JOURNAL OF GEOPHYSICAL RESEARCH VOLUME 67, No. 6 JUNE 1962

Graphic Data on the Earth's Main Magnetic Field in Space

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Abstract. The applicability of a dipole representation of the earth's main magnetic field at high altitudes is discussed. Useful characteristics of the centered dipole approximation are presented in simple graphical form. Data that can be obtained from the charts include location of field lines, arc length along a field line, electron gyrofrequency, inclination (or dip), magnetic-field intensity, magnitude and direction of the gradient of the earth's field, and radius of curvature of the lines of force. The resulting accuracy is believed sufficient for many purposes.

Introduction. The study of magnetism is one of the oldest fields of science, and the fact that a compass needle aligns itself in a north-south direction has been known for many centuries. Experiments that indicate that the earth acts as a great magnet date back to about 1600 and the work of William Gilbert. Terrestrial magnetism has been under continual study ever since. The wealth of knowledge that has been accumulated is impressive, but by no means complete, since the basic phenomena are believed to occur either at great depths in the earth or at great heights in the atmosphere [*Fleming*, 1939]. It has been found that, in general, the secular component of the main field arises from magnetization of the solid earth, and that the very much smaller ephemeral field arises from atmospheric effects [*Chapman and Bartels*, 1951].

Terrestrial measurements early indicated that, supposedly as a result of internal eddies and surface anomalies, the surface field was not that of a simple magnet. Spherical-harmonic analysis, based on observations of the field vectors made anywhere over the earth's surface, shows that the first, and by far the largest, term represents the magnetic dipole at the center of the earth whose field would give the best fit to the actual irregular field over the surface.

Continuing world-wide measurements have provided, and continue to provide, current information on the main field at the earth's surface [*Vestine, Laporte, Lange, Cooper, and Hendrix*, 1947]. Such empirical information has been utilized by researchers, in fields other than geomagnetism per se, whenever knowledge of the earth's main field at the surface was found to be significant in interpreting their measurements. Information on the field at points above the earth may be obtained by extrapolation of the surface values. A tabulation of extrapolated field values has been given for several altitudes up to 5000 km for 1945 [*Vestine, Lange, Laporte, and Scott*, 1947]. Extrapolation provides accurate magnetic-field data for such studies as earth-based radar investigations of auroras [*Chapman*, 1952] and field-aligned ionization irregularities [*Egan*, 1959], and Faraday-polarization-rotation studies of integrated electron content between a low-altitude-orbit satellite and an earth observing station [*Garriott*, 1960]. Several geometric properties of a dipole field as applied to specific problems have been given [*Cohen and Dwarkin*, 1961; *Millman*, 1959].

In space experiments performed at altitudes of the order of thousands of kilometers, a first-order knowledge of the earth's field is often required. The higher-order, spherical-harmonic-analysis terms decrease more rapidly with increasing altitude than does the dipole field itself [*U. S. Air Force*, 1960]. Thus, a dipole approximation of the earth's main field becomes acceptably accurate for such high altitudes. To facilitate first-order computations and to provide a useful geometrical model, graphic presentations of certain characteristics of the earth's main dipole field have been prepared at Stanford. It is hoped that other workers, to whom knowledge of the gross characteristics of the earth's main field is important, will also find these data useful.

Centered dipole approximation. The eccentric dipole provides a better approximation to the main field than does the centered dipole.

