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A New Mechanism for Accelerating Electrons in the Outer Ionosphere[1]

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We propose here to study the phenomena of high-energy trapped particles in the earth's outer ionosphere using a new mechanism for particle acceleration. The phenomenon of gyroresonance would be employed to accelerate electrons in the earth's outer ionosphere with circularly polarized VLF electromagnetic waves generated on the ground. According to the theory of whistling atmospherics [*Storey,* 1953], the ionosphere can propagate a circularly polarized plane electromagnetic wave whose frequency is less than that of the plasma and gyrofrequencies. Large reductions in the phase and group velocities take place, and the ray paths tend to follow the earth's magnetic field.

In the outer ionosphere, where collisions are infrequent, the free electrons travel in spiral orbits, the guiding center of which lies along the direction of the earth's magnetic field [*Alfvén,* 1950]. Any electron which 'sees' a wave frequency of proper phase equal to its natural frequency of rotation (gyrofrequency) about the static magnetic field will tend to be accelerated by the wave along a spiral-like path of ever-increasing cross section. The action is roughly similar to that which takes place in a cylotron, except that the electric field is constant in magnitude and continually changing in direction. The energy absorbed by the electron is determined by the duration of synchronism and the amplitude of the wave.

Since the electrons, in general, have a longitudinal velocity (except at their mirror points), they will see a Doppler-shifted wave frequency. If there is a continuous longitudinal velocity distribution, in any region, there will be particles with a longitudinal velocity (oppositely directed to that of the wave) such that the Doppler-shifted wave frequency equals the electron gyrofrequency. These particles will be selectively accelerated by the wave. Since the number density of such particles is likely to be small, acceleration to high energies is possible without appreciable attenuation of the wave (in the space of a few wavelengths). Thus we can conceive an experiment for the production of high-energy electrons in the ionosphere using electromagnetic waves from a ground-based low-frequency transmitter. An appropriate name for the device with which to perform the experiment might be the 'geocyclotron.'

The essential condition of synchronism between the gyrofrequency f_H of the electron and the apparent frequency of the wave field is determined by the following three factors: (*a*) strength of the local static magnetic field H_0; (*b*) energy of electron, which determines relativistic change in f_H; (*c*) Doppler shift of exciting wave frequency due to longitudinal velocity (in direction of earth's field) of electron.

In general these factors will vary with time in a complicated manner. Thus as an electron spirals down the lines of force its longitudinal component of velocity will decrease because of convergence of the lines of force of the earth's field. At the same time, the gyrofrequency increases because of this convergence. If the electron is gaining energy from the wave, the effect of the increasing field strength of the earth on the gyrofrequency tends to be offset by the relativistic increase in the mass of the electron. To obtain synchronism over an appreciable period of time it is necessary to vary the frequency of the exciting wave in the proper way.

Low-energy particles. It is theoretically possible to accelerate low-energy electrons (1 kev) to energies of the order of 1 Mev utilizing a ground-based transmitter whose frequency de-

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creases with time in a unique manner. There is, however, some evidence that the wave will propagate with low loss only at frequencies less than one-half the local gyrofrequency. Thus it is probably necessary that the Doppler-shifted frequency seen by the particles be at least twice the wave frequency.

For available transmitter locations and powers (~100 kw) the minimum gyrofrequency along the field line is of the order of 10^4 cps and the interaction time is of the order of 1 second. Thus the frequency matching must be of the order of 1 part in 10^4 for good efficiency. Because the resonance is sharp and the required Doppler shift is large, it is evident that the magnitude of the interaction will depend markedly on variations in the longitudinal velocity of the particle. Since the longitudinal velocity is field sensitive, we would have to know the magnetic field strength to much better accuracy than we do now. Thus the acceleration interaction for low-energy electrons most probably will not take place for the length of time necessary to increase the particle energy by 1 Mev. An appreciable increase in transmitter power over the present 100 kw is necessary to produce the desired effect.

There are possibilities for proton acceleration by this mechanism. However, since the protons must have a longitudinal velocity slightly greater than the wave velocity, interaction occurs only with protons of energy of the order of 100 kev or more.

