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Propagation Characteristics of Whistlers Trapped in Field-Aligned Columns of Enhanced Ionization

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Abstract. Evidence from whistlers shows that the outer ionosphere contains columns or ducts of enhanced ionization. The theory of propagation in these ducts shows an upper cutoff frequency at one-half the gyrofrequency. The average propagation velocity for whistlers trapped in the ducts can be approximated by assuming that the energy follows along the ionization maximum with wave normals aligned with the magnetic field.

INTRODUCTION

It has been shown that whistler energy may be trapped by field-aligned columns or ducts of enhanced ionization in the outer ionosphere [*Smith, Helliwell, and Yabroff,* 1960]. In this paper we shall discuss the evidence from whistlers which supports the hypothesis that such ducts exist. The theory of propagation in the ducts will be expanded to show the time-delay focusing effects and an upper cutoff frequency for whistler propagation. The results show that, for frequencies below half the minimum gyrofrequency, the propagation velocity may be closely approximated by assuming that the energy travels along the ionization maximum with the wave normals aligned with the magnetic field. This result greatly simplifies the analysis of nose whistlers [*Helliwell, Crary, Pope, and Smith,* 1956]. A cutoff frequency of approximately one-half the minimum gyrofrequency is predicted for whistlers propagating in the ducts.

EVIDENCE FOR DUCTS

The data from whistler observations that indicate the presence of columns of enhanced ionization are listed below. The succeeding section shows that these data cannot be explained if the outer ionosphere is smoothly varying and void of irregularities capable of supporting discrete propagation paths. We will show, however, that the principal features of whistlers can be explained by assuming the existence of ducts of field-aligned ionization.

1. *Pure gliding tones.* Some whistlers sound like pure gliding tones. The spectrograms of such whistlers often reveal very thin traces. The time delay of a whistler from its originating atmospheric is approximately 1 second. Spectrograms may indicate trace widths of the order of 0.01 second at a given frequency.

On spectrographic analysis most whistlers show a number of distinct components whose traces are often very thin: The components usually have a common origin. The collection of components may then be designated a 'whistler group.' In a nose whistler group the nose frequency or frequency of minimum time delay usually decreases systematically with time. Nose whistler components usually exhibit sharply defined traces. Such behavior would not be expected on the basis of the theory of propagation of whistlers in a homogeneous medium [*Smith,* 1960*a*].

2. *Repetition of components.* Different whistlers in the same recording period (2 minutes every hour is the usual sampling schedule) almost always have components that show exactly the same time delay and are similar in general appearance. These components may have varying ratios of intensity from one whistler to the next, but a strong whistler will almost invariably contain all the components of a weaker whistler in the same period. The same components often appear in succeeding hours [*Carpenter,* 1960]. The data thus suggest many isolated and persistent paths of propagation.

3. *Low decrement of echoes.* Whistler echo trains with very little decrease in amplitude from one echo to the next have been observed on occasion. The whistler is apparently confined so that the energy does not diverge in space.

Such a confinement might result from ducts of enhanced ionization.

4. *Multiple echo delays.* The time-delays of certain components of whistlers are, within experimental error, usually related to those of their echoes by exact even or odd integral multiples. The experimental error is often less than 1 per cent. Furthermore, in a given 2-minute period of observation the ratio of time delays of components of short whistlers to those of long whistlers is nearly always 1:2. A component that is found to echo is, however, not necessarily the strongest component of the one-hop, or short, whistler.

5. *Echo coupling.* The only exception to the above rule of multiple echo delays is that coupled echoes, which appear to be combinations of two or more components of the one-hop whistler, are sometimes observed. For example, a short whistler may contain two strong components with time delays of A and B at a given frequency. The three-hop group may contain not only components at $3A$ and $3B$ but also at $2A + B$ and $2B + A$. Examples of such echo combinations were first published by *Morgan, Curtis, and Johnson* [1959]. Other examples have been noted on records from the IGY Whistlers-West program [*Helliwell and Carpenter,* 1961]. Echo coupling strongly suggests the existence of isolated and unique paths of propagation in the outer ionosphere.

6. *Nose whistler upper cutoff frequencies.* For all nose whistlers that have been analyzed, the ratio of the upper cutoff frequency to the estimated minimum gyrofrequency has never exceeded 0.56 [*Smith,* 1960*b*].