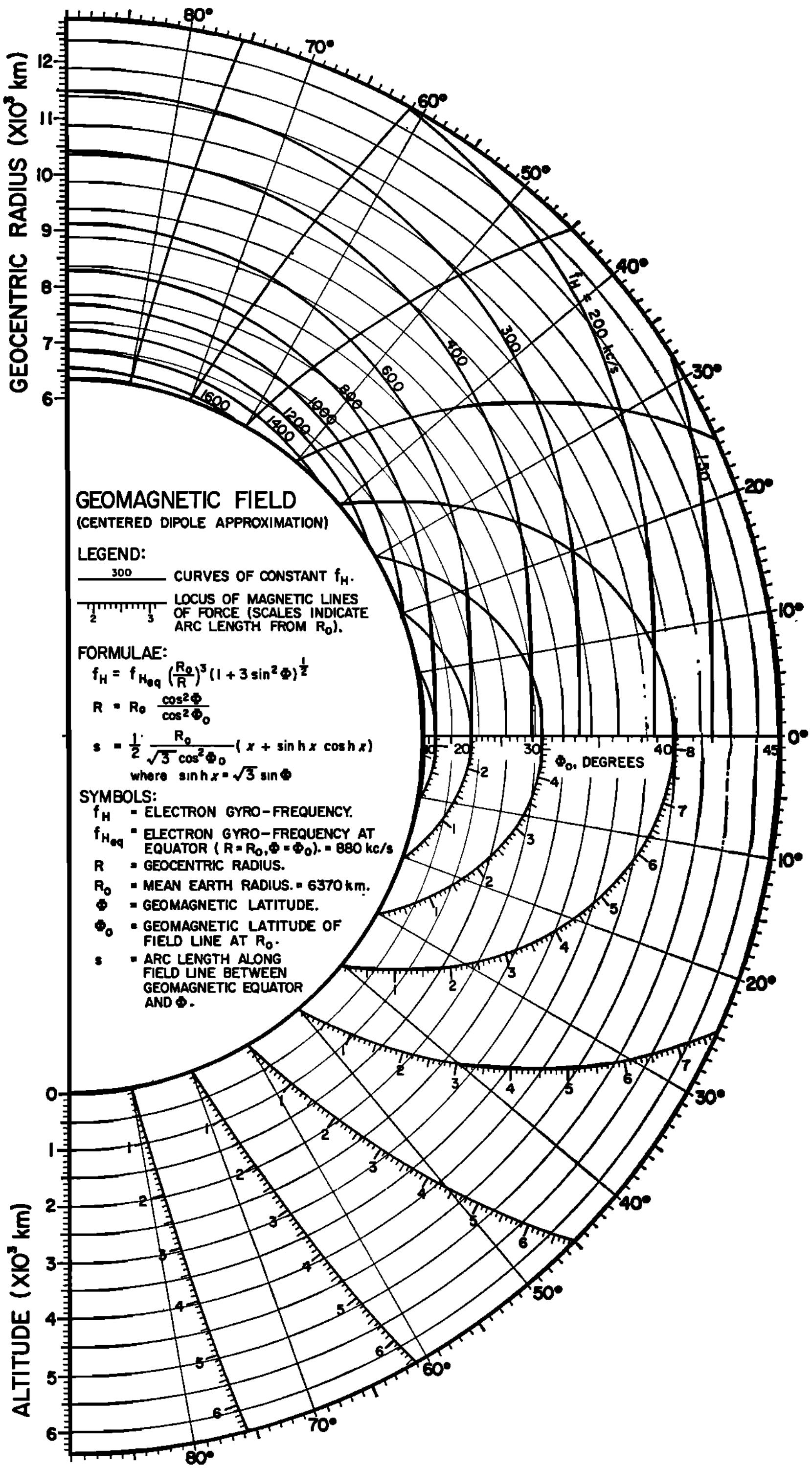

Fig. 1. Lines of force, arc lengths, and electron gyrofrequency of the earth's main magnetic field plotted in geomagnetic coordinates out to 1 earth radius of altitude.

