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Extension of Nose Whistler Analysis

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Recent work by *Smith* [1960] on the theory of nose whistlers suggests several methods of obtaining nose frequency and nose time delay information from ordinary whistlers: i.e., whistlers that do not exhibit a 'nose' on the spectrographic records. Since the nose frequency and time delay provide information on whistler path location and on electron density, and since ordinary whistlers occur far more frequently than do nose whistlers, such methods are potentially of great value as tools of research.

The purpose of this letter is to outline one of several possible methods to obtain the nose frequency and the time delay to the nose when the nose is not directly observable. The method is based on the observation that the dispersion characteristics of whistlers are nearly independent of frequency and ionization distribution when normalized with respect to the nose frequency and the low frequency dispersion. This suggests that the dispersion can be written in terms of a universal dispersion function that is a function of f/f_n only:

$$D(f) = tf^{1/2} = D_0 S_1(f/f_n) \quad (1)$$

where

D_0 is the low frequency dispersion.
f is the frequency.
f_n is the nose frequency.
t is the time delay.
S_1 is a universal dispersion function.

Where f_0 and f_H are the local electron plasma and gyro frequencies, respectively, D_0 may be defined as

$$D_0 = \frac{1}{2C} \int_{\text{path}} \frac{f_0}{f_H^{1/2}} \, ds$$

From a spectrographic analysis one can measure two time delays at two well-separated frequencies on one whistler trace. Now, forming two equations in two unknowns, D_0 and f_n, we solve for f_n either graphically or analytically, depending on the form of S_1. From f_n and a known value of t at frequency f, t_n is easily obtained through equation 1.

The basic details of the method follow. Consider an approximation to $D(f)$ suggested by Smith:

$$D(f) = tf^{1/2} = D_0 S_0(f/f_{H_1}) = D_0 \frac{2}{\pi} \frac{E(f/f_{H_1})}{1 - f/f_{H_1}} \quad (2)$$

where f_{H_1} is the minimum gyro frequency along the path of propagation (assumed to coincide with a line of force of the earth's magnetic field), and E is the complete elliptic integral of the second kind with modulus f/f_{H_1}. To obtain D as a function of f/f_n, we set $\partial t/\partial f = 0$ in equation 2 (the nose frequency condition), and then solve for $\Lambda_n = f_n/f_{H_1}$. We then have

$$S_1(f/f_n) = \frac{2}{\pi} \frac{E(\Lambda_n f/f_n)}{1 - \Lambda_n f/f_n}, \quad \Lambda_n = 0.386 \quad (3)$$

We now consider two time delays t_u and t_L at two frequencies f_u and f_L. Figure 1 shows schematically a middle-latitude 'one-hop' whistler with upper and lower frequencies of measurement 8 and 4 kc/s, respectively. (The 30-ms delay from $t = 0$ to the atmospheric represents the approximate time required for the atmospheric to propagate at the speed of light from the causative lightning flash to the whistler receiver.) To eliminate D_0 we form the ratio

$$\frac{D(f_u)}{D(f_L)} = \frac{t_u f_u^{1/2}}{t_L f_L^{1/2}} = \frac{S_1(f_u/f_n)}{S_1(f_L/f_n)} = S_2(f_L/f_u\ ;\ f_u/f_n) \quad (4)$$

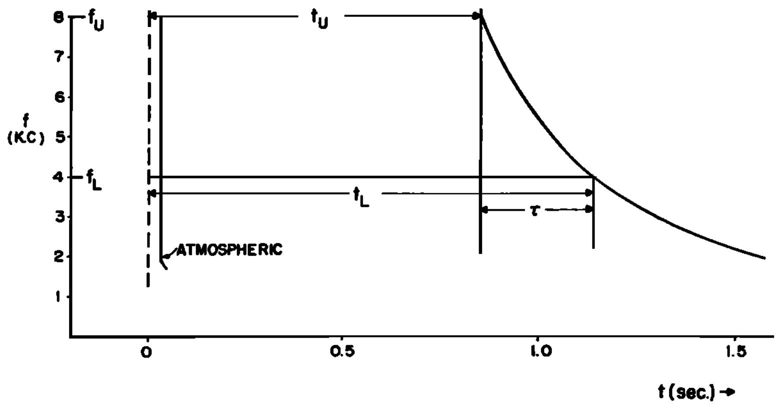

Fig. 1. Schematic illustration of a middle-latitude 'one-hop' whistler, showing measurements for f_n analysis and source identification.