7. *Similar whistlers from different source locations. Helliwell, Taylor, and Jean* [1958] found the location of whistler-causing atmospherics by direction-finding and triangulation. The results showed that whistlers at Stanford and Boulder with similar component structure were caused by lightning from the Pacific Ocean, the Gulf of Mexico, and North Dakota. The dispersion properties of particular components does not depend on the location of the source.

A similar result can be deduced from the 'hybrid' whistler [*Helliwell,* 1959], which consists of a short whistler and an associated long whistler, both excited by the same lightning stroke. The lightning source occurs in the same hemisphere as the observing station. When the proper propagation delays are taken into account, the time delays of certain of the components of the long whistler are related by exactly 2:1 to the components of the short whistler.

8. *Whistler occurrence rate as a function of geomagnetic latitude.* The rate of whistler occurrence is known to increase with increasing geomagnetic latitude from the equator to at least 50° [*Helliwell and Carpenter,* 1961; *Crouchley,* 1961]. The occurrence rate is highly variable at higher latitudes and depends in part on the particular observing site.

9. *Absence of leading components in echoes.* It has been noticed in the study of whistler echoes that the expected echoes of the leading components of short whistlers are sometimes absent in the echo structure, although echoes of weaker components showing greater time delay are often present.

SMOOTH OUTER IONOSPHERE MODELS

To explain the above observations we will first consider the possibility that the density in the outer ionosphere decreases smoothly with distance, with no irregularities capable of trapping whistler energy. A number of studies have been made using ray-tracing techniques to determine the paths of whistlers for various initial conditions and different smooth models of the outer ionosphere. Using the results of these studies, let us consider various factors that might account for the fine structure noticed in whistler spectrograms.

Yabroff [1959] has shown for a certain model that over a wide range of initial latitudes the resulting low-frequency whistler tends to focus at one latitude. Furthermore, the energy tends to arrive at the same time for the same variation in initial latitude. However, this type of behavior could not explain multiple components.

One suggested explanation for multiple-component whistlers is that the excitation conditions vary with distance from the source. The intensity of the signal from a vertical radiator is zero directly overhead. A maximum signal intensity would be expected at the lower ionosphere in the neighborhood of 100 km from the source. It is expected that multiple reflections between earth and ionosphere will cause the total whistler-mode signal to fluctuate with distance from the source. However, variation in excitation intensity cannot account for the repetition of components of whistlers that have sources in different places. The existence of hybrid whistlers clearly shows that the variation in source excitation is not the cause of discrete whistler components.

Helliwell, Crary, Pope, and Smith [1956] have suggested, as another possibility, that the whistler fine structure might be related to irregularities in the lower ionosphere that focus the whistler energy. The irregularities may be assumed to exist near the receiving end or the transmitting end of the main whistler path. The existence of a structure of similar effect may be hypothesized from a recent paper by *Budden* [1959]. He suggests that whistler energy will not penetrate the ionosphere in the presence of irregularities of ionization that have dimensions that are small when compared with a wavelength unless the irregularities are elongated in the direction of the magnetic field. We may assume that these elongated irregularities are bunched in localized regions, thus producing 'holes' in the lower ionosphere through which whistler energy may pass.

Since we are assuming for the moment that the outer ionosphere is smooth, we would expect the location of the irregularities in the lower ionosphere to be uncorrelated in the two hemispheres. Then the components of long and short whistlers would be uncorrelated. This is in contradiction to the observed facts.

Calculations made by Yabroff for smooth ionosphere models indicate that the ray path depends on the initial wave normal angle, and that the final wave normal angle will usually be very different from the initial value. For many of Yabroff's cases the final value of the wave normal at the end of the path approached 90° with respect to the magnetic field. The initial value of the wave normal for the first-hop whistler is taken to be approximately vertical. As *Storey* [1953] has shown, the high refractive index of the ionosphere refracts all the waves from below the ionosphere to a small cone around the vertical. The new value of initial wave normal for a second-hop whistler is found by considering the reflection of the final value of the first-hop whistler. This new value will usually not be close to the vertical. Thus the ray paths of the echoes could not be the same as that of the first-hop whistler. Furthermore, coupled echoes would not exhibit the simple relationship described previously.

The work of *Yabroff* [1959] does not indicate any sharp upper-frequency limit for whistlers when the outer ionosphere is smooth. The observed upper-cutoff frequency limit is easily explained by the theory presented below.