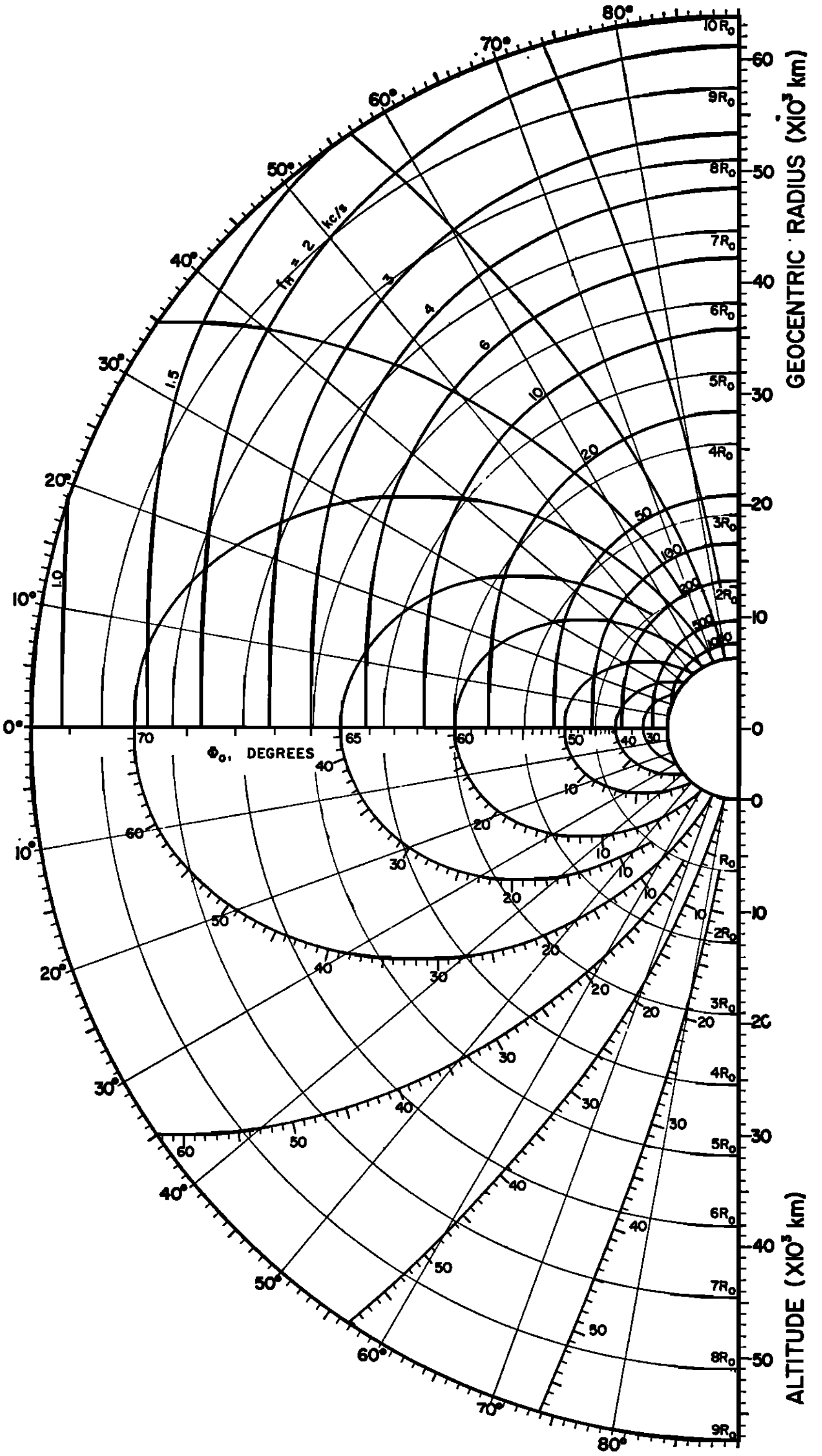

Fig. 2. Lines of force, arc lengths, and electron gyrofrequency of the earth's main magnetic field plotted in geomagnetic coordinates out to 10 geocentric earth radii.

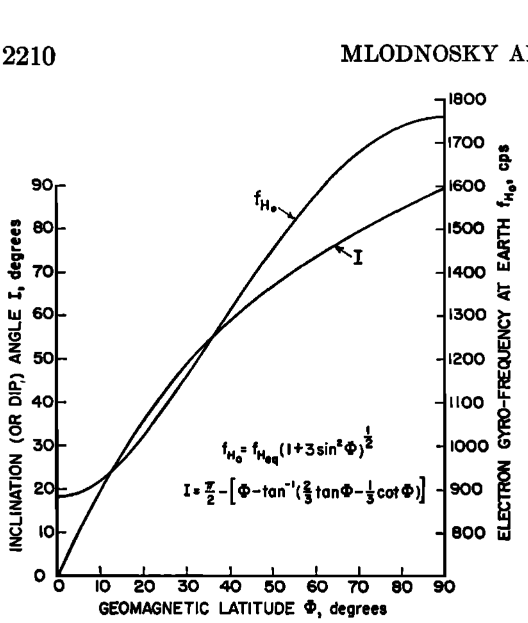

Fig. 3. Inclination and electron gyrofrequency at earth's surface as a function of geomagnetic latitude for the earth's dipole field.