where the subscripts refer to the upper and lower frequencies of measurement and S_2 is a new function of the ratios f_L/f_u and f_u/f_n. Figure 2 shows a plot of S_2 vs. f_u/f_n for several values of the parameter f_L/f_u.

When the measurements t_u and t_L are actually made, a value of S_2 may be calculated and a value of f_u/f_n read from the graph. Since f_u is known, f_n is thus obtained.

To find t_n, we note from (1) and (3) that

$$t_n = \frac{D_0 S_1(1)}{f_n^{1/2}} = \frac{D_0}{f_n^{1/2}} 1.456$$

We then have

$$\frac{t}{t_n} = \frac{0.678}{(f/f_n)^{1/2}} \frac{2}{\pi} \frac{1}{1 - \Lambda_n f/f_n} E(\Lambda_n f/f_n) \qquad (5)$$

which is illustrated in Figure 3. With f_u/f_n known, the appropriate value of t/t_n, and thus t_n, is easily obtained from the graph.

Considerations of theoretical and practical error in this method are complex and will be discussed in detail at a later date. That there are limitations to the method is most obvious from Figure 2, where it is clear that a small

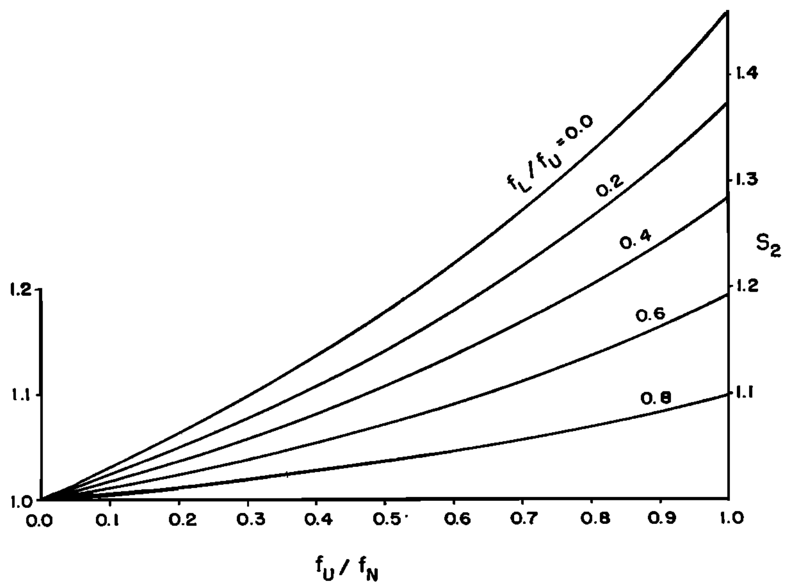

Fig. 2. Graph of S_2 vs. f_u/f_n for various values of the parameter f_L/f_u.

