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A New Very Low Frequency Phenomenon: Whistlers Trapped below the Protonosphere

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Abstract. A new whistler phenomenon has been identified through measurements at ground stations, on an Aerobee rocket between 100 and 200 km, and on the Alouette satellite at 1000 km. The new phenomenon is called the 'subprotonospheric' or 'SP' whistler, since most of its path appears to be restricted to the region below about 1000 km. The first example of an SP whistler was reported by Barrington and Belrose. In the present report a large number of observations are summarized, and the basic characteristics of the new phenomenon are described. Experimental results are presented which suggest that the whistler ray path is confined to the region between roughly 100- and 1000-km altitude, and that the whistler energy can echo back and forth between these levels. The SP phenomenon occurs mostly at night, typically within a few hours after sunset. SP events are often observed over a period of one or two hours in duration and, for a single Alouette pass, have been observed over a north-south range as great as 2000 km in extent. The evidence suggests that the SP phenomenon occurs mostly near sunspot minimum and at dipole latitudes greater than 45 degrees.

INTRODUCTION

Most reported whistler observations support a propagation model in which the whistler energy penetrates the ionosphere and propagates along approximately field-aligned paths to the opposite hemisphere. Travel times over the paths are typically of the order of 1 sec. Recently however, whistlers with travel times of the order of 0.1 sec have been found. Various properties of these events suggest that the ray paths are confined to some ionospheric region near the point of observation. An example of this type of whistler was first reported by *Barrington and Belrose* [1963], who showed Alouette satellite records of a whistler with two components. The first component was identified as a conventional fractional hop whistler, with travel time corresponding to propagation once through the ionosphere to 1000 km. The travel time of the second component was approximately three times that of the first, and it was suggested that the second component had propagated three times through the ionosphere, having been partially reflected near 1000 km, returned to the earth, re-reflected, and finally returned to the altitude of the satellite.

We have recently examined many examples of this new phenomenon and find that it occurs frequently and has certain well-defined characteristics. It is the purpose of this paper to describe these characteristics and to discuss certain aspects of the whistler propagation path. Because most of the postulated path lies below that part of the magnetosphere in which hydrogen is the principal ionic constituent, we shall for convenience call the new phenomena 'subprotonospheric' or 'SP' whistlers.

Recordings of SP whistlers have been made at ground stations, in the Alouette satellite near 1000 km, and on board an Aerobee rocket in the height range roughly 100 to 200 km. The observations bear out the suggestion of *Barrington and Belrose* [1963] that the whistler propagates through the ionosphere to a height of roughly 1000 km and is then reflected back to the lower ionosphere or ground. Frequently there are repeated echoes of the whistler, suggesting reflection back and forth between the bottom of the ionosphere and the upper region. Figure 1 shows a diagram of this interpretation, and indicates the altitudes at which SP whistlers have been observed. Numbers indicate the successive order of observations. In a case involving repeated reflections, the even-ordered appearances of the whistler energy are recorded near the lower boundary of the ionosphere or on the ground, and the odd-ordered hops are observed well above the peak of the *F* layer.

It should not be thought that the actual ray

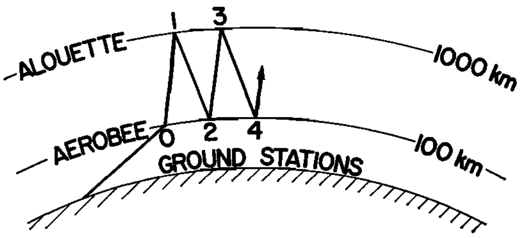

Fig. 1. Diagram of the path of a subprotonospheric whistler, showing the regions in which events described in the text were received.

path of an SP whistler is as simple as Figure 1 suggests. While the experimental data support a model in which propagation over most of the path is in the conventional right-handed circularly polarized whistler mode and travel time of a given component is approximately proportional to the number of traversals of the ionosphere, the data also suggest that certain details of the ray path may be quite complex.

Some of the experimental results on SP whistlers will be described in the following paragraphs. The extent to which the data support a simple propagation model will be indicated. Since we are dealing with a new class of phenomena, a relatively wide variety of detail will be presented. At the end of the paper, the experimental observations will be summarized, and some comments on the propagation model will be made. In a companion paper, R. L. Smith presents a possible theoretical explanation of the mechanism by which the whistler energy is reflected from the upper ionosphere.