Smith [1960*a*], after studying the propagation of whistlers in a homogeneous medium, concludes that isolated nose whistler traces can be explained only if there is present some additional feature of the medium that can focus the energy and restrict the wave normal variation.

DUCTS OF ENHANCED IONIZATION

From the evidence presented above it appears unlikely that the fine structure of whistlers is caused by excitation effects or by irregularities in the lower ionosphere. Some mechanism in the outer ionosphere must, therefore, cause the discrete components in whistlers. A likely mechanism is the presence of field-aligned columns of enhanced ionization which can trap whistler energy. *Smith, Helliwell, and Yabroff* [1960] show that only a small increase of ionization in the column over the background level is sufficient to trap completely all whistler energy within a given range of wave normal angles. The limit of the ray path is then defined by the boundaries of the enhanced column. Furthermore, as we shall show later, the effective group velocity of the energy is almost completely independent of the wave normal angles, ray paths, and trapping conditions within the column. Each column or duct can then easily account for an isolated component of a whistler group. Each component would be observed as a thin trace on the spectrogram.

The explanation for the absence of leading components in echoes may be simply that for the latter components a more favorable trapping angle is encountered upon reflection. The latter components represent whistlers traveling at higher geomagnetic latitudes. The magnetic field lines are more nearly vertical at higher latitudes, and hence the reflected wave normal angle would be expected to be nearly equal to the incident wave normal angle that presumably was within the trapping angle. At low geomagnetic latitudes the reflected wave normal may be far different from the incident wave normal. The reflected wave normal will probably lie outside

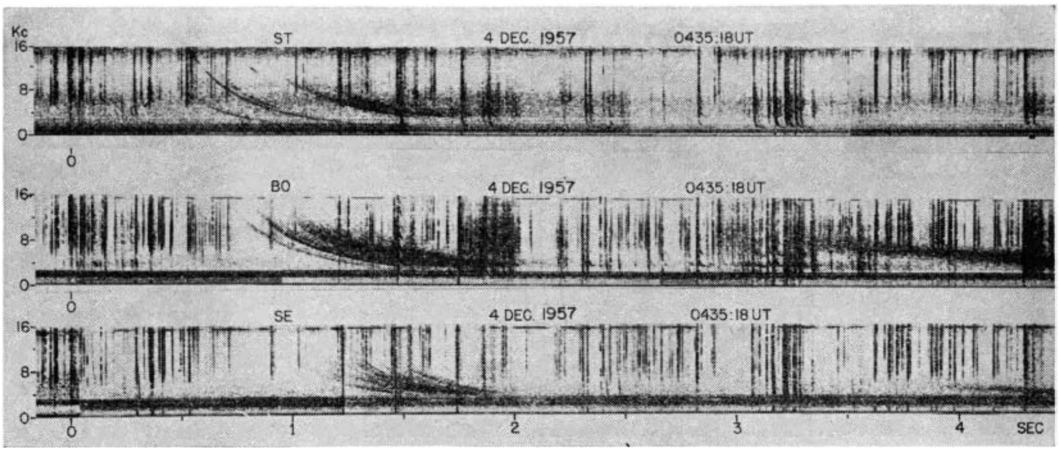

Fig. 1. Coincident whistler at Stanford, Boulder, and Seattle.

the maximum trapping angle. The above results are obtained very simply by the application of the Snell's law construction [*Smith, Helliwell, and Yabroff,* 1960], assuming that the ionization is horizontally stratified at the reflection point.

The delays of components in a whistler group are found to be independent of either the source location or the receiver location. The intensity of the components will, however, be a function of source and receiver location. In general, if a whistler is received simultaneously at two stations, the one at the higher geomagnetic latitude will observe relatively greater intensities in the components having greater time delays. Figure 1 demonstrates this behavior. The short whistlers shown here were received on December 4, 1957, at Stanford, California; Boulder, Colorado; and Seattle, Washington. The geomagnetic latitudes of these stations are 44°, 49°, and 54°, respectively. Examination of the atmospherics will show that the whistler was coincident at these three stations. Note that the components with greater time delay are received with more intensity at the higher geomagnetic latitudes.

The repetition of fine structure of whistlers indicates that the duct structure is relatively stable over periods of a few hours. This duct lifetime of a few hours appears to be consistent with the rate of migration of individual charged particles subjected to collisions in a magnetic field [*Smith,* 1960*b*].

