we would then require less horizontal gradient. The above equation predicts that if all frequencies followed the same path the reflected whistlers would have in addition to the normal Eckersley law behavior ($T \propto f^{-1/2}$) a slight added time delay and an increased dispersion at the higher frequencies. This additional dispersion has not yet been confirmed by measurement. The effect is probably quite small for two reasons. First, the above dispersion law holds only for wave normal directions within a few degrees of the transverse direction. Beyond these few degrees the dispersion is essentially as predicted by the Eckersley law. Second, only a very small amount of time is spent in this transverse region.

Since the altitude at which hydrogen ions predominate increases with increasing temperature, the subprotonospheric whistler can probably be observed more frequently during the night at sunspot minimum.

One test of the hypothesis presented here would be to calculate the transverse refractive index on the basis of actual ion and electron densities in the reflection region. A rocket which could record SP whistlers and measure the densities of the electron and ion components as a function of altitude to a height of at least 1000 km would be very useful for this purpose. The wave-normal direction of the SP whistler could be measured by the technique of *Cartwright* [1964] or *Brice and Smith* [1963] to determine the height of reflection. Subprotonospheric whistlers may prove to be useful for sounding the ionosphere above the *F* layer. The dispersion gives the integrated electron content up to the reflection height. The mere existence of the whistler indicates the presence of hydrogen ions, and the upper frequency limit of the whistler gives a lower bound on the electron density in the reflection region.

Note added in proof. An explanation of subprotonospheric whistlers similar to that described above has been advanced independently by D. G. Cartwright (private communication).

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