EXPERIMENTAL RESULTS

Observations between 100 and 200 km. A large number of SP whistlers were received on an Aerobee rocket fired from Wallops Island (50°N dipole latitude) on July 9, 1963, at 2146 EST. One purpose of the firing was to test the performance of the broad-band (0.2–12.5 kc/s) receiver developed by Stanford Research Institute for use by Stanford University in the Eogo satellite. The antenna consisted of a loop mounted vertically inside the nose-cone section. The rocket reached a peak altitude of 204 km and made broad-band VLF observations for approximately 7 minutes after launch. The first whistler event was recorded at about 108 km, at the approximate height of a pronounced sporadic-*E* layer. From this point to peak altitude and back down to 102 km where the final event was recorded, the observed whistler rate was relatively uniform at about 15 per minute, with a single burst of relatively high activity in the vicinity of 200 km. Ground recordings at Greenbank, West Virginia, roughly 500 km from Wallops Island, revealed no detectable whistler events during the entire rocket flight.

Nearly all 75 whistlers observed on the rocket exhibited the type of component structure illustrated in Figure 2. The upper left spectrogram, with frequency range 0–8 kc/s, shows a whistler received at an altitude of 201 km. The two right-hand records, with frequency range 0–16 kc/s, show the early part of the same event as observed on the rocket and on the ground at Greenbank. On each record the origin of the time scale indicates the leading edge of the essentially impulsive first signal, or 0th hop of the whistler. Following this impulsive signal are two dispersed traces with travel times at 3 kc/s of approximately 0.09 sec and 0.18 sec, respectively. These are the 2nd and 4th hops indicated in Figure 1. Though these components are well defined in the lower ionosphere, they are not detected on the ground recording. The ground recording can conveniently be compared to the Aerobee record above it by noting the presence on both spectrograms of NAA signals at 14.7 kc/s.

The lower left-hand record in Figure 2, with frequency range 0–8 kc/s, shows a whistler event recorded at an altitude of 204 km. In this record the SP traces are well defined, and two later components can also be seen, with travel times at 5 kc/s of about 1.0 sec and 1.7 sec. (These later traces should not be confused with the two additional SP events identified by the arrows in the lower margin of the record. The arrows indicate the position of the 0th hop for these SP events.) The later traces, produced by the lightning source that initiated the first SP event, represent parts of the energy that followed magnetospheric paths extending from hemisphere to hemisphere.

All but one or two of the 75 Aerobee whistler events exhibited at least one SP component, but there was great variety in the definition of the later traces. This is illustrated by a comparison of the two left-hand records in Figure 2. In the upper record the SP components are intense,

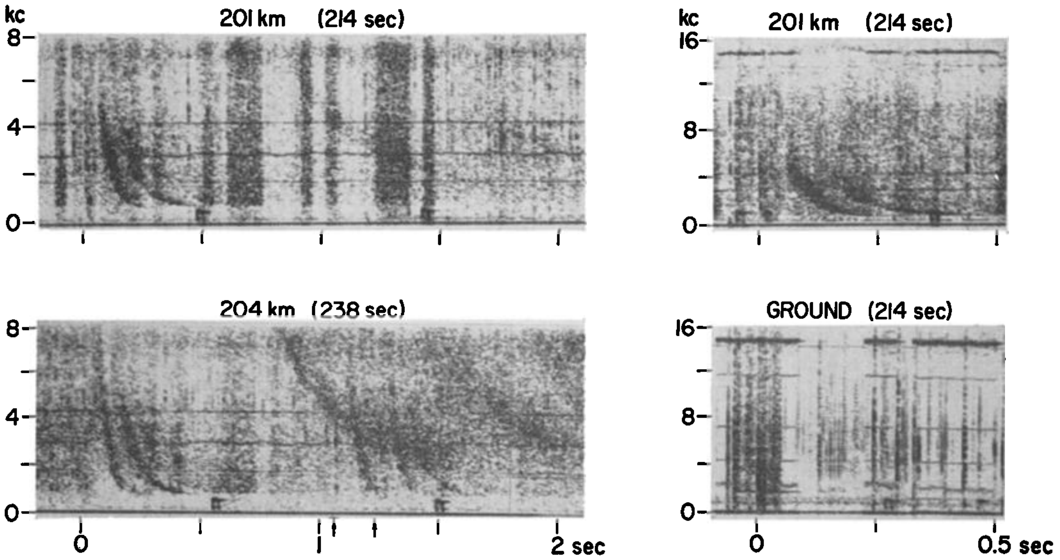

Fig. 2. Spectrographic records of whistlers received on an Aerobee rocket on July 9, 1963, at 2146 EST. The horizontal lines near and below 4 kc/s are of instrumental origin. The vertical bars of enhanced background noise in the upper left-hand record tend to follow the code pattern of station NAA at 14.7 kc/s. The dark region is produced when NAA is keyed off and there is no strong whistler signal present.