Ducts with high enhancement factors should trap all the whistler energy and thus allow propagation of whistler echo trains with little transmission loss. When the lower ionospheric layers are smooth and relatively lossless, and the lowest is sharply bounded, most of the whistler energy should suffer total internal reflection, except for the energy having wave normals lying within a few degrees of vertical. A roughly analogous situation obtains on a lossless transmission line with very high resistance terminations at either end.

The rule of multiple echo delays follows logically from the duct hypothesis. Echo coupling probably requires that the effective duct paths terminate sufficiently high above the reflection level so that energy leaving one duct and reflecting from the lower layers will have sufficient space to couple to another duct. Recall that the ray direction at low frequencies must lie within a cone of 19°29′ around the magnetic field direction.

The relation of trapping in ducts of enhanced ionization to whistler occurrence rate as a function of latitude has been discussed in a previous paper [*Smith, Helliwell, and Yabroff,* 1960].

VALIDITY OF RAY THEORY

In the following discussion we will assume that ray theory applies for propagation in the ducts. This requires that the dimensions of a duct be large compared to a wavelength. If the dimensions of a duct were less than a wavelength, one might expect a waveguide dispersion effect at low frequencies. Such effects have not been observed. At higher whistler frequencies, ray theory should be applicable simply because of the increased number of wavelengths per unit dimension across the ducts.

EFFECTIVE GROUP-RAY VELOCITY OF A TRAPPED WHISTLER

The influence of the duct on the effective group-ray velocity should be examined. The wave normal angle of whistler energy trapped in a duct is continuously changing as the ray path executes snakelike oscillations from one side of the duct to the other. The group-ray velocity is therefore continuously changing. A given duct is likely to contain a wide variety of initial wave normal angles. Furthermore, a range of electron densities is encountered by the ray on its side-to-side excursions.

If the ray path contains many side-to-side oscillations, the average group velocity along the duct will be nearly equal to the average group velocity of one ray path oscillation. We will consider here only ducts of enhanced ionization. At the center of the duct the wave normal angle has its largest value, and the longitudinal component of group velocity is usually somewhat less than that of an electromagnetic wave traveling in the longitudinal direction. Near the edge of the duct the wave normal angle is close to zero, but, since the energy is now traveling in regions of lower ionization density, the group velocity is somewhat greater than that of a wave traveling down the center of the duct in the longitudinal direction. Thus the average group-ray velocity will be given approximately by the group velocity of a strictly longitudinal wave traveling down the center of the duct.

The true average group velocity can be determined only by integrating the ray-path equations and will be a function of the shape of the ionization distribution as well as the initial wave normal and frequency of the wave. The ray-path equations for an inhomogeneous anisotropic medium are given by *Haselgrove* [1954]. We will consider first the two-dimensional case for reasons previously discussed [*Smith, Helliwell, and Yabroff*, 1960]. Let the constant magnetic field be directed along the x axis, and let the refractive index vary only in the y direction. The ray-path equations can be written

$$\frac{dx}{dt} = \frac{c}{\mu^2}\left(\mu \cos\theta + \frac{\partial\mu}{\partial\theta}\sin\theta\right) \qquad (1)$$

$$\frac{dy}{dt} = \frac{c}{\mu^2}\left(\mu \sin\theta - \frac{\partial\mu}{\partial\theta}\cos\theta\right) \qquad (2)$$

$$\frac{d\theta}{dt} = \frac{1}{\mu^2}\frac{\partial\mu}{\partial y}\cos\theta \qquad (3)$$

$$\frac{dt_G}{dt} = \frac{\mu_G}{\mu} \qquad (4)$$

In the above equations, dx/dt and dy/dt are the components of the ray velocity, c is the velocity of light, θ is the angle between the magnetic field and the wave normal, and t_G is the time of propagation of the energy. The phase refractive index and the group refractive index are given respectively by μ and μ_G.