However, graphic presentation of the asymmetric eccentric-dipole approximation would be complicated by the necessity for including geomagnetic longitude as a variable. The eccentric dipole is displaced from the earth's center by about 340 km for 1922 and adopted internationally at present [*Chapman and Bartels,* 1951]. The accuracy of the eccentric dipole as compared to the centered dipole, though important at ionospheric heights, is less important at higher altitudes. Therefore, the centered-dipole approximation (poles at 78.5°N, 291°E, and 78.5°S, 111°E) will be adopted here. Useful graphic charts giving the relationship between geographic coordinates and geomagnetic coordinates for the centered-dipole approximation are given by *McNish* [1938] and in the USAF Handbook of Geophysics [*U. S. Air Force,* 1960]; tabular conversion values at 1° intervals have appeared [*Vestine, Laporte, Lange, Cooper, and Hendrix,* 1947, p. 28].

A value of 8.1×10^{25} cgs units has been taken for the total magnetic moment of the first-degree harmonics of the earth's main field [*Fleming,* 1939]. This magnetic moment yields a value of $B = 0.314$ gauss at the equator. This corresponds to $H = 25$ amp-turn/m and 31,400 γ, and to an electron gyrofrequency of 880 kc/s. The data to be presented here may be made more accurate by modifying the data obtained from the graphs by the proportion of the magnetic moment given above to the corresponding secular moment for any time period in question.

The dipole representation breaks down where the earth's field merges with the interplanetary magnetic field. However, it has been suggested that the earth's relative motion and solar winds distort the earth's field significantly before the field decreases to interplanetary magnitudes. This distortion is dependent upon solar aspect, but probably occurs at distances greater than about 10 geocentric radii [*Dessler,* 1958; *Coleman, Sonett, Judge, and Smith,* 1960].

Field strength, direction, and arc length along lines of force. A concise development of the equations of a magnetic line of force and the field strength along a given line of force for a magnetic dipole is given by *Alfvén* [1950]. The equation (and some tabulated values) for the arc length along a line of force of a dipole field, between a given latitude and the equator, has been given by *Chapman and Sugiura* [1956]. The above equations, together with the list of symbols, are included in Figure 1.

Plots of the lines of force originating at every 10° of latitude are shown in Figures 1 and 2 in hemispherical sectors of geomagnetic coordinates out to 1 earth radius in altitude and 10 geocentric radii, respectively. Scales of geocentric radial distance and altitude above earth are included in both figures, and the coordinate loci of constant radius are for increments of 500 km

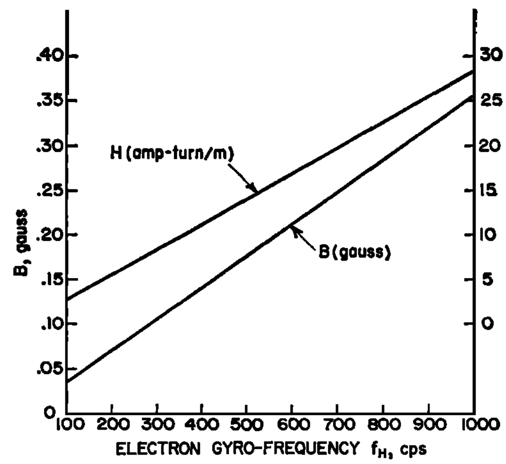

Fig. 4. Relationship between electron gyrofrequency and magnetic-field strength in amp-turn/m, and the magnetic induction in gauss/cm.