but the later traces are only faintly indicated near 4 kc/s.

The frequency-versus-time properties of eleven of the best defined Aerobee whistlers have been studied in detail. In addition, certain basic properties of all the events have been investigated through examination of real-time Rayspan records. In the following summary of the results, travel-time measurements are expressed in terms of dispersion $D = tf^{1/2}$ sec$^{1/2}$, where t is the travel time at frequency f. For convenience we shall indicate measurements on a specific SP component by using the hop number as a subscript; that is, D_2 and D_4 will represent measurements on the 2nd- and 4th-hop components. Most travel time measurements were made with respect to a fixed time at the leading edge of the 0th hop.

1. Within the experimental error of about ±5%, the SP components follow the dispersion law $D(f) = tf^{1/2} =$ constant. This is the expected frequency-versus-time behavior for quasi-longitudinal propagation in the right-handed circularly polarized whistler mode when wave frequency is well below the electron plasma frequency and gyrofrequency. Most of the dispersion measurements were made in the frequency range between about 1.5 and 4 kc/s. Below 1.5 kc/s there is some indication of an increase in dispersion with decreasing frequency, but this point is not yet well established.

2. For 11 cases, the value of D_2 ranged from 4.4 sec$^{1/2}$ to 5.4 sec$^{1/2}$. (The value of D_2 for an event was determined by averaging the results of measurements at several frequencies.) The scatter of ±10% around $D_2 = 4.9$ sec$^{1/2}$ is probably attributable to experimental uncertainty in identfying the leading edge of a trace or impulse.

3. There is no apparent systematic variation in D_2 throughout the flight. The electron-density profile obtained at the time of the shot shows that electron density increased quite rapidly only above 175 km. The systematic dispersion variations that might be present under these conditions are approximately of the order of some of the experimental uncertainties involved.

4. The travel time at a given frequency between the 2nd and 4th hops is approximately the same as that between the 0th and 2nd hops. The ratio of D_4 to D_2, averaged for each event, remained within ±10% of 2.0 throughout the flight. This uncertainty is approximately the same as that associated with experimental error.

5. The upper cutoff frequency of the 2nd-hop or 1st dispersed SP component is about 6.5 kc/s. Most values are in the range 6–8 kc/s. The upper cutoff frequency appeared to be somewhat higher near peak altitude. The cutoff of the 4th-hop SP component is about 5.5 kc/s, and the range throughout the flight is roughly 5–6 kc/s.

6. The lower cutoff frequencies of both the 2nd-hop and 4th-hop SP components lie between about 700 and 750 cps throughout the flight.

A number of other aspects of the results can be mentioned. One interesting point is the lack of VLF activity on the ground. A corresponding lack of VLF activity on the ground was noted by *Cartwright* [1964] during a rocket flight from Wallops Island on April 12, 1963, at 0030 LT. It appears likely that a factor contributing to the absence of ground activity was the observed sporadic-*E* layer. In addition to causing the subprotonospheric components to be re-reflected upward, this layer may have provided a means of reflecting conventional downcoming whistlers. The lack of activity on the ground at Greenbank, some 500 km from Wallops Island, is supported by the rocket experiment itself, which produced records during the subionospheric segments of the flight. During these periods no whistler events were observed.

Although primary attention here is devoted to the SP components, a few comments should be made about the higher-dispersion traces illustrated in the lower left-hand part of Figure 2. The propagation path of these events has not been studied in detail, but an examination of their dispersion properties and a comparison with the known properties of ground-observed whistlers suggests that the first component, with dispersion at 5 kc/s of about 68 $\sec^{1/2}$, made a single south-to-north traversal of a magnetospheric path. The path was probably excited by an impulse which reached the southern hemisphere by transequatorial subionospheric propagation from the source. This 'hybrid' mode of excitation has been previously observed on ground records [*Helliwell,* 1959; *Helliwell and Carpenter,* 1963], and it appears to be relatively common in observations made in the ionosphere. The second component, with dispersion of about 118 $\sec^{1/2}$, appears to be a two-hop whistler, with path entrance in the northern hemisphere.