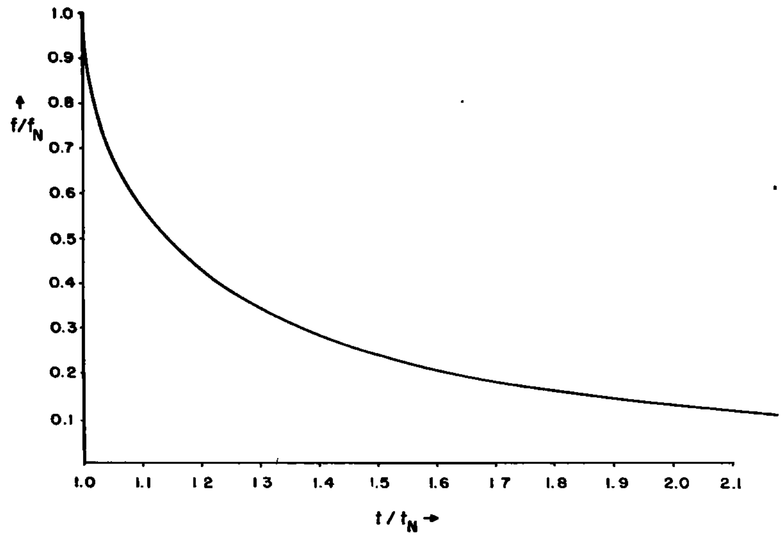

Fig. 3. Graph of f/f_n vs. t/t_n, based on the universal dispersion function S_1.

percentage error in S_2 will cause a very large error in f_u/f_n for low values of f_u/f_n.

Experience shows that it is necessary to have good frequency-time calibration of the spectrographic records, and that the whistler traces should be well defined over the range f_u to f_L. Under such conditions it is possible to obtain statistically reliable values of f_n that range as high as 50 kc.

The best way of checking the effectiveness of the method is to analyze actual nose whistler traces. An example in which three traces are measured is shown in Figure 4. The results for this whistler and for several others are given in

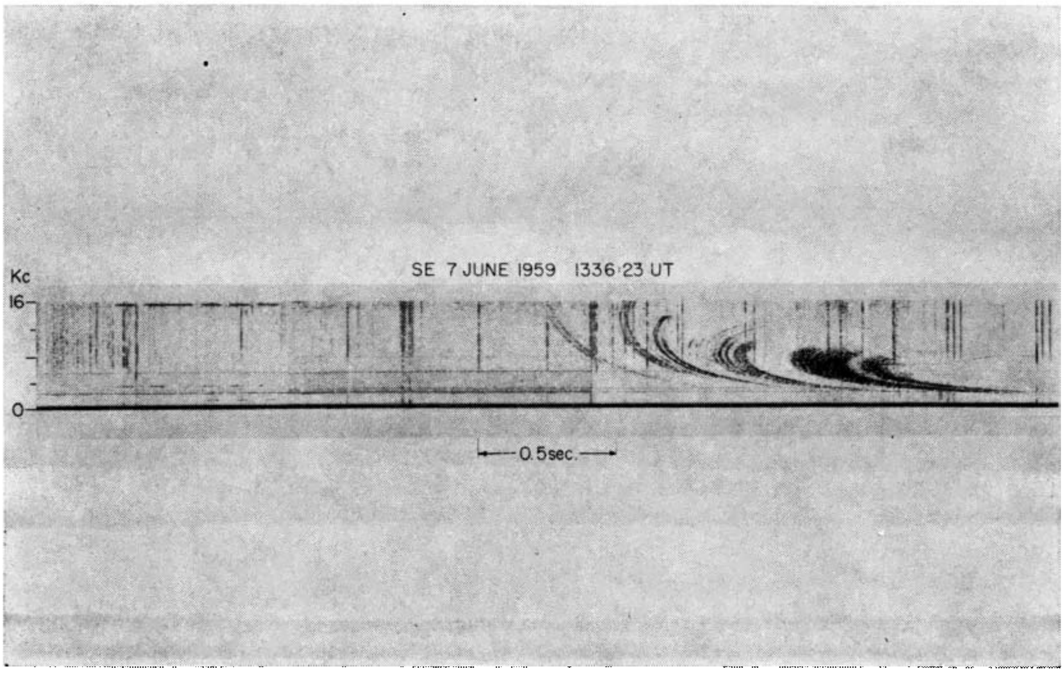

Fig. 4. Two closely spaced nose whistlers recorded at Seattle on June 7, 1959.