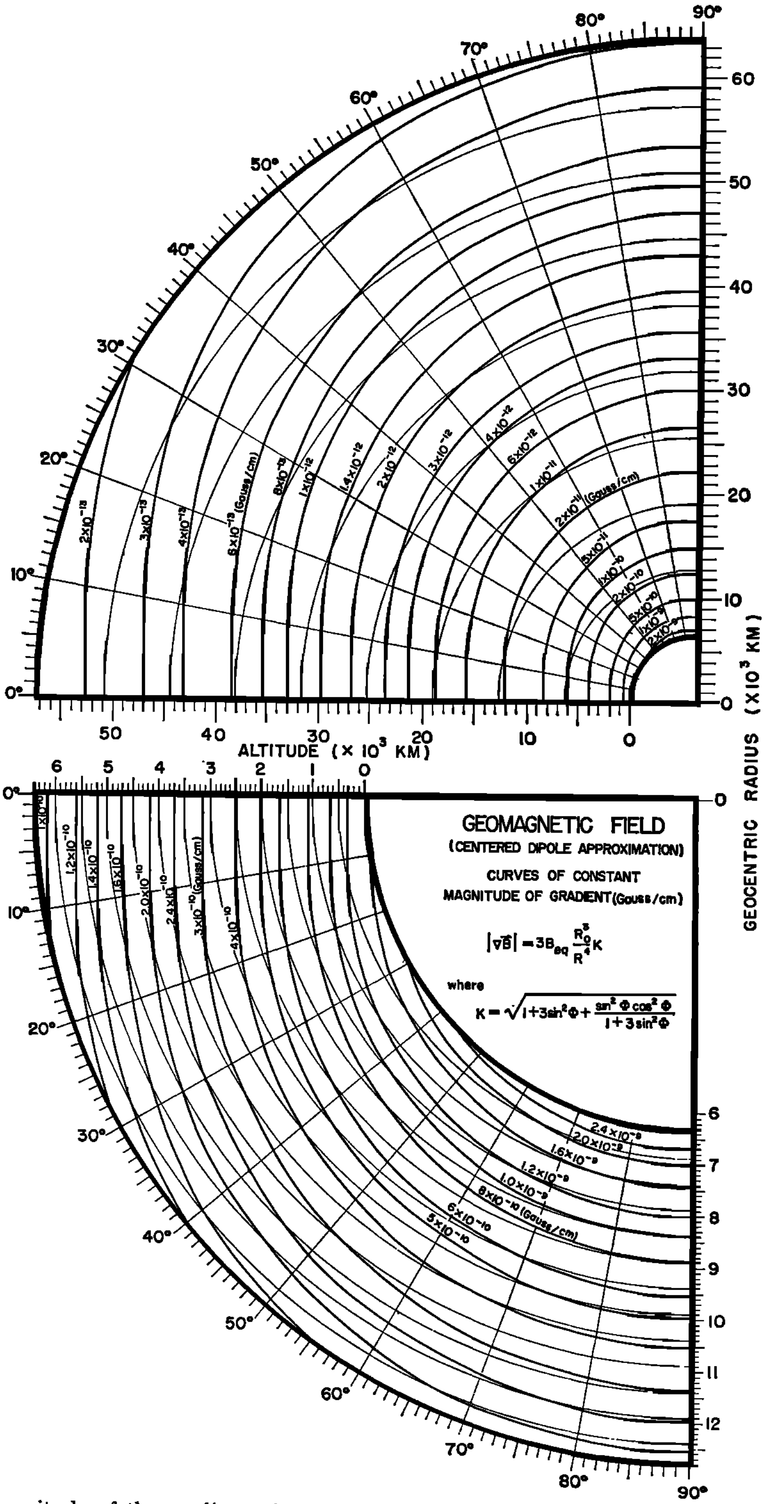

Fig. 5. Magnitude of the gradient of the earth's main magnetic field plotted in geomagnetic coordinates.