Relativistic case. When electrons are relativistic it is possible to obtain synchronism without recourse to Doppler shift. For example, consider relativistic electrons trapped between mirror points located close to the geomagnetic equator. The longitudinal velocities are then very small, and the electrons travel along essentially circular paths. Because of the relativistic increase in mass, the gyrofrequency is given by

$$f_H = f_{H_0}[1 - v^2/c^2]^{1/2} = f_{H_0} m_0/m \qquad (1)$$

where

f_{H_0} = the gyrofrequency of the electron with rest mass m_0.
v = velocity of electron.
c = velocity of light.
m = relativistic mass.

Now consider a situation in which initially the wave and gyrofrequencies are equal and the phase of the electron velocity vector and the electric field vector is such that energy is transferred to the electron. The electron's mass then increases, causing its gyrofrequency to decrease, and the velocity vector lags behind the electric field vector. Eventually the phase will reverse and the wave will receive energy from the electron, whose effective mass then decreases, causing the gyrofrequency to increase. The phase continues to drop back until the gyrofrequency again equals the wave frequency, at which point the phase lag begins to diminish. After a time the electron is again abstracting energy from the wave, and the process is repeated. This oscillatory behavior continues with no significant increase in electron energy so long as the wave frequency remains constant.

To effect an increase in electron energy the wave frequency must decrease with time. If the rate of mass increase is sufficiently large, the gyrofrequency will tend to drop below the wave frequency. However, the resulting phase shift will reduce the rate of mass increase so that on an average the gyrofrequency equals the wave frequency. If, on the other hand, the wave frequency decreases too fast, the rate of mass increase will not be sufficient to maintain synchronism. This behavior is similar to the principle of phase stability which is the basis of the synchrocyclotron [*McMillan*, 1945]. The condition for phase stability is then

$$f_H \leq f \qquad (2)$$

where f = wave frequency.

We now need f_H as a function of time t and the magnitude of electric intensity E. For low rates of energy absorption per cycle we find, to a good approximation, that

$$f_H = f_{H_0} \frac{1}{\sqrt{1 + (k_i + [Et/k])^2}} \qquad (3)$$

where

$$k = \frac{m_0 C}{|e|} = +1.71 \times 10^{-3} \; \frac{\text{meter-kilogram}}{\text{coulomb-sec}}$$

$$k_i = \sqrt{\left(\frac{f_{H_0}}{f_{H_i}}\right)^2 - 1}$$

f_{H_i} = gyrofrequency at $t = 0$.

The optimum exciting wave will have a frequency variation with time given by (3). As can be seen, this function depends on the electric field strength of the wave. We shall call it the geocyclotron limit.

The increase in energy that can be achieved is limited primarily by the range of frequencies that can be transmitted to the interaction region. The frequency of middle-latitude whistlers is known to range from 500 to 35,000 cps, and it is thought that both limits can be extended appreciably. It is likely, therefore, that the effective mass of the electron can be increased by about two orders of magnitude by this method. This corresponds to an energy of roughly 50 Mev. Such an increase should be easily measured with the aid of satellite counters.

The time required to produce a given energy increase will depend on the field intensity as given in (3). For radiated powers of the order of 100 kw, it is believed that field intensities of the order of 10^{-3} volt/m could be produced at a distance of 1 earth radius from the surface of the earth. Thus an increase of energy from 0.5 Mev to 50 Mev requires, according to (3), a minimum time of about 3 minutes. This is short compared with the expected lifetimes of such particles. We cannot expect, however, to achieve the full energy in one pass of the wave because of the east-west drift velocity imparted by the gradient of the earth's field. The drift period for particles at 1 earth radius from the surface, and mirroring on the equator, is given by *Welch and Whitaker* [1959] as

$$T_D \cong 30/(E\ \text{Mev}) \quad \text{min}$$

The time available for acceleration on one pass depends on the energy of the particle and the fraction of the interaction belt illuminated by the transmitter. For a transmitter at 45° latitude this fraction is about 2 per cent. Thus the time the particle spends within the beam of the transmitter is

$$T_i = 0.02 T_D = 0.6/(E\ \text{Mev}) \quad \text{min}$$

Consequently, during the first pass through the illumination region, 0.5-Mev particles will acquire an energy increment of approximately 5 Mev. During the second and succeding passes, the particles that arrive at the interaction region with the proper phase with respect to the wave will receive an additional energy increment. (There is a possibility that a significant fraction of the particles originally accelerated in the first pass will rapidly acquire the proper phase with respect to the wave during the initial part of each successive pass. Calculations are now in progress to determine whether or not this is the case).