The appropriate approximation for the phase refractive index for whistlers is

$$\mu = \frac{f_0}{f^{1/2}f_H^{1/2}(\cos\theta - \lambda)^{1/2}} \qquad (5)$$

where

f = wave frequency

f_0 = plasma frequency = $\sqrt{80.6N}$

f_H = gyrofrequency

$\lambda = f/f_H$ = normalized frequency

θ = wave-normal angle

N = electron density (number per cubic meter)

The group refractive index is

$$\mu_G = \frac{f_0\cos\theta}{2f^{1/2}f_H^{1/2}(\cos\theta - \lambda)^{3/2}} \qquad (6)$$

Equations 6 and 7 can be integrated immediately for all values of λ between 0 and 1, giving

$$\frac{f_0(y)}{f_0(0)} = \frac{\cos\theta_0}{(\cos\theta_0 - \lambda)^{1/2}}\,\frac{(\cos\theta - \lambda)^{1/2}}{\cos\theta} \qquad (7)$$

where θ_0 is the wave normal angle at $y = 0$, the ionization maximum. This result could have been predicted by the use of the Snell's law construction. The trapping criteria can be obtained immediately by inserting the values of θ which gave a ray parallel to the magnetic field.

Let us consider a plasma frequency distribution across the duct given by

$$f_0(y) = f_0(0)(\cos y)^{1/2} \qquad (8)$$

If this value of plasma frequency is inserted into the refractive index equation for low frequencies, and the indicated differentiations and substitutions are performed, the average group velocity can be shown to be

$$V_G = \tfrac{1}{2} V_{GL} (\cos \theta_0)^{3/2} \cdot \left[\frac{1}{\cos^2 \theta_0} + \frac{K(\sin \theta_0)}{E(\sin \theta_0)} \right] \qquad (9)$$

where V_{GL} is the longitudinal group velocity at the ionization maximum, and K and E are complete elliptic integrals of the first and second kind respectively.

In this example the difference between the effective group velocity and the longitudinal group velocity is less than 1 per cent for initial wave normal angles less than 54°. The lowest value of electron density encountered by the trapped ray for an initial wave normal angle of 54° is less than 60 per cent of the maximum ionization density. The group-ray velocity for energy traveling in a homogeneous medium with a wave normal angle of 54° differs by 7.5 per cent from the longitudinal value.

Other models of ionization distributions and other frequencies will give somewhat different values for the effective group velocity. In the absence of any knowledge of the ionization distributions in an actual duct, the following model was believed useful for calculation purposes:

$$f_0(y) = f_0(0)[(1 - A) + A \exp(-y^2/B)] \qquad (10)$$

This is a model of ionization with a constant background level and a superimposed duct having a gaussian distribution. Using this model, equations 5 to 8 were integrated on a digital computer. Values of A chosen were 0.05 and 0.25. The values of θ_0 were taken at 5° intervals, until the ray was no longer trapped. Appropriate values of normalized frequency between 0 and 0.49 were chosen. The ratios of the longitudinal group velocity to the effective group velocity for the different parameters are shown in Table 1. Notice that the error in assuming strictly longitudinal propagation is usually less than 1 per cent.

For the examples given the effective group velocity is thus seen to be reasonably approxi-

TABLE 1. Correction Factors for Trapped Whistlers

λ	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
A. 5 Per Cent Plasma Frequency Enhancement													
0.0	1.000	1.000	0.999	0.994	0.973								
0.05	1.000	1.000	1.000	0.997	0.982								
0.10	1.000	1.001	1.001	0.999	0.989								
0.15	1.000	1.001	1.002	1.002	0.995								
0.20	1.000	1.002	1.003	1.005	1.002								
0.25	1.001	1.002	1.005	1.008	1.009	0.999							
0.30	1.001	1.003	1.007	1.012	1.016	1.017	0.990						
0.35	1.001	1.004	1.009	1.016	1.024	1.034	1.044	1.055	1.097				
0.40	1.001	1.005	1.012	1.021	1.034	1.053	1.086						
0.45	1.002	1.006	1.015	1.028	1.054								
0.47	1.002	1.007	1.017										
0.49	1.002	1.008											
B. 25 Per Cent Plasma Frequency Enhancement													
0.0	1.000	1.000	1.000	0.999	0.998	0.995	0.991	0.983	0.968	0.941	0.858		
0.05	1.000	1.000	1.001	1.001	1.001	1.000	0.997	0.992	0.981	0.961	0.914		
0.10	1.000	1.001	1.002	1.003	1.004	1.004	1.004	1.002	0.996	0.984	0.958		
0.15	1.000	1.001	1.003	1.005	1.007	1.010	1.012	1.014	1.014	1.010	1.001	0.977	
0.20	1.000	1.002	1.004	1.007	1.011	1.016	1.022	1.027	1.034	1.041	1.050	1.067	1.123
0.25	1.001	1.003	1.006	1.010	1.016	1.024	1.033	1.044	1.058				
0.30	1.000	1.003	1.014	1.032	1.046	1.064	1.090	1.134					
0.35	1.000	1.004	1.017	1.043	1.063	1.092	1.155						
0.40	1.001	1.005	1.012	1.022	1.036	1.057	1.093						
0.45	1.002	1.006	1.015	1.029	1.055								
0.47	1.002	1.007	1.017										
0.49	1.002	1.008											