TABLE 1. Comparison of Measured and Calculated Values of f_n and t_n of Nose Whistlers

		Nose Frequency, kc		Nose time delay, sec,			
Station	Date	Measured	Calculated	Measured	Calculated	f_u/f_n	f_L/f_u
Seattle	June 7, 1959	13.4 ± 0.3	13.3 ± 1.5*	0.823 ± 0.017	0.821 ± 0.041	0.600	0.6
	1335:83 UT	10.7 ± 0.3	10.9 ± 1.1	0.947 ± 0.019	0.938 ± 0.042	0.640	0.5
		7.5 ± 0.2	8.0 ± 0.9	1.182 ± 0.019	1.16 ± 0.055	0.623	0.6
Seattle	May 21, 1958 1335:57 UT	9.9 ± 0.4	10.6 ± 1.1	1.197 ± 0.023	1.189 ± 0.053	0.660	0.5
Boulder	Feb. 5, 1958 1235:97 UT	12.6 ± 0.5	13.3 ± 1.1	1.204 ± 0.021	1.19 ± 0.042	0.750	0.4
Seattle	Aug. 18, 1959 1235:59 UT	15.0 ± 0.7	15.0 ± 1.0	0.403 ± 0.019	0.402 ± 0.02	0.801	0.5
Unalaska	April 27, 1959	19.3 ± 0.6	19.6 ± 1.4	0.927 ± 0.019	0.927 ± 0.03	0.818	0.5
	0535:15 UT	18.5 ± 1.0	19.1 ± 1.3	0.987 ± 0.021	0.986 ± 0.03	0.787	0.5
		21.0 ± 1.5	20.9 ± 1.2	0.889 ± 0.021	0.889 ± 0.027	0.863	0.722
Stanford	May 17, 1958	31.0 ± 2.0	28.8 ± 9.5	0.512 ± 0.02	0.528 ± 0.098	0.278	0.5
	1435:87 UT	31.0 ± 2.0	30.7 ± 4.5	0.512 ± 0.02	0.514 ± 0.039	0.522	0.5
		31.0 ± 2.0	31.2 ± 1.6	0.512 ± 0.02	0.510 ± 0.02	0.899	0.5
	1435:88 UT	31.0 ± 2.0	32.9 ± 11.5	0.512 ± 0.02	0.504 ± 0.11	0.260	0.5
		31.0 ± 2.0	30.4 ± 4.6	0.512 ± 0.02	0.515 ± 0.041	0.494	0.6

* The errors assigned to the calculated results are not the specific errors associated with the particular whistlers in question, but are 'typical' errors associated with the 'typical' whistler for which f_n, f_u, and f_L are as indicated. In order to determine the feasibility of calculating f_n and t_n in the manner described above, it was necessary to make a preliminary study of measuring error, based on the appearance of typical whistlers in the relatively large group of whistlers exhibiting at least a fairly well defined edge over the range f_u to f_L. The expected errors derived from this preliminary study (and applied to the whistlers listed in the table) must be distinguished from the actual measuring errors associated with any particular whistler. In several cases listed, the actual measuring errors are smaller than those assigned, owing to extremely good definition of the traces. Only 'typical' errors were recorded, so as to present a cautious picture that is suggestive of more general measuring conditions.