At each pass the amount of energy the particle receives becomes less as the energy increases. In our example the energy of the electron increases approximately at the rate of 18 Mev/min, according to (3). But interaction takes place only 2 per cent of the time because of the drift effect.[2] Thus the 50-Mev energy level would require at least $50/18 \times 1/0.02 =$ 140 minutes, assuming optimum interaction conditions.

We have concluded that large increases in energy can be achieved per particle. Now we must compare the energy we propose to add with that already present to determine whether we can increase particle energies by a measurable amount. The energy density for particles of energy 1 Mev and above can be estimated from Explorer IV and Pioneer III [*Dyce and Nakada*, 1959] results. These lead to an energy of 10 Mev/m^3 or 10 Mev/m^3 $\times$ 1.6 $\times$ 10^{-13} $\times$ joule/Mev $=$ 1.6 $\times$ 10^{-12} joule/m^3. The power density in the electromagnetic wave is estimated at

$$\left(\frac{10^{-3}\ \text{volt/m}}{40\ \text{ohms}}\right)^2 = \frac{1}{4} \times 10^{-7}$$

The length of the path over which this power will be absorbed is not known, since it depends upon the particle paths, which are not known. A reasonable estimate might be 5000 km, giving a power flow per unit volume of

$$\frac{10^{-7}}{4 \times 5000 \times 10^{+3}} = \frac{1}{2} \times 10^{-14}\ \frac{\text{watt}}{m^3}$$

reduced by the fractional illumination ratio of 0.02 giving up to 10^{-16} watt/m^3. The total energy which can be added depends upon the lifetimes of the accelerated particles, which are not pre-

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[2] Since the gyroradius is about 50 km at 50 Mev, there is little chance for the electron to move out of the transmitter beam except in the drift direction.

cisely known. From present information on the Van Allen radiation we estimate the lifetime to be about 6 days. The total energy density is then 10^{-16} watt/m^3 × 60 × 60 × 24 × 6 seconds, or about $\frac{1}{2} \times 10^{-10}$ joule/m^3.

Thus our man-made experiment might increase the number density of high-energy electrons by more than an order of magnitude! Let us compare the total energy in the form of high-energy electrons introduced by a VLF transmitter over a 6-day period with that injected by a 2-kt A bomb in the Argus experiment. Assuming that 1 per cent of the radiated power is absorbed in accelerating electrons, the energy from the VLF transmitter is about 1000 watts × 6 days $\cong \frac{1}{2} \times 10^9$ joules, whereas that from the A bomb was 10^8 joules [*Christofilos*, 1959].

If our assumptions are reasonably near the truth, we should be table to increase significantly the number of high-energy electrons in the outer ionosphere. Therefore, we can study the efficiency of acceleration, the lifetimes of trapped particles, the existence and effects of ring currents (such as production of magnetic storms), the shape of the earth's magnetic field, the distributions of mirror points, and various related questions. A circularly polarized transmitter beamed vertically could be used. Arrays of such transmitters spaced along a line of geomagnetic latitude and properly phased would be useful. Total power output would be at least 100 kw.

We can also use this experiment to test the duct theory of whistler propagation. This theory says that whistler energy tends to be confined to enhanced columns of ionization [*Smith, Helliwell, and Yabroff*, 1960]. If this is true, the electromagnetic wave energy would be concentrated in the ducts and hence would increase the density of high-energy particles in the ducts relative to surrounding regions. The result would be a system of rings of high-energy particles around the earth separated by the duct spacing.

A further application of the geocyclotron theory provides an explanation of the production of high-energy particles in the Van Allen belts. Whistlers passing through the outer ionosphere produce a variation of frequency with time somewhat similar in form to that required in the geocyclotron experiment. It may be that absorption of whistler energy accounts for a substantial fraction of the observed high-energy particles.

The principle of the geocyclotron could be adapted readily to use in a satellite or probe. The vehicle would be provided with a suitable electron gun, and the accelerating electric field would be created by appropriate antennas. Such a device would have the advantage of creating a strong local field that could produce relativistic particles in relatively few turns and hence reduce the phase-stability requirement discussed previously.

In summary, the proposed geocyclotron experiment appears to provide a practical means for trapped-particle experimentation without the hazards and difficulties encountered in using nuclear detonations.

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