mated by assuming that the energy travels along the ionization maximum with the wave normals aligned along the magnetic field lines. Part of the explanation is that a duct of given enhancement inherently excludes wave normal angles exceeding a certain value. In addition, there is an averaging effect on the group ray velocity as previously discussed.

If the cross section of the duct is approximately circular and the ray does not cross the axis of the duct, the trapping properties are determined partly by the gradients of ionization. In the limiting case the ray path follows a helical path along the duct, and the trapping condition depends only on the gradient of ionization. The group ray is traveling in a region of ionization less dense than at the ionization maximum; thus it tends to travel faster. On the other hand, the wave normal is always directed away from the magnetic field, thus tending to make the group ray travel slower. There is again an averaging effect on the group-ray velocity.

VARIATIONS OF TRAPPING CONDITIONS ALONG A DUCT

Without knowing the exact size of the ducts or the variation of density along the ducts we can nevertheless make some further qualitative deductions about the behavior of propagation in the ducts.

We have seen [*Smith, Helliwell, and Yabroff*, 1960] that for frequencies below half the gyrofrequency trapping could occur in both troughs and crests (minima and maxima) of ionization for a uniform magnetic field and no variations of refractive index along the field. Consider the more realistic picture of ducts extending along the earth's dipole field. The gyrofrequency near the entrance to the ducts is approximately 1 Mc/s and is very high compared with even the highest observed whistler frequencies. The initial value of normalized frequency, $\lambda = f/f_H$, must be close to zero. For this condition the minimum required enhancement factor for trapping in a trough is very much larger than that required for trapping in a crest. It is therefore likely that if the whistler energy is trapped, it will be trapped initially in a crest of ionization.

As the whistler travels up the field lines in the duct, the gyrofrequency slowly decreases and hence the normalized frequency increases. The refractive index will vary along the duct because both gyrofrequency and electron density are changing. Furthermore, the enhancement factor may change. One may ask if the energy will remain in the duct. Figure 2, taken from a previous paper [*Smith, Helliwell, and Yabroff*, 1960], is a ray-path region diagram showing lines of constant enhancement factor for a uniform duct and for various values of normalized frequency and θ_0, the wave normal direction when the ray is at the maximum of ionization. For propagation in a crest of given enhancement factor, the cone of possible trapped wave normals increases with frequency. Alternatively, if the maximum wave normal angle is held constant and the frequency is increased, the ray will be restricted closer and closer to the maximum of ionization as the frequency increases. The example given below indicates that the maximum wave normal angle will actually decrease as the gyrofrequency increases. Then it appears that, unless the duct properties change radically, the whistler energy will remain in the duct. We will see, however, that the properties will change abruptly near a normalized frequency of one-half.

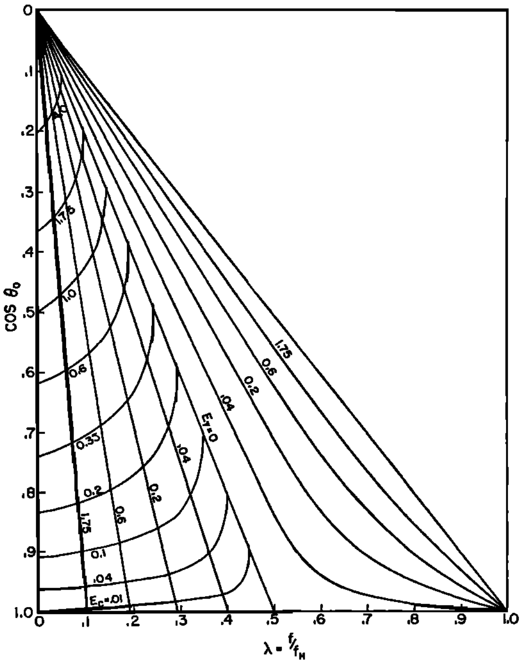

Fig. 2. Enhancement factors on the ray-path region diagram.