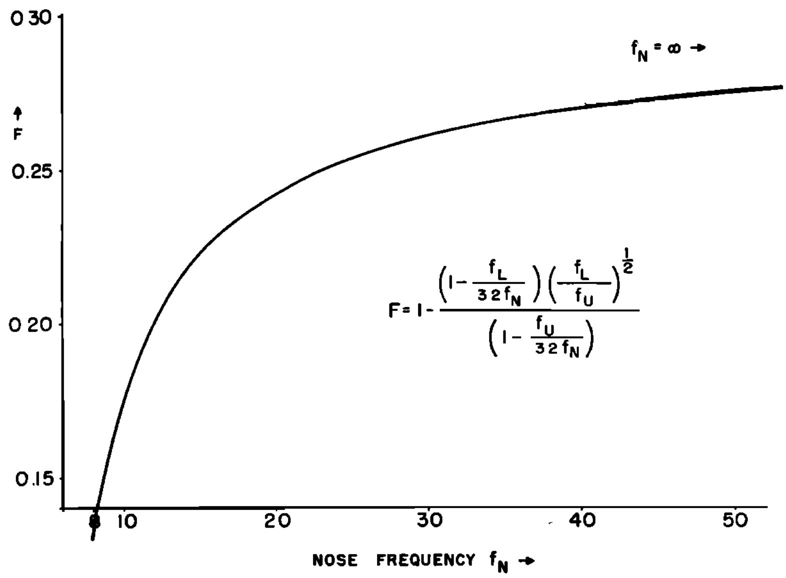

Fig. 5. Graph of F vs. f_n used in whistler source identification (case of $f_L = 4$ kc, $f_u = 8$ kc).

Table 1. In the cases for May 17, 1958, two or more measurements have been made at different frequencies f_u and f_L for a single nose whistler trace.

It is seen that the calculated and measured values agree well, although the former tend to exceed the latter by small percentages in the range near 10 kc/s.

Other methods of analysis may be developed using variations of the scheme outlined above, such as the use of measurements t and $\partial t/\partial f$ at frequency f. Another method depends upon the low frequency behavior of whistlers and requires the measurement of $\partial D/\partial f$ and D at a frequency f such that $f/f_{H_1} \ll 1$.

One serious problem in f_n analysis, and in whistler analysis in general, is the identification of the sources of short (one-hop) whistlers. Whenever possible, the identification should be made by methods that are essentially independent of any particular theory of the frequency-time behavior of whistlers, such as by comparison of whistlers recorded during the same 2-minute period, comparison of events recorded simultaneously in opposite hemispheres, etc. When such 'independent' methods are unsuccessful in a case that is of particular interest, it may be possible to apply the following approach: Instead of the more complicated relation (1), we write, as a reasonable approximation to it,

$$D = tf^{1/2} = D_0 \frac{1}{1 - f/3.2f_n} \tag{6}$$

For two frequencies f_u and f_L, we form

$$\frac{t_u f_u^{1/2}}{t_L f_L^{1/2}} = \frac{1 - f_L/3.2f_n}{1 - f_u/3.2f_n} \tag{7}$$

Then, writing $\tau = t_L - t_u$, we have (Fig. 1)

$$t_L = \tau/F \tag{8}$$

where

$$F = 1 - \frac{(1 - f_L/3.2f_n)(f_L/f_u)^{1/2}}{1 - f_u/3.2f_n} \tag{9}$$

Now, for convenience, we choose $f_u = 8$ kc, $f_L = 4$ kc/s, and obtain a graph of F vs. f_n (see Fig. 5). Then, in a particular case, we guess the value of f_n for the trace, measure τ, read F from the graph, and thus obtain t_L. If our guess is within 25 per cent of the true nose frequency, this calculated value of t_L will be within roughly 6 per cent of true t_L when the true nose frequency is near 30 kc/s. For higher values of f_n, the situation improves, so that the method is particularly useful in the case of middle latitude whistlers.

In the case of actual nose whistler traces, there is a very simple procedure for obtaining approximate source identification. Measure $\tau = t_L - t_n$ and from the graph of Figure 3 read $t/t_n = R$ (here t_L/t_n) corresponding to the known ratio f_L/f_n. Then we have

$$t_n = \frac{\tau}{R - 1}$$

If $f_L/f_n > \frac{1}{2}$, the error in τ tends to be the main source of error in t_n. For smaller f_L/f_n, the error in measuring f_n becomes the important factor. In any event, very careful measurements must be made, and attention must be restricted to traces that are well defined in the range f_L to f_n.

Reference

Smith, R. L., The use of nose whistlers in the study of the outer ionosphere, Ph.D. dissertation, July 11, 1960, *SEL Technical Report 6*, Stanford University.

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