Ground observations. During 1962 and 1963, a relatively large number of subprotonospheric whistlers were received at the ground stations Suffield Experimental Station (58°N dipole latitude) and Great Whale River (65°N dipole latitude). Figure 3 is a graph, based on aural monitoring of hourly two-minute recordings, of the number of recording periods containing SP whistlers versus local time at the observing station. On the graph are represented 55 two-minute runs from 26 recording days during 28 station months of operation. The number of cases exhibits an abrupt rise at about 2100 LT, and after remaining at a high value for several hours, decreases gradually and then falls rapidly near dawn. On most of the days represented in Figure 3, examples were found during only one or perhaps two successive runs. On a few occasions SP whistlers were detected for as many as four or five hours in succession.

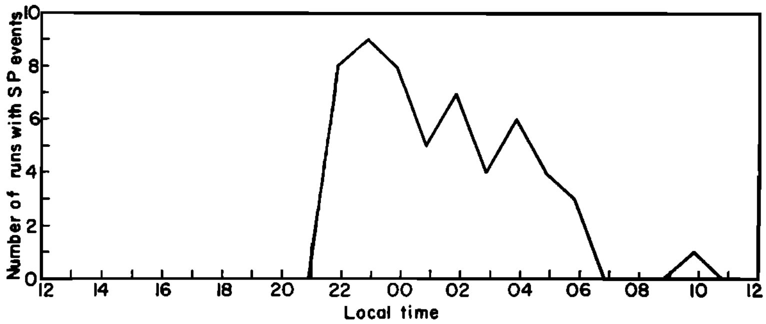

Fig. 3. Graph of the number of two-minute recording periods during which SP whistlers were observed as a function of local time at the station. The data were taken at Suffield Experimental Station (50°N, 111°W geographic; 58°N dipole latitude) and Great Whale River (55°N, 80°W geographic; 65°N dipole latitude).

The ground observations appear to be limited by the nature of source activity. More than half the events were seen in local summer, when lightning activity in neighboring areas is relatively high. In many of the runs containing SP whistlers, only one or perhaps two faint examples were observed following one or more of the loudest impulsive atmospherics in the run. On some occasions, however, many SP events were detected, and a few included 4th hops, as in the case of the Aerobee recordings. When SP events were observed on the ground, other forms of whistler activity were seldom detected. On a few rare occasions, the SP whistler was followed by a faint two-hop long whistler.

A preliminary study shows that the ground records are similar to the Aerobee records in several respects. The even-ordered hops are observed, and travel times are frequently such that $D_2 \approx 5$ sec$^{1/2}$ and D_4 (when observed) ≈ 10 sec$^{1/2}$. Most of the ground-observed dispersion values fall in the range $D = 3\text{–}6$ sec$^{1/2}$, a distribution to be expected if the corresponding spatial and temporal range of variation in ionospheric electron density is within a factor of about 4. The frequency range of the ground observations is usually relatively limited as compared to the range observed on the Aerobee. Often the detectable energy in the SP components is limited to the band from about 700 to about 2000 cps.

On the basis of ground observations, it appears that the subprotonospheric whistler is associated with sunspot minimum. An extensive amount of experimental work has been done with whistler records made since early in the IGY, and as yet no examples of SP events from this early period have been reported. Certain IGY records should be carefully sampled to clarify this situation.

Observations at 1000 km. The broad-band VLF experiment on the Alouette satellite provides a valuable source of information on the spatial distribution of SP whistlers, and on the general properties of SP events as observed at 1000 km. The first report of an SP whistler recorded on the Alouette was made by *Barrington and Belrose* [1963]. These authors showed a spectrogram of two events, each of which exhibited two low-dispersion traces, apparently corresponding to the 1st and 3rd hops of what we call a subprotonospheric whistler. The value of D_3 was reported to be three times the value of D_1. The time of the recording was about 2020 LT and the dipole latitude was about 51°N.