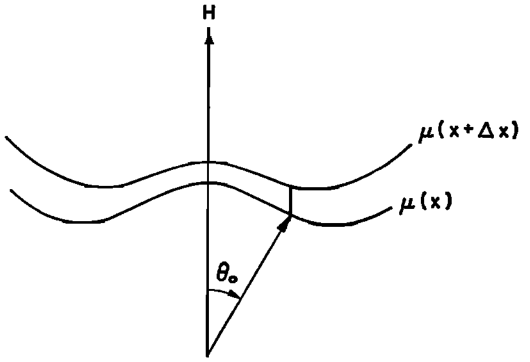

Fig. 3. Refractive index surfaces at two points along a duct.

If the whistler is trapped in a crest, the angle between the ray and the field will decrease for a given wave normal when the gyrofrequency is decreased. The maximum possible angle of the ray also decreases with decreasing gyrofrequency. If the plasma frequency decreases more rapidly than described above, it is still possible for the maximum wave normal angle to decrease with decreasing gyrofrequency. If the plasma frequency is constant, or even if it is proportional to the square root of the gyrofrequency, the refractive index will increase in the direction of decreasing gyrofrequency. This additional component of refractive index gradient tends to reduce the wave normal angle all along the ray path. The integrated effect is then a reduction in the maximum wave normal angle near the center of the column. If the plasma frequency decreases more rapidly than described above, it is still possible for the maximum wave normal angle to decrease with decreasing gyrofrequency, but this can be determined only by integration of the ray path equations.

If the ionization density does not decrease more rapidly than the gyrofrequency, the nature of the refractive index surfaces at two successive points along the duct is as indicated in Figure 3. The small change in gyrofrequency will not change the wave normal when $\theta = 0°$ near the edge of the ducts. The gradient of refractive index in this case will furthermore be nearly perpendicular to the field lines. However, when the ray is at the maximum of the ionization, the gradient of refractive index will be directed entirely along the field lines, neglecting the curvature of the field lines. The wave normal angle will be close to its largest value, θ_0, at this point. The Snell's law construction indicates that θ_0 will decrease. Notice that if the plasma frequency were constant along the duct, θ_0 would decrease even more rapidly. The degree of enhancement necessary for trapping thus will decrease with decreasing gyrofrequency. Unless the actual duct enhancements decrease rapidly, the energy will remain trapped.

UPPER CUTOFF FREQUENCY

Another feature of trapping in ducts is a high-frequency cutoff near a normalized frequency of one-half. Reference to the ray-path region diagram shows that trapping can take place only in troughs of ionization for normalized frequencies above $\frac{1}{2} \cos \theta_0$. The highest frequency for which the energy may be trapped in *crests* is thus one-half the local gyrofrequency. If energy at a frequency greater than one-half the minimum gyrofrequency along a duct is traveling up the duct, then, when a critical value of gyrofrequency is reached, the energy will tend to be refracted away from the maximum of ionization. If the energy escapes from the duct, very little would be expected to return and be trapped. The observed cutoff frequency of nose whistlers is somewhat greater than one-half the minimum gyrofrequency, however. This can be explained by the following considerations. As the energy approaches the critical region, the energy will tend to follow the crest of ionization. Furthermore, the ray will be directed very close to the magnetic field. When the critical gyrofrequency has been passed, the energy will slowly refract away from the ionization maximum. If the energy does not leave the duct before it passes over the top of the path and again reaches the critical value of gyrofrequency, then the energy may remain in the duct.

CONCLUSIONS

The evidence presented here shows that whistler energy is confined over most of its path by field-aligned ducts of enhanced ionization in the outer ionosphere. Furthermore, the average group velocity along the duct is given fairly accurately by the strictly longitudinal approximation; whistler energy following a snakelike path along a duct of enhanced ionization will have the same average velocity as energy directed along the field lines at the crest of ionization.

The theory presented indicates that this approximation may fail at wave frequencies above half the local gyrofrequency because the energy will no longer be trapped in the duct. The equations describing the time delay of nose whistlers as a function of frequency are very much simplified when the longitudinal group velocity can be used. These results can be used in the analysis of nose whistlers to obtain electron densities in the outer ionosphere [*Smith,* 1960*b*].

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