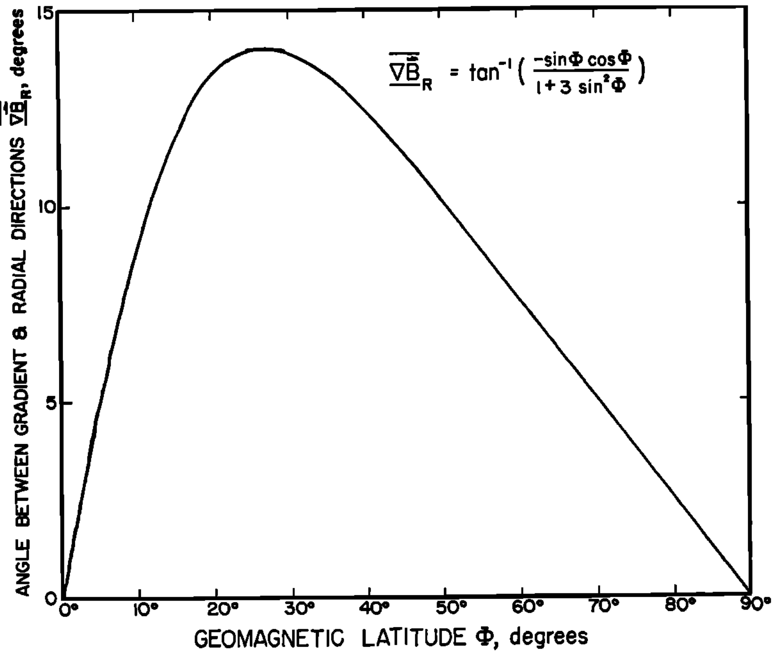

Fig. 6. Direction of the gradient of the centered-dipole approximation of the earth's main magnetic field as a function of geomagnetic latitude.

and 1 earth radius for Figures 1 and 2, respectively. The earth is represented as a sphere with a radius of 6370 km. Along the zero-latitude radial is shown a scale of earth-surface latitude origin of the lines of force. The following table provides a convenient rule of thumb for sketching additional lines of force (R_{eq} is the equatorial geocentric radial distance of the desired line):

Latitude	Geocentric Radius
0	1.00 R_{eq}
30	0.75 R_{eq}
45	0.50 R_{eq}
60	0.25 R_{eq}

Arc length is shown in the lower quadrants of the figures by the scales constructed along the lines of force with origins at the earth's surface. The direction of the dipole field in space may be scaled from the figures with reasonable accuracy as tangent to the relevant line of force. For greater accuracy, Figure 3 gives a plot of the inclination of the dipole field as a function of latitude. It should be noted that the inclination is independent of radial distance. Also shown in Figure 3 is the variation of the surface electron gyrofrequency versus latitude for the assumed magnetic moment.

For application at Stanford, electron gyrofrequency was selected as the most useful measure of field strength, and contours of constant gyrofrequency are shown in Figures 1 and 2. Figure 4 gives a graphic presentation of the relationship between electron gyrofrequency and magnetic-field strength in mks units and the magnetic induction in gauss. Figure 4 can be extended to include the complete range of gyrofrequencies encountered by appropriate application of factors of 10 to the scales shown.

Gradient of the field and radius of curvature of the lines of force. The field gradient and the curvature of the lines of force are important, for example, for their effects on charged-particle motion (and/or drifts) [*Spitzer*, 1956] and for their influence on the guiding of propagating electromagnetic energy [*Smith, Helliwell, and Yabroff*, 1960]. The relevant expressions are easily derivable from those already presented and are included in the figures. The coordinate systems and the symbols used are consistent with those of Figures 1 and 2.