A relatively large number of observations of SP whistlers have been made through telemetry of the Alouette receptions to a ground station at Stanford University. What appear to be SP whistlers have been observed on several passes during the hours after local sunset and on at least one pass near dawn. A particularly interesting set of records was obtained on April 19, 1963, at approximately 1920 local time at the satellite. The track was approximately north to south near the 105°W geographic meridian. During a 5-minute observing period, 7 well-defined examples of SP whistlers were recorded, the first at 53°N geographic (62°N dipole) latitude, and the last at approximately 37°N geographic (46°N dipole) latitude. Thus the north-south range of observations was of the order of 2000 km on this occasion. It is noteworthy that, though a substantial number of SP whistlers were observed over the range from about 53° to about 37°N geographic, no SP events were observed over the range from 37°N to the end of the pass at about 10°N. On this low-latitude segment there was a significant amount of source activity available, as evidenced by the presence on the Alouette records of a number of conventional low-dispersion whistlers with $D = 3\text{–}4$ sec$^{1/2}$.

Figure 4 shows spectrograms of SP whistlers recorded on the April 19, 1963, pass at 53°, 50°, 38°, and 37°N geographic latitude. (The event at 33°N will be discussed in a later paragraph.) Each of the four examples shows several odd-ordered SP components. The event at 38° exhibits 6 well-defined traces, corresponding to the 1st, 3rd, 5th, 7th, 9th, and 11th hops. This event also exhibits conventional whistler traces, with travel times of 2 sec or greater. There are actually three whistler events at 38°, although only the first exhibits well-defined dispersion properties. Arrows mark the approximate time of origin of the second and third flashes, while the origin of the time scale indicates the approximate time of origin of the principal whistler event. The horizontal lines were introduced by instrumentation on the satellite.

The results of a preliminary survey of the dispersion properties of the SP events illustrated in Figure 4 can be summarized as follows:

1. The dispersion D_1 of the 1st-hop com-

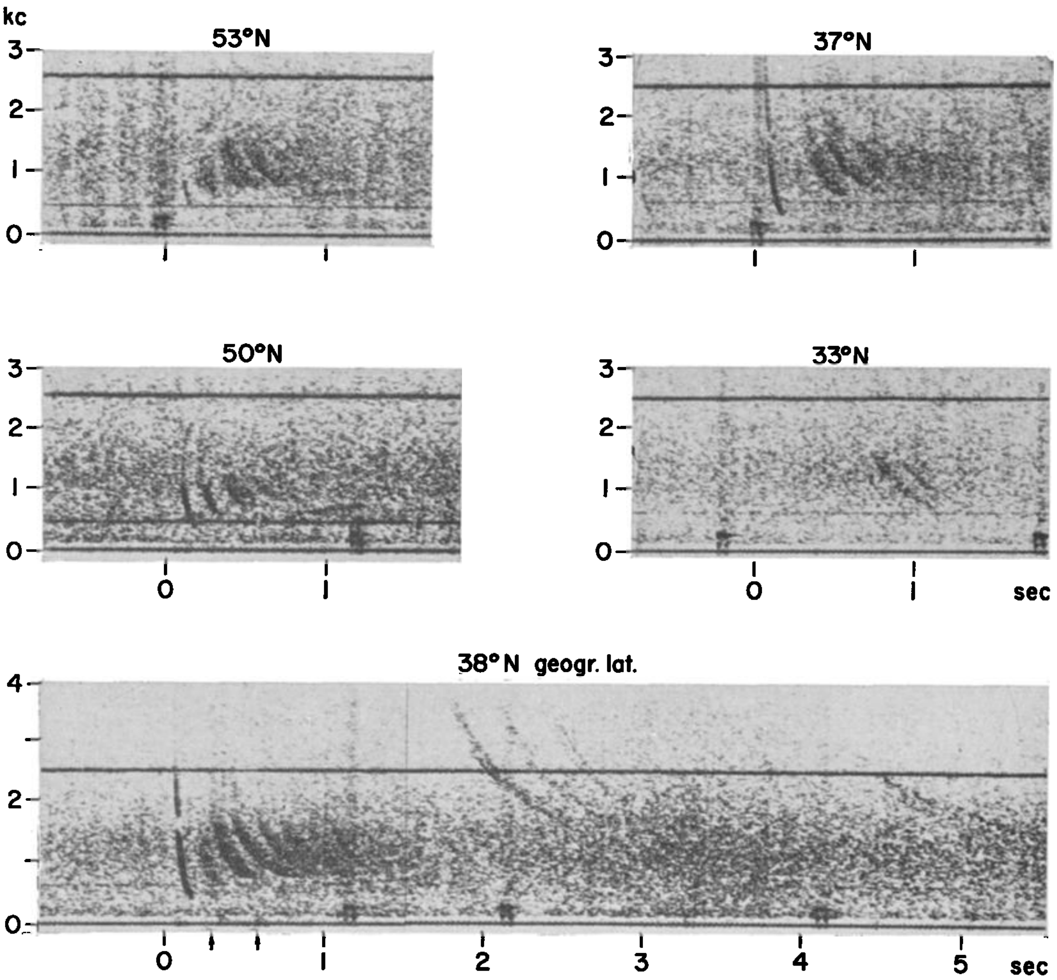

Fig. 4. Spectrograph records of whistlers received on the Alouette satellite on April 19, 1963, at approximately 1920 local time at the satellite. The approximate geographic latitude of the satellite is indicated above each record. The horizontal lines are of instrumental origin. The nose-like effect above 1 kc/s on the record for 50°N is due to intermodulation between the whistler signal and the static inverter frequency near 2.6 kc/s.

ponent is approximately 3 $\sec^{1/2}$. This value is consistent with previous nighttime observations of 1-hop propagation through the ionosphere to 1000 km [*Barrington and Belrose,* 1963]. It is also consistent with theoretical predictions of whistler dispersion for 1-hop propagation through a nighttime ionosphere [*Smith,* 1961; *Pope,* 1961]. The value of D_1 is very nearly the same for all events.

2. To a first approximation, successive odd-order hops are separated by twice the travel time of the 1st hop. There do appear to be small departures from strict integral relations. Preliminary measurements show that, to a degree depending somewhat on the frequency of measurement, the interval between the 1st- and 3rd-hop components tends to be of the order of 15% shorter than the intervals between the later traces. After the 3rd hop, the travel-time separation for most cases corresponds to a difference in dispersion of about $D = 5$–$6 \sec^{1/2}$.

3. To a first approximation, the dispersion $D = tf^{1/2}$ is constant for a given component. However, as in the case of the Aerobee records, there appears to be a trend toward increasing dispersion with decreasing frequency, particu-

larly below 1 kc/s. This trend is being investigated.

4. In several cases the upper cutoff frequency increases as hop number increases from 1 to 3 to 5. In all cases the upper cutoff frequency appears to increase from the 3rd to the 5th hop, and then remains roughly constant after the 5th hop at about 1500 cps. Note that the upper cutoff frequency of the 5th hop is near the upper cutoff frequency of a faint noise band.

5. The lower cutoff frequency of the SP whistler increases with increasing trace order until the 5th hop.

6. In several of the events there appears to be a systematic variation in intensity from component to component. The most striking variation is the low intensity of the 3rd hop and the apparent gradual increase in intensity from the 3rd to the 5th and possibly to the 7th hop, after which the intensity appears to decrease again. In the case of the event at 37°N, the 3rd hop is barely detected. Beginning with the event at about 50°N, the relative intensity of the 3rd hop decreases with decreasing latitude.

A number of the features of the Alouette records for April 19, 1963 (Figure 4), were repeated in a striking way on a recording made a year later on April 25, 1964, at roughly the same time and location. The satellite track was approximately north-south along the 122°W geographic meridian at approximately 1855 LT, shortly after ground sunset. Figure 5 shows spectrographic records of two SP events, one