The magnitude of the gradient of the earth's

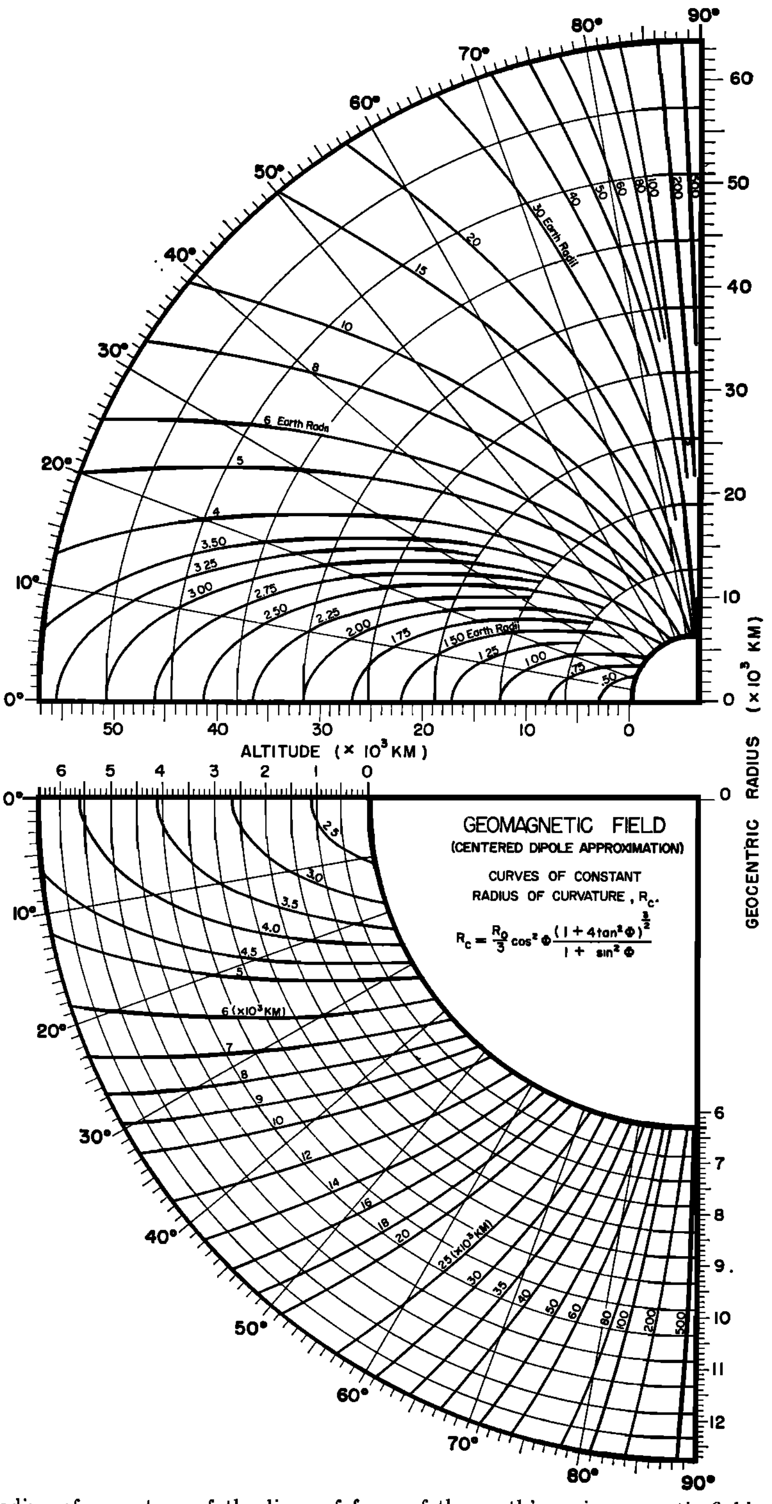

Fig. 7. Radius of curvature of the lines of force of the earth's main magnetic field plotted in geomagnetic coordinates.

dipole field, in gauss per centimeter, is shown in Figure 5. The direction of the gradient is independent of radial distance and is shown in Figure 6 as a plot of the angle between the gradient direction and the radial direction versus latitude. This angle is always in the direction of the equator.

The radius of curvature of the lines of force of the earth's dipole field is shown in Figure 7 in units of earth radii and thousands of kilometers for the far and near coordinate systems, respectively.

Acknowledgments. We are indebted to colleagues for helpful suggestions; to K. K. Shih, W. Nute, and S. Foiles for assistance with calculations; and to J. Lancaster of the Stanford Research Institute for the very careful drafting of Figures 1 and 2.

The work reported in this paper was supported in part by the National Aeronautics and Space Administration under Research Grant NSG 174-61 to Stanford University.

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(Manuscript received March 9, 1962.)