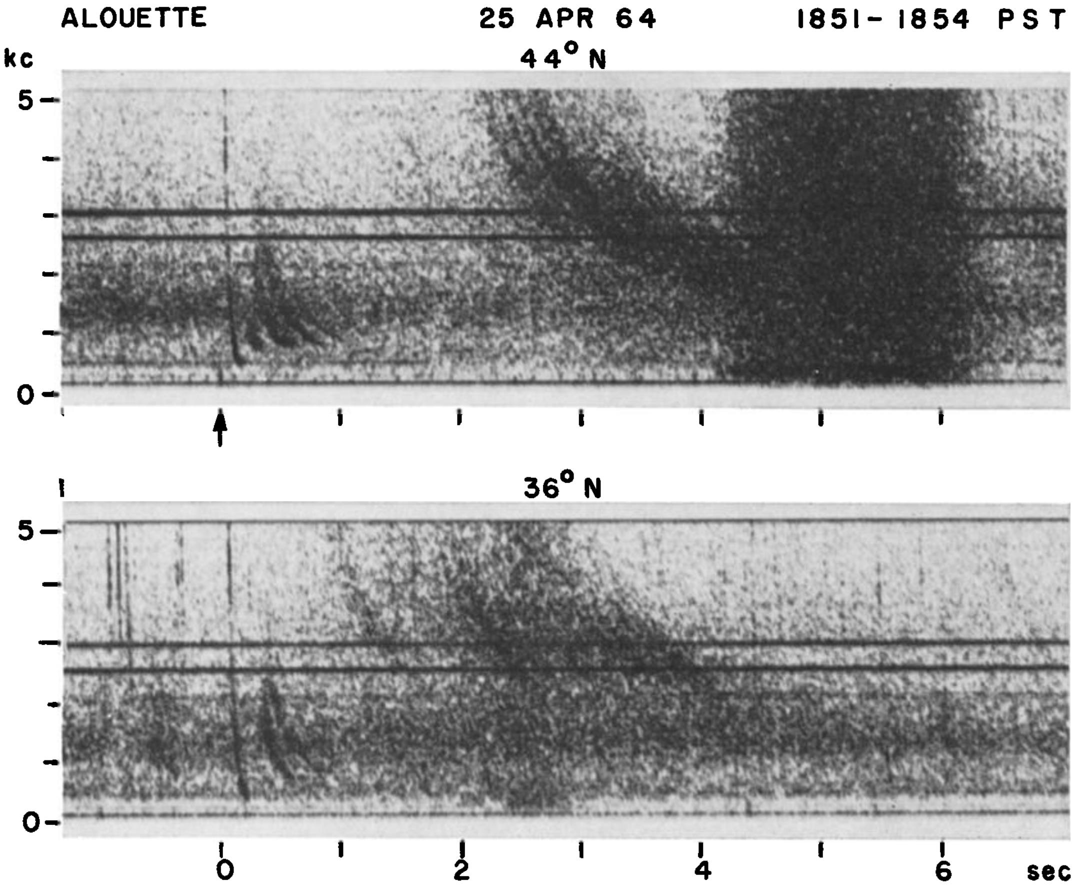

Fig. 5. Spectrographic records of whistlers received on the Alouette satellite on April 25, 1964, at approximately 1850 LT at the satellite. The approximate geographic latitude of the satellite is indicated above the record. The horizontal lines and the wide vertical band at the upper right are of instrumental origin.

recorded at 44°N geographic (50°N dipole) latitude, and the other at about 36°N geographic (43°N dipole) latitude. A comparison of Figures 4 and 5 reveals similarity both in the main features and in several details. The simple model of reflection back and forth between an upper and lower region is supported. Once again the travel time between the 1st and 3rd hops is slightly less than the interval between later hops. The 3rd-hop component is again relatively faint and limited in frequency range, and its intensity decreases with decreasing latitude. In the upper record of Figure 5 the 3rd hop is detected, whereas on the bottom record it is not observed.

It is tempting to speculate on some of the details of the propagation path as they might be deduced from the Alouette records, but it would seem preferable to await both further detailed measurements and theoretical studies of possible ray paths. It does seem reasonable to suggest that for observations by the Alouette the effective area of the 3rd hop can be relatively small. This is suggested by several forms of evidence, including the relatively limited frequency range of the 3rd hop, as well as the tendency, on at least two occasions, for the 3rd hop to disappear as the satellite moves southward.

Several additional remarks should be made about Figure 4. The event at 33°N clearly differs from the usual SP configuration. The time of origin of the event has been determined only approximately, within about 50 msec. Two traces are in evidence, with travel-time separation corresponding to a difference in dispersion of about 5 $\sec^{1/2}$. The lack of evidence of a 1st-hop component near $t = 0$ suggests that the originating flash was at a considerable distance from the subsatellite point. Because the other events in Figure 4 exhibit a decrease in 3rd-hop intensity with decreasing latitude, it may be suggested that the two traces at 33°N are by analogy merely the late hops of an SP whistler, perhaps the 11th and 13th. Another possibility is that the traces are part of a conventional whistler propagating from an origin in the southern hemisphere. In this case the separation of the traces may be due to return of whistler energy to the satellite after reflection near the bottom of the ionosphere.

An additional comment should be made in connection with the records for 53°N and 50°N. On both records a triggered rising tone appears at approximately the same place with respect to the SP traces. The riser begins at roughly 600 cps, not far from the low-frequency tail of the 5th or 7th hop, and it rises about 150 cps in frequency over a period of about 0.5 sec. The confined nature of the whistler path makes it possible to conclude that the rising tone was triggered within the region between the lower ionosphere and roughly 1000 km.

CONCLUSIONS

Under certain conditions whistler energy can follow a ray path that is confined to the region between roughly 100- and 1000-km altitude. The observations suggest that the energy echoes back and forth between these levels. The mechanism of lower reflection can probably be explained as reflection either from a sporadic-*E* layer or reflection at the lower boundary of the ionosphere, in the manner of conventional whistler reflection as discussed by *Helliwell* [1963]. The unique aspects of the new phenomenon seem to lie in the mechanism of upper reflection and in the details of the ray path. In a companion paper, R. L. Smith presents an explanation of the upper reflection involving refraction of the wave through the transverse region of propagation.

The altitude of upper reflection is not yet known precisely. The dispersion properties of many of the observed events, when compared to previous theoretical and experimental measurements, indicate that the upper reflection level is near 1000 km. Pending more detailed studies, it can be estimated to be in the range 700–1300 km.

The probability of occurrence of the SP phenomenon, given suitable lightning source activity, is estimated to be of the order of 0.5 during the hours after sunset, near sunspot minimum, and at middle to high latitudes. SP events have been detected on approximately half the Stanford recordings of Alouette whistlers made during the postsunset hours. The ground observations do not show such a high rate of observation, probably because of the strong blanketing effect of the ionosphere.

The data suggest that most of the propagation is in the right-handed circularly polarized whistler mode, and that each traversal of the path extends from roughly 100 to 1000 km. However, some of the Alouette results suggest that the ray path may show small but systematic departures from a simple model. An example of this is the limited effective area of observation of the 3rd-hop SP component at 1000 km. The striking repeatability of the Alouette records on different passes suggests that a relatively complex descriptive picture of the SP phenomenon can be obtained from further study of spectrographic records.

An explanation of the SP phenomenon must account for the following observational details: (1) most of the examples have been found at night, the majority within a few hours after sunset; (2) examples are often found during two consecutive hourly recording periods and occasionally appear for as many as four or five hours in succession; (3) the north–south range over which SP whistlers have been found is as great as 2000 km (for a single Alouette pass); (4) no examples of SP whistlers observed on the ground have been reported for the period before 1962; (5) examples have thus far been limited to observations at dipole latitudes greater than about 45°.

Note added in proof. We have recently heard that D. G. Cartwright (personal communication) observed what appears to be an SP whistler during a VLF rocket experiment at Wallops Island in 1963.

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References

Barrington, R. E., and J. S. Belrose, Preliminary results from the very-low-frequency receiver aboard Canada's Alouette satellite, *Nature, 198* (4881), 651–656, May 18, 1963.

Cartwright, D. G., Rocket observations of very low frequency radio noise at night, *Planetary Space Sci., 12*(1), 11–16, January 1964.

Helliwell, R. A., Whistler paths and electron densities in the outer ionosphere, Proc. Symp. Physical Processes Sun-Earth Environment, *Defence Res. Board, DRTE Pub. 1025,* 165–175, July 20–21, 1959.

Helliwell, R. A., Coupling between the ionosphere and the earth-ionosphere waveguide at very low frequencies, pp. 452–460, *Proceedings of the International Conference on the Ionosphere, London, July 1962,* Institute of Physics and the Physical Society, London, Bartholomew Press, Dorking, England, 1963.

Helliwell, R. A., and D. L. Carpenter, Whistlers excited by nuclear explosions, *J. Geophys. Res., 68*(15), 4409–4420, 1963.

Pope, J. H., An estimate of electron densities in the exosphere using nose whistlers, *J. Geophys. Res., 66*(1), 67–75, 1961.

Smith, R. L., Properties of the outer ionosphere deduced from nose whistlers, *J. Geophys. Res., 66*(11), 3709–3716, 1961.

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